UNIVERSITY OF INDONESIA

MODELING OF BLOOD CENTRIFUGATION AND COMPONENT SEPARATION

THESIS

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FACULTY OF ENGINEERING
DEPARTMENT OF CHEMICAL ENGINEERING
DEPOK
DECEMBER 2008

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THESIS
Completed as a requirement to achive a Bachelor degree of Engineering

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STATEMENT OF AUTHORSHIP

With this I stated that this thesis is original and is not a duplicate from published thesis, and every literature cited and used has been stated properly.

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ABSTRACT

Name : Priskila Hanata

Course : Chemical Engineering International Program

Title : Modeling of Blood Centrifugation and Component Separation

Blood salvage system, using the method of centrifugal sedimentation, is a highly complex process currently designed primarily empirical and based on trial-and-error. The aim of this project is to study the flow behavior of blood and develop approaches to its modeling and numerical simulation when it is subjected to a strong centrifugal field. When the blood is centrifugated at 1500 rpm, the sedimentation occur almost instantaneously in a narrow region near the chamber inlet (26% of the chamber volume). The blood then separates into three phases, which are plasma, buffy coat, and red blood cells. Three different set of equations are developed to describe the flow of each phase through the chamber. These equations were used to develop a numerical solution of the fundamental model using the excel software tools.

Keywords :
Blood, Centrifugation, Sedimentation, Flow behavior.

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NOMENCLATURE

τ_o	Bingham model coefficient for red blood cells	Pa
ΔP	Pressure drop across the chamber	Pa
τ_{xz}	Shear stress	Pa
$\mu^{(P)}$	Viscosity of plasma	Pa.s
μ_{37}	Viscosity of plasma at 37 ^{O}C	Pa.s
A	Cross sectional area of a settling chamber	m^2
a	Acceleration due to centrifugal force	m^2/s
C_D	Dimensionless drag coefficient	
D_p	Particle diameter	μm
F_b	Buoyant force	N
F_c	Centrifugal force	N
F_d	Drag force acting in an opposite direction to the particle motion	N
F_g	Gravity forces	N
g	Gravitational acceleration	m^2/s
H	Height at plasma-wall interface	mm
h_-	Height when $\tau_{xz} = -\tau o$	mm
h_+	Height when $\tau_{xz} = \tau o$	mm
h_{BC}	Height of the buffy coat layer	mm
h_{BCR}	Height at buffy coat-red blood cells interface	mm
h_o	Height at V_z maximum	μm

h_{PBC}	Height at plasma-buffy coat interface	mm
h_T	Total height of the chamber	mm
L	Length of the chamber	m
m	Mass of the falling particle	kg
$m^{(B)}$	Power law coefficient for buffy coat	$Pa.s^n$
$m^{(R)}$	Power law coefficient for red blood cells	$Pa.s^n$
$n^{(B)}$	Dimensionless power coefficient for buffy coat	
$n^{(R)}$	Dimensionless power coefficient for red blood cells	
Q_c	Flow rates of buffy coat	mL/min
Q_{in}	Flow rates of whole blood	mL/min
Q_o	Flow rates of plasma	mL/min
Q_R	Flow rates of red blood cells	mL/min
r	Radial distance from the center of rotation	m
t	Residence time in the separation chamber	min
T	Temperature of the separation process	^OC
t_T	Time of settling	min
v	Velocity of the settling particle	m/s
V_T	Total volume of the chamber	mL
v_t	Settling velocity in the radial direction	m/s
V_z	Velocity	m/s
x	Height of the chamber	m
δ	Smallest radius at which the sedimentation of a particle may travel	m
η	Temperature coefficient, taken as 0.021	$^OC^{-1}$

ρ	Plasma density	kg/m^3
ρ_p	Particle density	kg/m^3
ω	Angular velocity	rad/s

CHAPTER 1
INTRODUCTION

1.1 BACKGROUND

Blood is a life sustaining fluid that has a very complex rheological behavior. The volume of blood inside a body has to be maintained, because the shortage of blood or its component could lead to a serious damage.

Due to safety hazard, the use of third person blood has shown a continuous decline in the past few years [1]. Today, the blood needed is supplied by a blood salvage system where the blood is collected from the patient continuously and returned to the patient body. This process also used in the apheresis process.

The purpose of therapeutic apheresis is to remove a component of blood, which cause disease, and replace it with healthy cells from donor [2]. Apheresis process using *"on-line"* blood therapy instruments begins with removal of blood continuously from the patient/donor into a centrifuge device. The blood is centrifugated to separate the cells into its component in a matter of seconds before it treated, re-mixed, and returned to the patient body.

The development of these devices has been played a significant role for treated a blood-related disease such as leukemia. The example is when granulocyte infections occur at the person who has leukemia. Granulocyte replacement therapy is used to reverse the course of infection; and so the development of the device helps to obtain a higher yield of leucocytes [3].

The method used in blood salvage systems is centrifugal sedimentation. Since blood is supplied continuously from the patient, thus the design of the centrifuge is continuous as well. One of the companies that are a frontrunner in the blood bank technology industry is Gambro BCT. The company develops the technology of blood collection and processing, as well as the instrument for therapeutic apheresis.

The continuous process of the blood salvage system is a highly complex process depending on many parameters. Up to now, the design of the centrifuge has been

primarily empirical and based on trial-and-error. For example, in Gambro BCT, the control of the blood interfaces for their instruments is manually adjusted. The outlet streams have to be monitored all the time by an operator. The method is to observe the change of colour in the outlet streams and adjust the inlet flow rates. This is non-effective and expensive procedure, which leads to non-optimal results. In order to move in the direction of more efficient design, it is necessary to simulate the flow of blood and its separation into components in a centrifugal field. This project presents an analytical investigation for continuous flow centrifuge. The flow behavior of blood was discussed, as well as modeling the numerical simulation when it is subjected to a strong centrifugal field (approx. 200 G's). The result of this project provide a basic to the development of blood therapy instruments that provide more efficient, better treatment for patients.

1.2 AIMS OF THE PROJECT

The aims of this project are as follow:

A. To study the blood composition
B. To review an existing centrifugal device
C. To study sedimentation and centrifugal separation
D. To determined the terminal settling velocity and residence time, in order to ensure the sedimentation process occurs in almost instantaneously
E. To developed the fundamental equations that describe blood flow in a typical separation chamber
F. To perform an attempt at numerical solution of the fundamental model using the excel software tools
G. To formulate the modelling result into a velocity distribution graph showing important parameters

1.3 SCOPE OF THE PROJECT

The report was divided into four main points: nature of blood, sedimentation, blood flow behavior after the separation, and flow simulation inside the chamber. This project was focused on the blood flow behavior after the separation and the flow simulation inside the chamber. The other research subject such as nature of

blood and sedimentation was studied by the other student, Melissa. The results from Melissa were used as input data in this project calculation.

There are several assumptions made to simplify the project, it is assumed that sedimentation occurs almost instantaneously thus no flow involved. The project also only consider two dimension, that is sedimentation acting downward and flow of blood in axial direction. For the flow behavior, the blood is assumed has fully separated into components: plasma, buffy coat, and red blood cell. The three different components having different types of flow are also discussed further. Computational modeling using Microsoft Excel is then used to optimize the techniques. The modeling used in centrifugal sedimentation and separation can enhance performance, reduce time, and savings in equipment and energy costs. Noted that due to very expensive, time consuming and safety reasons, experimental work was not required during the project.

