



APPENDIX

Appendix 1: The complete list of alignment variables for Example 2.1.

(an alignment edge $\{v_l^i, v_m^j\}$ is represented as $\{i v l, j v m\}$ in the result)

$x\{1v1, 2v1\}$ $x\{1v3, 2v1\}$ $x\{1v2, 3v2\}$ $x\{2v1, 3v4\}$ $x\{2v4, 3v2\}$
 $x\{1v1, 2v2\}$ $x\{1v3, 2v2\}$ $x\{1v2, 3v3\}$ $x\{2v2, 3v1\}$ $x\{2v4, 3v3\}$
 $x\{1v1, 2v3\}$ $x\{1v3, 2v3\}$ $x\{1v2, 3v4\}$ $x\{2v2, 3v2\}$ $x\{2v4, 3v4\}$
 $x\{1v1, 2v4\}$ $x\{1v3, 2v4\}$ $x\{1v3, 3v1\}$ $x\{2v2, 3v3\}$ $x\{2v5, 3v1\}$
 $x\{1v1, 2v5\}$ $x\{1v3, 2v5\}$ $x\{1v3, 3v2\}$ $x\{2v2, 3v4\}$ $x\{2v5, 3v2\}$
 $x\{1v2, 2v1\}$ $x\{1v1, 3v1\}$ $x\{1v3, 3v3\}$ $x\{2v3, 3v1\}$ $x\{2v5, 3v3\}$
 $x\{1v2, 2v2\}$ $x\{1v1, 3v2\}$ $x\{1v3, 3v4\}$ $x\{2v3, 3v2\}$ $x\{2v5, 3v4\}$
 $x\{1v2, 2v3\}$ $x\{1v1, 3v3\}$ $x\{2v1, 3v1\}$ $x\{2v3, 3v3\}$
 $x\{1v2, 2v4\}$ $x\{1v1, 3v4\}$ $x\{2v1, 3v2\}$ $x\{2v3, 3v4\}$
 $x\{1v2, 2v5\}$ $x\{1v2, 3v1\}$ $x\{2v1, 3v3\}$ $x\{2v4, 3v1\}$

Appendix 2: The complete list of gap variables for Example 2.1.

(a gapped trace $(v_l^i, v_m^j)^j$ is represented as $(i v (l, m), j)$ in the result)

$y(1v(1,1))2$ $y(1v(2,3))3$ $y(2v(2,5))1$ $y(2v(1,4))3$ $y(2v(4,5))3$ $y(3v(3,4))1$ $y(3v(3,4))2$
 $y(1v(1,2))2$ $y(1v(3,3))3$ $y(2v(3,3))1$ $y(2v(1,5))3$ $y(2v(5,5))3$ $y(3v(4,4))1$ $y(3v(4,4))2$
 $y(1v(1,3))2$ $y(2v(1,1))1$ $y(2v(3,4))1$ $y(2v(2,2))3$ $y(3v(1,1))1$ $y(3v(1,1))2$
 $y(1v(2,2))2$ $y(2v(1,2))1$ $y(2v(3,5))1$ $y(2v(2,3))3$ $y(3v(1,2))1$ $y(3v(1,2))2$
 $y(1v(2,3))2$ $y(2v(1,3))1$ $y(2v(4,4))1$ $y(2v(2,4))3$ $y(3v(1,3))1$ $y(3v(1,3))2$
 $y(1v(3,3))2$ $y(2v(1,4))1$ $y(2v(4,5))1$ $y(2v(2,5))3$ $y(3v(1,4))1$ $y(3v(1,4))2$
 $y(1v(1,1))3$ $y(2v(1,5))1$ $y(2v(5,5))1$ $y(2v(3,3))3$ $y(3v(2,2))1$ $y(3v(2,2))2$
 $y(1v(1,2))3$ $y(2v(2,2))1$ $y(2v(1,1))3$ $y(2v(3,4))3$ $y(3v(2,3))1$ $y(3v(2,3))2$
 $y(1v(1,3))3$ $y(2v(2,3))1$ $y(2v(1,2))3$ $y(2v(3,5))3$ $y(3v(2,4))1$ $y(3v(2,4))2$

$$y(1v(2,2))_3 \quad y(2v(2,4))_1 \quad y(2v(1,3))_3 \quad y(2v(4,4))_3 \quad y(3v(3,3))_1 \quad y(3v(3,3))_2$$