1.4 REPORT OUTLINE

Chapter one discuss the background, aims, scope, and outline of this project. Chapter two of this report provides detailed information on the Background Theory on the Nature of human blood, Flow Centrifugal Device and Sedimentation. It includes the critical functions of blood, blood composition and its characteristics. The existing continuous flow centrifugal device is also explained by the aid of a schematic diagram. The theory of Sedimentation process, centrifugal separation and the application to blood separation is described. A calculation to find the terminal settling velocity and residence time was also performed. Chapter three discusses the methodology of the project, it will includes the flow behavior of blood and the development of fundamental equations to describe blood flow in a typical separation chamber. Besides, the modeling development are also shown in chapter three. Chapter 4 will present the results of the modeling and the discussion on its importance. The report concludes with Conclusion and limitations of the project as well as suggestions for future work.

CHAPTER 2
BACKGROUND THEORY

2.1 NATURE OF HUMAN BLOOD

2.1.1 Blood Critical Functions

Blood is a complex fluid with fascinating rheological behavior.

There are four critical functions of blood:

1. Delivers oxygen, hormones, nutrients and picks up the waste products from the cells
2. Prevents blood loss by healing wounds
3. Primary carrier of immunity
4. Helps to control body temperature

2.1.2 Blood Composition

2.1.2.1 Plasma

Plasma is the liquid medium of blood in which blood cells are suspended. The main function of plasma is transportation. Plasma contains of 90% water and 10% solutes. The solutes consist of organic substances and mineral substances. The organic substances include glucose, lipids, proteins (globulins, albumins, and fibrinogen), glycoproteins, hormones (gonadothropins, erythropoietin, thrombopoietin), amino acids and vitamins. The mineral substances are dissolved in ionic form or electrolytes.

2.1.2.2 Buffy Coat

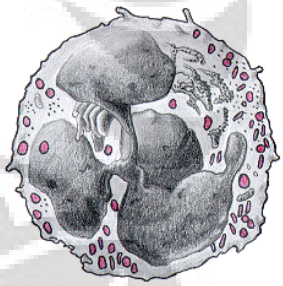
Buffy coat contains of white blood cells and platelets.

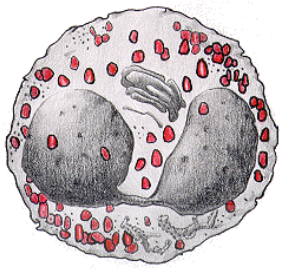
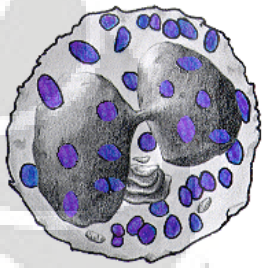
- White blood cells

  White blood cells or leukocytes play a very important role in the immune system. Leukocytes have various types of cells that vary in size, shape and function. As those cells circulate throughout the body, they guard all other cells and tissues against foreign organisms and matter.

  Granulocytes, monocytes, and lymphocytes are the three main groups of white blood cells. Three types comprise granulocytes: neutrophils, eosinophils, and basophils. The properties are as in table 1 below.

Table 2.1. White blood cells types and specifications [4]

Cells types	**Percentage in leukocytes**	**Diameter Sizes(μm)**	**Shapes**
Granulocytes: 1. Neutrophils	50 – 70	10 – 15	+ 

2. Eosinophils	2 – 4	12 – 17	+ 
3. Basophils	0.5 – 1	5 – 7	+ 
Monocytes	3 – 8	12 – 20	+ 
Lymphocytes	20 – 40	5 – 12	+ 

- Platelets

  Platelets or thrombocytes are important for proper blood clotting in order to prevent blood loss when wounds take place. Each cubic millimeter of blood should contain 250,000 to 500,000 platelets. If the number is too high, spontaneous clotting may occur. If the number is too low, clotting may not occur at all. Besides that, platelets are also involved in wound healing and inflammation. Platelets are small, colorless, enucleated bodies ranging from 2 to 4 μm in size.

2.1.2.3 Red Blood Cells

Red blood cells or erythrocytes are responsible for providing oxygen to the tissues and partly delivering carbon dioxide as a waste from the tissues to the lungs. Red blood cells are flexible biconcave disks, giving maximum surface area needed for the transfer of oxygen and carbon dioxide. Red blood cells have diameter of ~7 μm and thickness of ~2 μm.

One interesting phenomenon is that red blood cells do not contain a nucleus once they are in the circulation. There are three reasons why that happens: a nucleus (1) would decrease the quantity of space available for oxygen and carbon dioxide; (2) would add significantly to blood's weight and increase the workload of the heart; and (3) is not required, since red cells are fully differentiated and can transport oxygen and carbon dioxide without a nucleus.

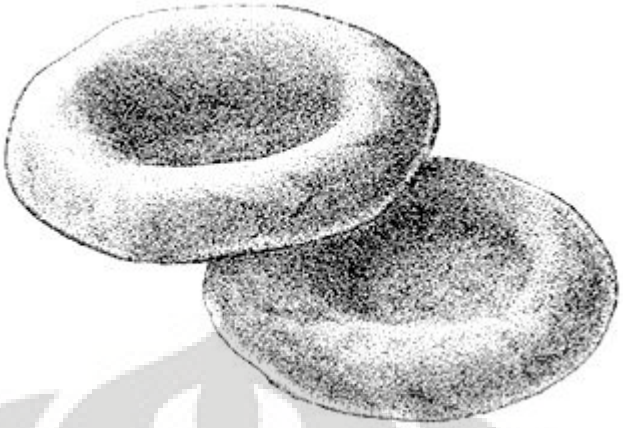


Figure 2.1. Red blood cells (erythrocyte)

[+] *Figure taken from www.funsci.com*

In summary, the appearance of the different blood cells is shown below,

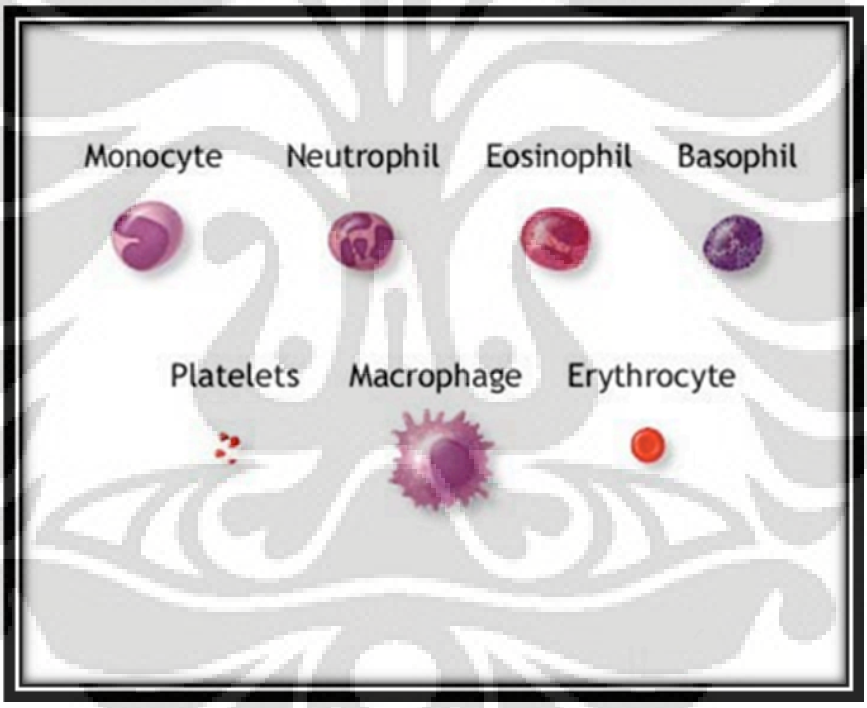


Figure 2.2. Shape of various blood cells (Source: www.adam.com)