Appendix 3: The complete objective function for Example 2.1.

$$\begin{aligned} \max & (-1)*x\{1v1, 2v1\} + (-1)*x\{1v1, 2v2\} + (-1)*x\{1v1, 2v3\} + (-1)*x\{1v1, 2v4\} + (1.79176)*x\{1v1, 2v5\} + (-1)*x\{1v2, \\ & 2v1\} + (1.79176)*x\{1v2, 2v2\} + (-1)*x\{1v2, 2v3\} + (-1)*x\{1v2, 2v4\} + (-1)*x\{1v2, 2v5\} + (-1)*x\{1v3, 2v1\} + (-1)*x\{1v3, \\ & 2v2\} + (1.79176)*x\{1v3, 2v3\} + (1.79176)*x\{1v3, 2v4\} + (-1)*x\{1v3, 2v5\} + (1.79176)*x\{1v1, 3v1\} + (-1)*x\{1v1, 3v2\} + \\ & (-1)*x\{1v1, 3v3\} + (1.79176)*x\{1v1, 3v4\} + (-1)*x\{1v2, 3v1\} + (-1)*x\{1v2, 3v2\} + (-1)*x\{1v2, 3v3\} + (-1)*x\{1v2, 3v4\} + (- \\ & 1)*x\{1v3, 3v1\} + (-1)*x\{1v3, 3v2\} + (1.79176)*x\{1v3, 3v3\} + (-1)*x\{1v3, 3v4\} + (-1)*x\{2v1, 3v1\} + (1.79176)*x\{2v1, \\ & 3v2\} + (-1)*x\{2v1, 3v3\} + (-1)*x\{2v1, 3v4\} + (-1)*x\{2v2, 3v1\} + (-1)*x\{2v2, 3v2\} + (-1)*x\{2v2, 3v3\} + (-1)*x\{2v2, 3v4\} + \\ & (-1)*x\{2v3, 3v1\} + (-1)*x\{2v3, 3v2\} + (1.79176)*x\{2v3, 3v3\} + (-1)*x\{2v3, 3v4\} + (-1)*x\{2v4, 3v1\} + (-1)*x\{2v4, 3v2\} + \\ & (1.79176)*x\{2v4, 3v3\} + (-1)*x\{2v4, 3v4\} + (1.79176)*x\{2v5, 3v1\} + (-1)*x\{2v5, 3v2\} + (-1)*x\{2v5, 3v3\} + \\ & (1.79176)*x\{2v5, 3v4\} + (0.693147)*y(1v(1,2))_2 + (1.09861)*y(1v(1,3))_2 + (0.693147)*y(1v(2,3))_2 + \\ & (0.693147)*y(1v(1,2))_3 + (1.09861)*y(1v(1,3))_3 + (0.693147)*y(1v(2,3))_3 + (0.693147)*y(2v(1,2))_1 + \\ & (1.09861)*y(2v(1,3))_1 + (1.38629)*y(2v(1,4))_1 + (1.60944)*y(2v(1,5))_1 + (0.693147)*y(2v(2,3))_1 + \\ & (1.09861)*y(2v(2,4))_1 + (1.38629)*y(2v(2,5))_1 + (0.693147)*y(2v(3,4))_1 + (1.09861)*y(2v(3,5))_1 + \\ & (0.693147)*y(2v(4,5))_1 + (0.693147)*y(2v(1,2))_3 + (1.09861)*y(2v(1,3))_3 + (1.38629)*y(2v(1,4))_3 + \\ & (1.60944)*y(2v(1,5))_3 + (0.693147)*y(2v(2,3))_3 + (1.09861)*y(2v(2,4))_3 + (1.38629)*y(2v(2,5))_3 + \\ & (0.693147)*y(2v(3,4))_3 + (1.09861)*y(2v(3,5))_3 + (0.693147)*y(2v(4,5))_3 + (0.693147)*y(3v(1,2))_1 + \\ & (1.09861)*y(3v(1,3))_1 + (1.38629)*y(3v(1,4))_1 + (0.693147)*y(3v(2,3))_1 + (1.09861)*y(3v(2,4))_1 + \\ & (0.693147)*y(3v(3,4))_1 + (0.693147)*y(3v(1,2))_2 + (1.09861)*y(3v(1,3))_2 + (1.38629)*y(3v(1,4))_2 + \\ & (0.693147)*y(3v(2,3))_2 + (1.09861)*y(3v(2,4))_2 + (0.693147)*y(3v(3,4))_2 \end{aligned}$$

Appendix 4: The complete first type constraints for Example 2.1.

$$\begin{aligned} & x\{1v1, 2v1\} + x\{1v1, 2v2\} + x\{1v1, 2v3\} + x\{1v1, 2v4\} + x\{1v1, 2v5\} + y(1v(1,1))_2 + y(1v(1,2))_2 + y(1v(1,3))_2 = 1 \\ & x\{1v2, 2v1\} + x\{1v2, 2v2\} + x\{1v2, 2v3\} + x\{1v2, 2v4\} + x\{1v2, 2v5\} + y(1v(1,2))_2 + y(1v(1,3))_2 + y(1v(2,2))_2 + \\ & y(1v(2,3))_2 = 1 \\ & x\{1v3, 2v1\} + x\{1v3, 2v2\} + x\{1v3, 2v3\} + x\{1v3, 2v4\} + x\{1v3, 2v5\} + y(1v(1,3))_2 + y(1v(2,3))_2 + y(1v(3,3))_2 = 1 \\ & x\{1v1, 3v1\} + x\{1v1, 3v2\} + x\{1v1, 3v3\} + x\{1v1, 3v4\} + y(1v(1,1))_3 + y(1v(1,2))_3 + y(1v(1,3))_3 = 1 \\ & x\{1v2, 3v1\} + x\{1v2, 3v2\} + x\{1v2, 3v3\} + x\{1v2, 3v4\} + y(1v(1,2))_3 + y(1v(1,3))_3 + y(1v(2,2))_3 + y(1v(2,3))_3 = 1 \\ & x\{1v3, 3v1\} + x\{1v3, 3v2\} + x\{1v3, 3v3\} + x\{1v3, 3v4\} + y(1v(1,3))_3 + y(1v(2,3))_3 + y(1v(3,3))_3 = 1 \\ & x\{1v1, 2v1\} + x\{1v2, 2v1\} + x\{1v3, 2v1\} + y(2v(1,1))_1 + y(2v(1,2))_1 + y(2v(1,3))_1 + y(2v(1,4))_1 + y(2v(1,5))_1 = 1 \end{aligned}$$