2.2 CENTRIFUGAL DEVICE

A schematic diagram of the continuous flow centrifuge is shown in Figure 2.3 below. The inlet is whole blood from the patient/donor body and is located at the bottom of the rotating device. The blood enters the chamber in which the separation into plasma, buffy coat, and red blood cells takes place. Three outlets were located at the top of the chamber in different radial positions to remove separated components. Peristaltic pumps are used to remove the plasma and white cells phases. The position of the interfaces is controlled by varying the flow rates into and out of the device. The residence time has to be large enough to allow complete separation to occur.

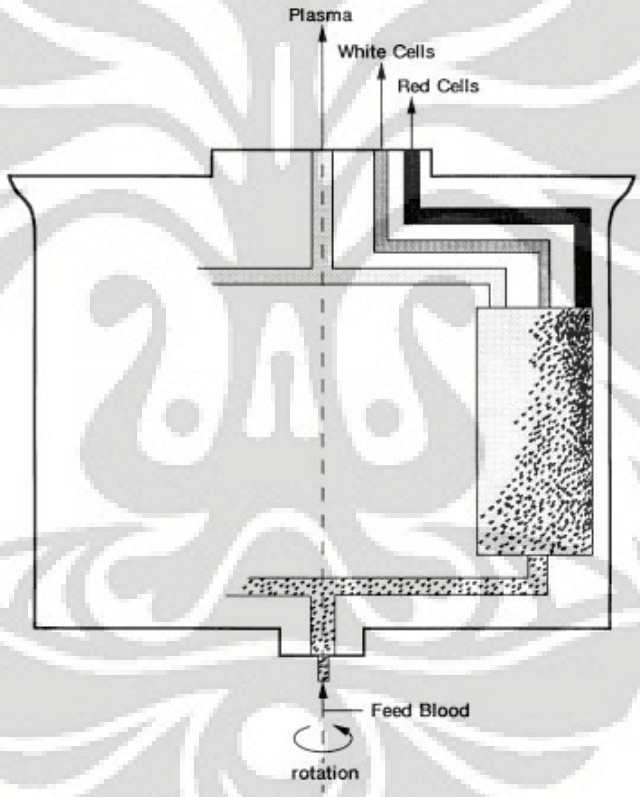


Figure 2.3. Continuous flow centrifuge design (Source: Zydney, A.L., 2000)

2.3 SEDIMENTATION

2.3.1 Sedimentation Theory

Sedimentation, also known as *settling*, is the removal of a particle in a suspension by settling under gravity. The purpose of sedimentation is to remove the particles from a fluid so the fluid is free from the particle

contaminants. The fluid separates into clear fluid and slurry of higher solid content. The particles are suspended in the fluid and are separated according to their size and density.

Free settling or *unhindered settling* occurs when a particle is at a distance from the walls and other particles, so that its movement is not interfered by others. On the other hand, when the concentration of the particles increases they will settle at a lower rate. The particles will be close enough together that they can no longer settle independently. This process is called *hindered settling*.

For a particle moving in a fluid, there are three forces acting on the body: gravity acting downward (F_g), buoyant force acting upward (F_b), and drag force acting in an opposite direction to the particle motion (F_d). The diagram of settling for a spherical particle is as shown below (Fig.2.4).

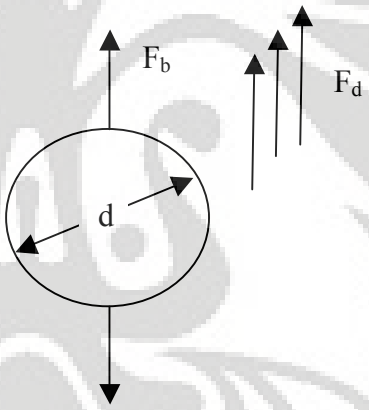


Figure 2.4. Particle settling diagram

For particle settling at velocity v [m/s], the buoyant force F_b acting in upward direction is

$$F_b = \frac{m\rho g}{\rho_p} = V_p \rho g \qquad [2.1]$$

The gravitational force F_g is

$$F_g = mg \qquad [2.2]$$

Where m [kg] is the mass of the falling particle, g [m^2/s] is the gravitational acceleration, ρ_p [kg/m^3] and ρ [kg/m^3] are the density of the solid particle and density of the fluid respectively.

The drag force F_d, also known as frictional resistance, on a particle is given as

$$F_d = C_D \frac{v^2}{2} \rho A \qquad [2.3]$$

Where A [m^2] is the area of the particle, and C_D is the drag coefficient and is dimensionless.

Adding the resultant forces together for the moving particles:

$$m\frac{dv}{dt} = F_g - F_b - F_d \qquad [2.4]$$

Substituting Eq. 2.1, Eq. 2.2, and Eq. 2.3,

$$m\frac{dv}{dt} = mg - \frac{m\rho g}{\rho_p} - \frac{C_D v^2 \rho A}{2} \qquad [2.5]$$

Integrating the equation with $dv/dt = 0$, and solving for the terminal velocity v_t,

$$v_t = \sqrt{\frac{2g(\rho_p - \rho)m}{A\rho_p C_D \rho}} \qquad [2.6]$$

For spherical particles, $m = \pi D_p^3 \rho_p / 6$ and $A = \pi D_p^2 / 4$. Substituting m and A to Eq. 2.6, v_t becomes,

$$v_t = \sqrt{\frac{4(\rho_p - \rho)g D_p}{3 C_D \rho}} \qquad [2.7]$$

The drag coefficient C_D for rigid sphere is a function of the Reynolds number. For particles in the laminar-flow region (Stokes' law region), the drag coefficient is

$$C_D = \frac{24}{D_p v \rho / \mu} \qquad [2.8]$$

where μ [Pa.s] is the viscosity of the fluid. Substituting this into Eq. 2.6, the equation of the terminal velocity for a rigid sphere is

$$v_t = \frac{g D_p^2 (\rho_p - \rho)}{18\,\mu} \qquad [2.9]$$

2.3.2 Centrifugal Separation

Centrifugation acts based on the principle that an object moving in a circle at a steady angular velocity is acted on by an outward force. The particles are subjected to centrifugal forces which make them move radially through the liquid in outward or inward direction, depending on whether they are heavier or lighter than the liquid. The magnitude of this force depends on is the angular velocity ω [rad/s] and the radial distance from the center of rotation r [m]. The acceleration a [m/s^2] due to the centrifugal force is

$$a = \omega^2 r \qquad [2.10]$$

and so the centrifugal force F_c [N] acting on the particle is

$$F_c = mr\omega^2 \qquad [2.11]$$

For centrifugal sedimentation, the gravitational field in Eq. 2.9 is replaced by centrifugal field. Now v_t is the settling velocity in the radial direction. The equation becomes

$$v_t = \frac{\omega^2 r D_p^2 (\rho_p - \rho)}{18\,\mu} \qquad [2.12]$$

Since $v_t = dr/dt$, and integrating between the limit $r = r_1$ at $t = 0$ and $r = r_2$ at $t = t_T$, the time of settling t_T is

$$t_T = \frac{18\,\mu}{\omega^2 (\rho_p - \rho) D_p^2} \ln\left(\frac{r_2}{r_1}\right) \qquad [2.13]$$

Where r_2 can also be written as $r + \delta$, for δ determines the smallest radius at which the sedimentation of a particle may travel. The equation can also be written as

$$t_T = \frac{18\,\mu}{\omega^2 (\rho_p - \rho) D_p^2} \ln\left(\frac{r+\delta}{r}\right) \qquad [2.14]$$

In order to obtain the separation in a continuous flow centrifuge, the residence time in the separation chamber must be large enough to allow the red cells to move to the outer region of the device. The residence time in the separation chamber is inversely proportional to the blood flow rate Q_{in},

$$t = \frac{Ah_T}{Q_{in}} \qquad [2.15]$$

Where h_T [m] and A [m^2] are the length and cross-sectional area of the chamber, respectively.

2.3.3 Application To Blood Separation

In the case of blood, the suspended particle is the blood cells and the fluid is the plasma solution. A method for determining the sedimentation rate of erythrocytes (ESR) under ambient gravity conditions by centrifugation was investigated by Wardlaw, 2001, and is described in U.S. Pat. No. 6,204,066. The method used by Wardlaw to determine the erythrocyte sedimentation rate, is to take the erythrocyte layer/plasma interface position readings at known time intervals during centrifugation of the blood sample in the tube. The results were plotted to get the settling curve of blood sedimentation (Fig. 2.5).

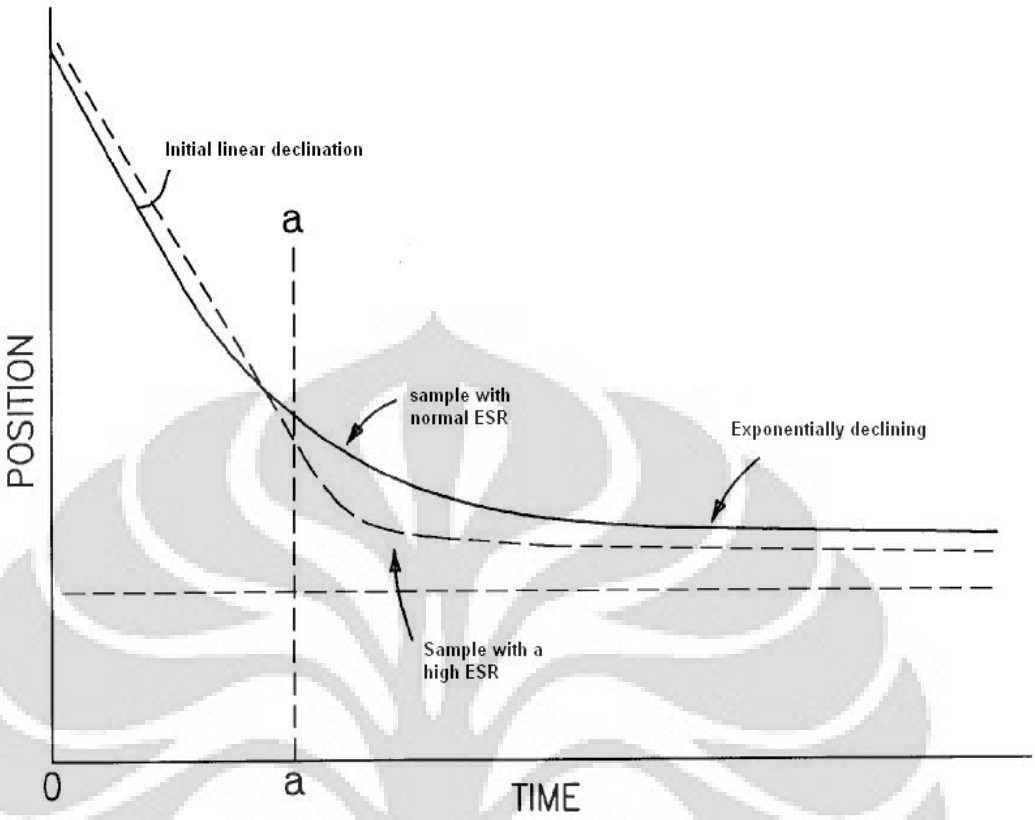


Figure 2.5. Plot of various positions of the plasma/erythrocyte interface in the blood sample during centrifugation (Source: Wardlaw, 2001)

Figure 2.5 shows a linear declination initially followed by exponentially declining slope. The linear declination is the region where free settling takes place. The erythrocyte particles are in low concentration thus settle freely without interference from adjacent particles. The settling velocity is described in Eq. 2.12 for Stokes' Law. Only after centrifugated at a given time, the concentration of particles in suspension in the bottom of the chamber is increased. The distance between particles is small (erythrocyte layer compaction), so the settling velocity would be less than would be calculated from Stokes' Law. This is where the slope is exponentially declining, and also known as hindered settling. The above studies shown that for the sample of whole blood centrifugated at 4000 rpm (1 G), it took 5 minutes for a full settling. In this research however, the same method is used, but the centrifuge is operated at much higher rotational speed.

To determine the settling velocity and time to settling at higher speed, the blood properties used are listed below,

Table 2.2. Properties of various blood cells

Properties	Value	Source
Plasma viscosity, μ	1.7×10^{-3} Pa.s	Gruttola, S.D., et al, 2005
Plasma density, ρ	1025 kg/m^3	Benson, K., 1999
Particle density, ρ_p	1125 kg/m^3	Benson, K., 1999
Particle diameter, d_p	7 μm	Whitmore, 1968

The centrifuge is assumed to be operated at rotational speed of 1500 rpm in order to obtain a full separation between the red cells and buffy coat. This is done to avoid the formation of highly viscous region of cells at the outer edge of the chamber, and to minimize excessive heating around the rotating seals (Rock, 1983). The blood flow rate is estimated to be 75 mL/min, with chamber size given in Appendix A.

By using Eq. 2.12 and Eq. 2.13, it was found that the terminal settling velocity is equals to **7.9×10^{-4} m/s**, and the time of settling is **2.1 seconds**.

The residence time in the separation chamber for the moving fluid was calculated using Eq. 2.15, and is equal to **8 seconds**.

Comparing the residence time and time of settling, the sedimentation occurs in a narrow region near the chamber inlet (26% of the chamber volume) before it separates into three components, plasma, buffy coat, and red blood cell. Thus the sedimentation happens at almost instantaneously as the blood flows into the chamber. Flow behavior of the three different phases is discussed in the next section.

Detailed calculation are attached on Appendix B.

CHAPTER 3
METHODOLOGY

Blood is a non-Newtonian fluid; therefore, the generalized rheological models are used to describe the flow systems. One particular model has been selected to express the blood flow behaviour. That is the Ostwald de Waele model also known as the power law model. An assumption was made that the flow is laminar, and it is proven with the calculation of Reynolds number equal to 48.3625.

The flow of blood in the centrifugation chamber is divided into three phases: which are plasma, buffy coat, and red blood cells. Plasma contains 90% water and so behaves as a Newtonian fluid. Buffy coat contains a high concentration of white blood cells and platelets. It should be described as non-Newtonian fluid. The red blood cells phase is extremely dense slurry that is described as a Bingham fluid with power law.

3.1 GEOMETRICAL CONFIGURATION

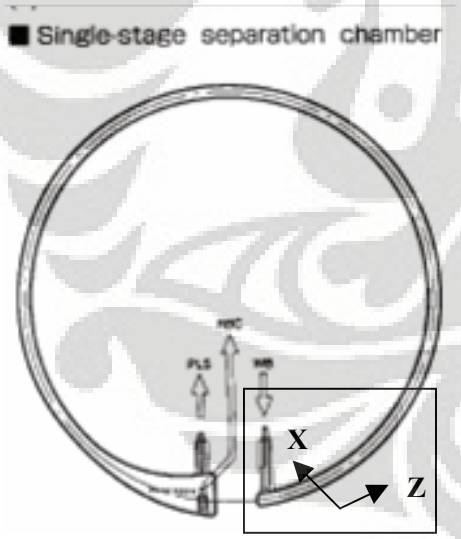


Figure 3.1 The geometrical configuration of the system

Figure 3.2 below describes how the flow system inside the centrifugation chamber looks like.

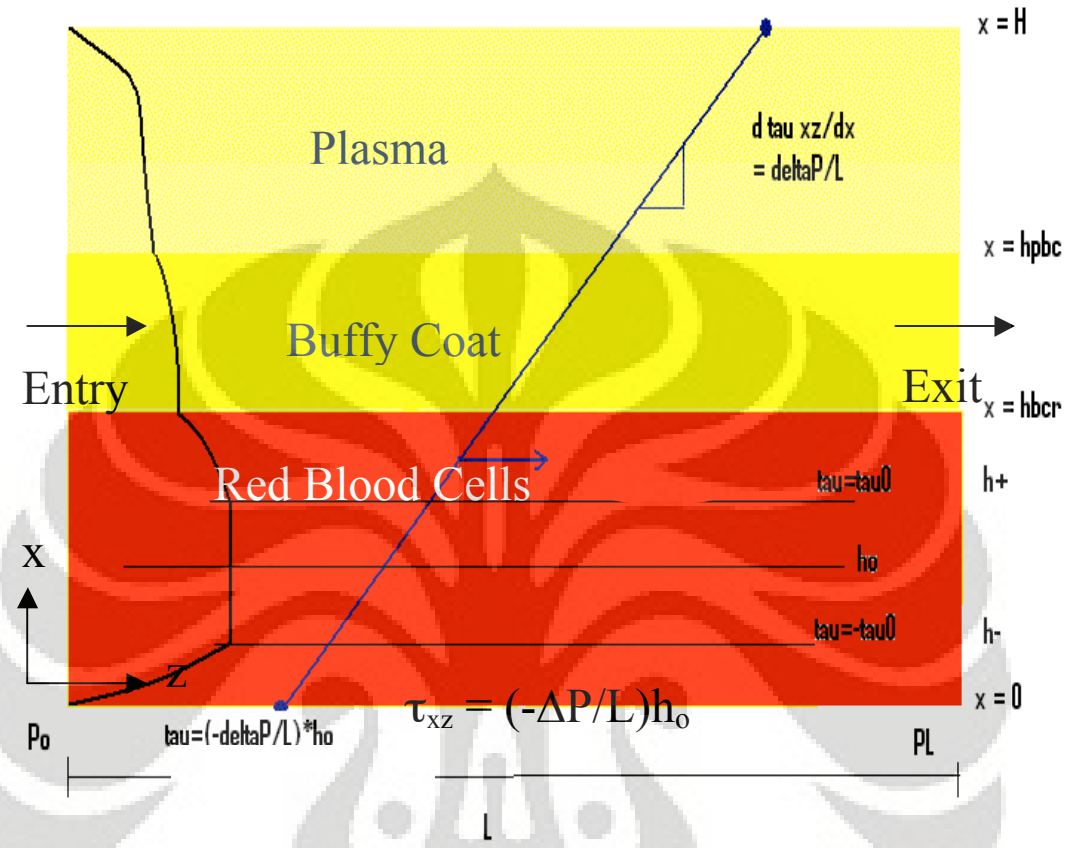


Figure 3.2. Blood flow in the separation chamber with velocity profile and shear-stress curve

This figure shows schematically the velocity profile and the three regions of plasma, buffy coat and red blood cells. Also shown are the shear-stress curve that is linear and three areas for the red blood cells region.

The following equations are developed to describe the flow through the chamber:

3.2 PLASMA

Shear stress equation describing the system

$$\tau_{XZ} = \frac{\Delta P}{L}(x - h_o) \qquad [3.1]$$

Newtonian law of viscosity

$$\tau_{xz} = -\mu^{(P)} \frac{dV_z^{(P)}}{dx} \qquad [3.2]$$

Combining the equations of shear stress

$$-\mu^{(P)} \frac{dV_z^{(P)}}{dx} = \frac{\Delta P}{L}(x - h_o) \qquad [3.3]$$

The derivative equation describing the plasma region

$$\frac{dV_z^{(P)}}{dx} = -\frac{\Delta P}{\mu^{(P)} L}(x - h_o) \qquad [3.4]$$

Expression describing the velocity of plasma

$$V_z^{(P)} = -\frac{\Delta P}{2\mu^{(P)} L} x^2 + \frac{\Delta P}{\mu^{(P)} L} h_o x + C_1 \qquad [3.5]$$

Boundary condition 1

$$V_z^{(P)}(x = H) = 0 \qquad [3.6]$$

Applying boundary condition 1, to obtain the first integral constant (C1)

$$0 = -\frac{\Delta P}{2\mu^{(P)} L} H^2 + \frac{\Delta P}{\mu^{(P)} L} h_o H + C_1 \qquad [3.7]$$

The first integral constant (C1)

$$C_1 = \frac{\Delta P}{\mu^{(P)} L} H\left(\frac{H}{2} - h_0\right) \qquad [3.8]$$

At interface with buffy coat

$$V_z^{(P)}(x = h_{PBC}) = -\frac{\Delta P}{2\mu^{(P)} L} h_{PBC}^{\ 2} + \frac{\Delta P}{\mu^{(P)} L} h_o h_{PBC} + \frac{\Delta P}{\mu^{(P)} L} H\left(\frac{H}{2} - h_o\right) \qquad [3.9]$$

3.3 BUFFY COAT

Shear stress equation describing the system

$$\tau_{XZ} = \frac{\Delta P}{L}(x - h_o) \qquad [3.10]$$

Power law model for buffy coat

$$\tau_{xz} = -m^{(B)}\left[\frac{dV_z^{(B)}}{dx}\right]^{n^{(B)}} \qquad [3.11]$$

Region of buffy coat in the chamber

$$h_{BCR} \le x \le h_{PBC} \qquad [3.12]$$

Combining the equations of shear stress

$$-m^{(B)}\left[\frac{dV_z^{(B)}}{dx}\right]^{n^{(B)}} = \frac{\Delta P}{L}(x - h_o)$$

$$\left[\frac{dV_z^{(B)}}{dx}\right]^{n^{(B)}} = -\frac{\Delta P}{m^{(B)}L}(x - h_o) \qquad [3.13]$$

The derivative equation describing the plasma region

$$\frac{dV_z^{(B)}}{dx} = \left[-\frac{\Delta P}{m^{(B)}L}(x - h_o)\right]^{1/n^{(B)}} \qquad [3.14]$$

Boundary condition 2

$$V_z^{(B)}(x = h_{PBC}) = V_z^{(P)}(x = h_{PBC}) \qquad [3.15]$$

3.4 RED BLOOD CELLS

Shear stress equation describing the system

$$\tau_{XZ} = \frac{\Delta P}{L}(x - h_o) \qquad [3.16]$$

Power law with Bingham describing red blood cells

$$\tau_{xz} = -m^{(R)}\left[\frac{dV_z^{(R)}}{dx}\right]^{n^{(R)}} \pm \tau_o \qquad [3.17]$$

First part of red blood cells region

$$h_+ \le x \le h_{BCR} \qquad [3.18]$$

Combining the equations of shear stress for the first part of red blood cells region

$$-m^{(R)}\left[\frac{dV_z^{(R)}}{dx}\right]^{n^{(R)}} - \tau_o = \frac{\Delta P}{L}(x - h_o)$$

$$-m^{(R)}\left[\frac{dV_z^{(R)}}{dx}\right]^{n^{(R)}} = \frac{\Delta P}{L}(x - h_o) + \tau_o$$

$$\left[\frac{dV_z^{(R)}}{dx}\right]^{n^{(R)}} = -\frac{\Delta P}{m^{(R)}L}(x - h_o) - \frac{\tau_o}{m^{(R)}} \qquad [3.19]$$

The derivative equation describing the first part of red blood cells region

$$\frac{dV_z^{(R)}}{dx} = \left[-\frac{\Delta P}{m^{(R)}L}(x - h_o) - \frac{\tau_o}{m^{(R)}}\right]^{1/n^{(R)}} \qquad [3.20]$$

Second part of red blood cells region

$$h_- \le x \le h_+ \qquad [3.21]$$

The derivative equation describing the second part of red blood cells region

$$\frac{dV_z^{(R)}}{dx} = 0 \qquad [3.22]$$

Third part of red blood cells region

$$0 \le x \le h_- \qquad [3.23]$$

Combining the equations of shear stress for the third part of red blood cells region

$$-m^{(R)}\left[\frac{dV_z^{(R)}}{dx}\right]^{n^{(R)}} + \tau_o = \frac{\Delta P}{L}(x - h_o)$$

$$-m^{(R)}\left[\frac{dV_z^{(R)}}{dx}\right]^{n^{(R)}} = \frac{\Delta P}{L}(x - h_o) - \tau_o$$

$$\left[\frac{dV_z^{(R)}}{dx}\right]^{n^{(R)}} = -\frac{\Delta P}{m^{(R)}L}(x-h_o)+\frac{\tau_o}{m^{(R)}} \qquad [3.24]$$

The derivative equation describing the third part of red blood cells region

$$\frac{dV_z^{(R)}}{dx} = \left[-\frac{\Delta P}{m^{(R)}L}(x-h_o)+\frac{\tau_o}{m^{(R)}}\right]^{1/n^{(R)}} \qquad [3.25]$$

3.5 MODELING DEVELOPMENT

The model is represented by three differential equations. These can be solved numerically in sequence from the bottom of the chamber to the top. The boundary conditions are at the bottom of the chamber, the two phase interfaces, and the top of the chamber. This sets up a split-boundary problem that must be solved iteratively. The approach used here is to solve the equations on an Excel spreadsheet using the Euler method and iterate the solution to meet the final boundary condition using Excel's Solver.

The parameters used in the modeling are:

ΔP	0.182	Pa
L	0.0066	m
dx	1	μm
$m^{(R)}$	0.021	$Pa.s^n$
$n^{(R)}$	0.2	
τ_o	0.002	Pa

$m^{(B)}$	0.02	$Pa.s^n$
$n^{(B)}$	0.5	
$\mu^{(P)}$	0.0017	Pa.s
μ	0.0027	Pa.s

Reasonable values were assumed for unknown parameters. These parameters are $m^{(R)}$, $n^{(R)}$), τ_o, $m^{(B)}$, and $n^{(B)}$. However, they were assumed based on the values obtained from the literature for other fluids. Values for m and n were assumed based on a comparison with effective viscosity values. For example, in the data obtained for n value, the more viscous the fluid is, the value of n is smaller. Therefore the value of n for red blood cells (R) is smaller from the value of n for buffy coat (B). Nevertheless, the value of m is almost the same for RBC and BC. This occurs because the most important parameter that should be different is the effective viscosity calculated in the modeling. The value for τ_o was assumed based on the diameter of red blood cell particle and on a comparison with fluids having different particle sizes. The value of ΔP was found by an approximation to Newtonian model for whole blood, using the Hagen-Poiseuille equation for pipe flow.

Following that, the equations used were identified in each range. The equations that were used are:

Shear stress equation describing the system

$$\tau_{XZ} = \frac{\Delta P}{L}(x - h_o) \qquad [3.26]$$

Equation used for velocity

$$V_z = V_z old + \frac{dV_z}{dx} old * dx \qquad [3.27]$$

$0 \le x \le 2312$

$$\frac{dV_z^{(R)}}{dx} = \left[-\frac{\Delta P}{m^{(R)} L}(x - h_o) + \frac{\tau_o}{m^{(R)}} \right]^{1/n^{(R)}} \qquad [3.28]$$

$2313 \le x \le 2457$

$$\frac{dV_z^{(R)}}{dx} = 0 \qquad [3.29]$$

$2458 \le x \le 3000$

$$\frac{dV_z^{(R)}}{dx} = \left[-\frac{\Delta P}{m^{(R)} L}(x - h_o) - \frac{\tau_o}{m^{(R)}} \right]^{1/n^{(R)}} \qquad [3.30]$$

$3001 \le x \le 5000$

$$\frac{dV_z^{(B)}}{dx} = \left[-\frac{\Delta P}{m^{(B)} L}(x - h_o) \right]^{1/n^{(B)}} \qquad [3.31]$$

$5001 \le x \le 6600$

$$\frac{dV_z^{(P)}}{dx} = -\frac{\Delta P}{\mu^{(P)} L}(x - h_o) \qquad [3.32]$$

Equation describing the viscosity throughout the system

$$effective\ \mu = -m\left[\frac{dVz}{dx}\right]^{n-1} \qquad [3.33]$$

The differential equations above were solved in sequence from the bottom of the chamber (RBC region) to the top of the chamber (Plasma region) using Euler method.

After all parameters and equations was specified, a value of h_o was also assumed. Because the value of h_o was first assumed, the value of V_z was not equal to zero at x = 6600 microns. Consequently, Excel's Solver was used to adjust the value of h_o to make V_z (at x = 6600 microns) = 0.

Then the value of h_o was found.

CHAPTER 4
RESULTS AND DISCUSSION

From the modeling process, h_o is found equal to 2385 μm.

The graph describing the velocity was generated as shown in Figure 4.1 below,

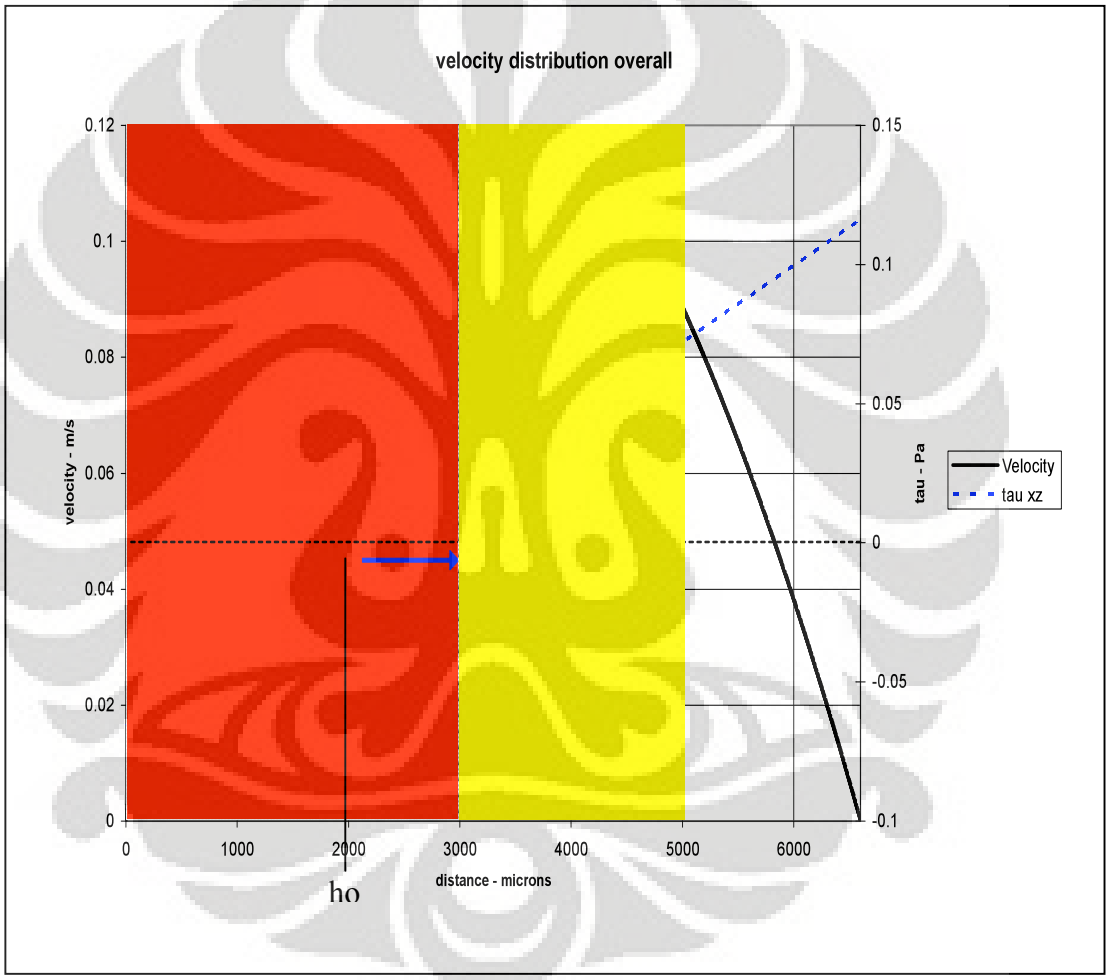


Figure 4.1. Velocity distribution for all data of x, τ_{xz} and V_z derived from Excel data sheet

This graph was constructed based on the data of x, V_z and τ_{xz}.

Some important features shown from Figure 4.1 are:

1. At $\tau_{xz} = 0$, $h_o = 2385$ microns = 0.002385 m, V_z max = 0.099 m/s
2. V_z maximum occurs at the red blood cells region
3. The velocity profile for plasma is nearly linear since it is a relatively inviscid Newtonian model

The reason why maximum temperature occurs at the red blood cell region is because the average velocity given by flow rate divided by area or by continuity of the flow. So, it does not only depend on the viscosity.

Since there are some parameters that were assumed, to illustrate the applicability of the model, these parameters were varied. The results are shown in the table 4.1 below,

Table 4.1. Comparison of h_o between the result and in the variation of the parameters

Differences	**h_o (microns)**
Result	2385
Minus 20%	2194
Minus 10%	2371
Plus 10%	2771
Plus 20%	2883

Table 4.2. Complete comparison of the important values between the result and in the variation of the parameters

	Result	**m, n, and T_o values**			
		minus 20%	minus10%	plus 10%	plus 20%
h_o (μm)	**2385**	**2194.0**	**2371.0**	**2711.0**	**2883.0**
$m^{(R)}$ (Pa.s^n)	0.021	0.0168	0.0189	0.0231	0.0252
$n^{(R)}$	0.2	0.2	0.2	0.2	0.2
τ_o (Pa)	0.002	0.0016	0.0018	0.0022	0.0024
$m^{(B)}$ (Pa.s^n)	0.02	0.016	0.018	0.022	0.024
$n^{(B)}$	0.5	0.4	0.45	0.55	0.6
$\mu^{(P)}$ (Pa.s)	0.0017	0.0017	0.0017	0.0017	0.0017
μ (Pa.s)	0.0027	0.0027	0.0027	0.0027	0.0027
effective viscosity					
rbc min(0)	0.00247	0.000175	0.000232	0.000374	0.000454
rbc max(3000)	0.0816	0.01192	0.06778	11.0677	176941.9
bc max(3001)	0.0235	0.01545	0.0215	0.055105	0.09787
bc min(5000)	0.00555	0.002383	0.00375	0.010164	0.014282
plasma	0.0017	0.0017	0.0017	0.0017	0.0017
blood	0.0027	0.0027	0.0027	0.0027	0.0027

As can be seen from the comparison of the results above, when the parameters were varied, h_o value also changed. However, the value of h_o did not change dramatically, since it is still in the red blood cells region.

There is one problem that occurred when the parameters were varied: the value of $n^{(R)}$ value could not be varied at all. When the value of $n^{(R)}$ was varied, the model solution would not compute, and the values of V_z, dV_z/dx, and effective viscosity became non-numeric (NUM) on the Excel spreadsheet. Therefore, the value of $n^{(R)}$ was not changed at all.

CHAPTER 5
CONCLUSION

The conclusion of this project are as follow:

a. Plasma behaves as a Newtonian fluid; buffy coat acts as non-Newtonian fluid and red blood cells as a Bingham fluid with power law.

b. From the graph of velocity distribution, it is shown that the maximum velocity occurs at the red blood cells region. The value of the maximum velocity is 0.099 m/s.

c. At shear stress (τ_{xz}) is equal to 0, height at the maximum velocity (h_o) is equal to 2385 microns or 0.002385 m.

d. The velocity profile for plasma is nearly linear since it is a relatively inviscid Newtonian model.

Due to the time limitation and the complexity of the project, the problem was simplified using 2-D model; sedimentation acting downward and flow behavior in axial direction. Some idea for future work is to solve the problem in 3-D model, which include the wall friction and the actual shape of the chamber. The process of sedimentation and flow at the entry of the chamber should also be combined since it happens at the same time. The fact that there are many different components in blood with different shape and sizes; it may be needed to model the differential sedimentation for each different species.

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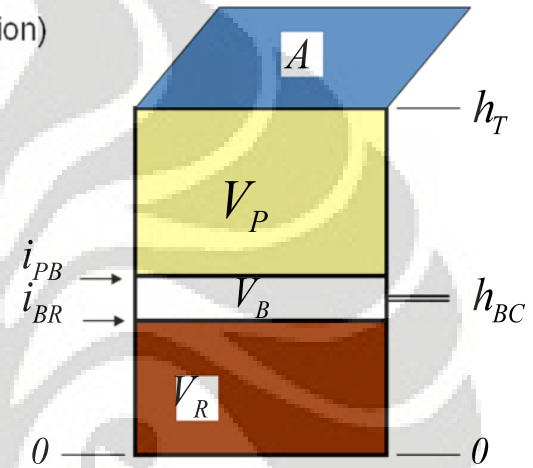
APPENDICES

Appendix A: Blood Flow Compartment Dimensions

Interface elevations (constant cross-section)

$$i_{BR} = \frac{V_R}{A} \qquad i_{PB} = \frac{V_B + V_R}{A}$$



Basic Data

Dimensions

$h_T = 0.262" = 6.66\,mm$

$h_{BC} = 0.178" = 4.52\,mm$

$A = 15\,cm^2$

$V_T = 9.98\,cm^3 \cong 10\,mL$

Nominal Compartments

$V_R = 4.5\,mL$ $\quad i_{BR} = 3\,mm$

$V_B = 3\,mL$ $\quad i_{PB} = 5\,mm$

$V_P = 2.5\,mL$

Nominal Flow Rates

$$Q_{in} = 75\frac{mL}{min}$$

$$Q_O = 35\frac{mL}{min}$$

$$Q_C = 4\frac{mL}{min}$$

$$Q_R = 36\frac{mL}{min}$$

Inlet Volume Fractions

$z_P = 0.47$

$z_B = 0.053$

$z_R = 0.48$

Appendix B: Detailed Calculation

Viscosity of Plasma:

$$\mu_{plasma} = \mu_{37} \exp[\eta(37 - T)]$$
$$= 1.4 \exp[0.021(37 - 29)]$$
$$= 1.7 cp$$
$$= 1.7 \times 10^{-3} Pa.s$$

where:

μ_{37} = viscosity of plasma at 37 ^{O}C

η = the temperature coefficient

T = the temperature of the separation process

Terminal settling velocity (Eq. 12):

$$\omega = 1500 \times \frac{2\pi}{60}$$
$$= 157.08 \, {}^{1}\!/_{\sec}$$

$$v_t = \frac{\omega^2 r \cdot d_p (\rho_p - \rho)}{18\mu}$$
$$= \frac{157.08^2 \times 0.2 \times (7 \times 10^{-6}) \times (1125 - 1025)}{18 \times 1.7 \times 10^{-3}}$$
$$= 7.9 \times 10^{-4} m/s$$

Time of settling (Eq. 14):

$$\tau_t = \frac{18\mu}{\omega^2 . (\rho_p - \rho) . d_p{}^2} . \ln\left(\frac{r + \delta}{r}\right)$$
$$= \frac{18 \times 1.7 \times 10^{-3}}{157.08^2 \times (1125 - 1025) \times (7 \times 10^{-6})^2} \ln\left(\frac{200 + 1.67}{200}\right)$$
$$= 2.10 \, \text{s}$$

Residence time (Eq. 15):

$$t = \frac{A \times h_T}{Q_{in}}$$
$$= \frac{15 cm^2 \times 0.666 cm}{75 \, mL/\min}$$
$$\cong 8 \sec$$

Appendix C: Parameters Raw Data and Development

Fluids	N	m(Pa s^n)
23.3%illinois yellow clay in water	0.229	5.55408
0.67%CMC in water	0.716	0.3035592
1.5%CMC in water	0.554	3.126564
3%CMC in water	0.566	9.28872
33%lime in water	0.171	7.182
10%napalm in kerosene	0.52	4.275684
4%paper pulp in water	0.575	20.01384
54.3%cement rock in water	0.153	2.508912
2%hydroxyethylcellulose(T=293K)	0.189	93.5
2%hydroxyethylcellulose(T=313K)	0.223	59.7
2%hydroxyethylcellulose(T=333K)	0.254	38.5
0.5%hydroxyethylcellulose(T=293K)	0.509	0.84
0.5%hydroxyethylcellulose(T=313K)	0.595	0.3
0.5%hydroxyethylcellulose(T=333K)	0.645	0.136
1%polyethylene oxide(T=293K)	0.532	0.994
1%polyethylene oxide(T=313K)	0.544	0.706
1%polyethylene oxide(T=333K)	0.599	0.486

ThO2	
dP (μm)	τ_o
0.03	0.72
0.75	0.45
1.6	0.1
2.4	0.1

As can be seen, the particle diameter (dP) equal to 7μm has τ_o far less than 0.1. From the graph derived, the equation was dP = -0.6395 ln(To)+5.0783. So, τ_o = 0.002.

$$Q = \frac{\pi \Delta P R^4}{8 \mu L}$$

$$\Delta P = \frac{8 \mu L Q}{\pi R^4}$$

$$A = height * width$$

$$A = 0.666x2.5 = 1.65cm^2$$

$$A = \pi R^2$$

$$R = 0.73x10^{-2}\, m$$

$$L = 0.06m$$

$$\mu blood = 0.0027Pa.s$$

$$Q = 75\frac{mL}{\min} = 1.25x10^{-6}\frac{m^3}{s}$$

$$\Delta P = \frac{8x0.0027x0.06x1.25x10^{-6}}{\pi x(0.73x10^{-2})^4}$$

$$\Delta P = \frac{1.62x10^{-9}}{8.917x10^{-9}}$$

$$\Delta P = 0.182Pa$$

The equations above are the development of ΔP value from approximation of Newtonian model, using Hagen-Poiseuille equation.

The Reynolds number was calculated with the equation below:

$$\mathrm{Re} = \frac{\rho \mathrm{V} D}{\mu} = \frac{\mathrm{V} D}{\nu} = \frac{\mathrm{Q} D}{\nu A}$$

where:

- V is the mean fluid velocity in (SI units: m/s)
- D is the diameter (m)
- μ is the dynamic viscosity of the fluid (Pa·s or N·s/m²)
- ν is the kinematic viscosity (ν = μ / ρ) (m²/s)
- ρ is the density of the fluid (kg/m³)
- Q is the volumetric flow rate (m³/s)
- A is the pipe *cross-sectional* area (m²)

It is known that:

- Density of blood (ρ) = 1060 kg/m^3
- Flow Rate (Q) = 1.25 x 10^{-6} m^3/s
- Diameter of pipe (D) = 1.46 x 10^{-2} m
- Blood Viscosity (μ) = 0.0027 Pa.s
- Pipe cross-sectional Area (A) = 0.000165 m^2

So Reynolds number = 48.3625

Which is laminar, because it is smaller than 2000.

Appendix D: Simulation Results

Vz	x	
0.099750917	3000	interface of RBC-BC
0.088578543	5000	interface of BC-plasma
-8.14536E-07	6600	plasma-wall
0.011753636	0.000247	

Effective vis		x
0.000246826	rbc min	0
0.081791343	rbc max	3000
0.023562587	bc max	3001
0.00554785	bc min	5000
0.0017	plasma	
0.0027	blood	

Excel spreadsheet

ΔP	0.182 Pa	mR	0.021 $Pa.s^n$	mB	0.02 $Pa.s^n$
L	0.0066 m	nR	0.2	nB	0.5
ho	2385.0 μm	To	0.002 Pa	μP	0.0017 Pa.s
dx	1 μm			μWhole blood	0.0027 Pa.s