$$\begin{aligned}
& x\{1v1, 2v2\} + x\{1v2, 2v2\} + x\{1v3, 2v2\} + y\{2v(1,2)\}1 + y\{2v(1,3)\}1 + y\{2v(1,4)\}1 + y\{2v(1,5)\}1 + y\{2v(2,2)\}1 + \\
& y\{2v(2,3)\}1 + y\{2v(2,4)\}1 + y\{2v(2,5)\}1 = 1 \\
& x\{1v1, 2v3\} + x\{1v2, 2v3\} + x\{1v3, 2v3\} + y\{2v(1,3)\}1 + y\{2v(1,4)\}1 + y\{2v(1,5)\}1 + y\{2v(2,3)\}1 + y\{2v(2,4)\}1 + \\
& y\{2v(2,5)\}1 + y\{2v(3,3)\}1 + y\{2v(3,4)\}1 + y\{2v(3,5)\}1 = 1 \\
& x\{1v1, 2v4\} + x\{1v2, 2v4\} + x\{1v3, 2v4\} + y\{2v(1,4)\}1 + y\{2v(1,5)\}1 + y\{2v(2,4)\}1 + y\{2v(2,5)\}1 + y\{2v(3,4)\}1 + \\
& y\{2v(3,5)\}1 + y\{2v(4,4)\}1 + y\{2v(4,5)\}1 = 1 \\
& x\{1v1, 2v5\} + x\{1v2, 2v5\} + x\{1v3, 2v5\} + y\{2v(1,5)\}1 + y\{2v(2,5)\}1 + y\{2v(3,5)\}1 + y\{2v(4,5)\}1 + y\{2v(5,5)\}1 = 1 \\
& x\{2v1, 3v1\} + x\{2v1, 3v2\} + x\{2v1, 3v3\} + x\{2v1, 3v4\} + y\{2v(1,1)\}3 + y\{2v(1,2)\}3 + y\{2v(1,3)\}3 + y\{2v(1,4)\}3 + \\
& y\{2v(1,5)\}3 = 1 \\
& x\{2v2, 3v1\} + x\{2v2, 3v2\} + x\{2v2, 3v3\} + x\{2v2, 3v4\} + y\{2v(1,2)\}3 + y\{2v(1,3)\}3 + y\{2v(1,4)\}3 + y\{2v(1,5)\}3 + \\
& y\{2v(2,2)\}3 + y\{2v(2,3)\}3 + y\{2v(2,4)\}3 + y\{2v(2,5)\}3 = 1 \\
& x\{2v3, 3v1\} + x\{2v3, 3v2\} + x\{2v3, 3v3\} + x\{2v3, 3v4\} + y\{2v(1,3)\}3 + y\{2v(1,4)\}3 + y\{2v(1,5)\}3 + y\{2v(2,3)\}3 + \\
& y\{2v(2,4)\}3 + y\{2v(2,5)\}3 + y\{2v(3,3)\}3 + y\{2v(3,4)\}3 + y\{2v(3,5)\}3 = 1 \\
& x\{2v4, 3v1\} + x\{2v4, 3v2\} + x\{2v4, 3v3\} + x\{2v4, 3v4\} + y\{2v(1,4)\}3 + y\{2v(1,5)\}3 + y\{2v(2,4)\}3 + y\{2v(2,5)\}3 + \\
& y\{2v(3,4)\}3 + y\{2v(3,5)\}3 + y\{2v(4,4)\}3 + y\{2v(4,5)\}3 = 1 \\
& x\{2v5, 3v1\} + x\{2v5, 3v2\} + x\{2v5, 3v3\} + x\{2v5, 3v4\} + y\{2v(1,5)\}3 + y\{2v(2,5)\}3 + y\{2v(3,5)\}3 + y\{2v(4,5)\}3 + \\
& y\{2v(5,5)\}3 = 1 \\
& x\{1v1, 3v1\} + x\{1v2, 3v1\} + x\{1v3, 3v1\} + y\{3v(1,1)\}1 + y\{3v(1,2)\}1 + y\{3v(1,3)\}1 + y\{3v(1,4)\}1 = 1 \\
& x\{1v1, 3v2\} + x\{1v2, 3v2\} + x\{1v3, 3v2\} + y\{3v(1,2)\}1 + y\{3v(1,3)\}1 + y\{3v(1,4)\}1 + y\{3v(2,2)\}1 + y\{3v(2,3)\}1 + \\
& y\{3v(2,4)\}1 = 1 \\
& x\{1v1, 3v3\} + x\{1v2, 3v3\} + x\{1v3, 3v3\} + y\{3v(1,3)\}1 + y\{3v(1,4)\}1 + y\{3v(2,3)\}1 + y\{3v(2,4)\}1 + y\{3v(3,3)\}1 + \\
& y\{3v(3,4)\}1 = 1 \\
& x\{1v1, 3v4\} + x\{1v2, 3v4\} + x\{1v3, 3v4\} + y\{3v(1,4)\}1 + y\{3v(2,4)\}1 + y\{3v(3,4)\}1 + y\{3v(4,4)\}1 = 1 \\
& x\{2v1, 3v1\} + x\{2v2, 3v1\} + x\{2v3, 3v1\} + x\{2v4, 3v1\} + x\{2v5, 3v1\} + y\{3v(1,1)\}2 + y\{3v(1,2)\}2 + y\{3v(1,3)\}2 + \\
& y\{3v(1,4)\}2 = 1 \\
& x\{2v1, 3v2\} + x\{2v2, 3v2\} + x\{2v3, 3v2\} + x\{2v4, 3v2\} + x\{2v5, 3v2\} + y\{3v(1,2)\}2 + y\{3v(1,3)\}2 + y\{3v(1,4)\}2 + \\
& y\{3v(2,2)\}2 + y\{3v(2,3)\}2 + y\{3v(2,4)\}2 = 1 \\
& x\{2v1, 3v3\} + x\{2v2, 3v3\} + x\{2v3, 3v3\} + x\{2v4, 3v3\} + x\{2v5, 3v3\} + y\{3v(1,3)\}2 + y\{3v(1,4)\}2 + y\{3v(2,3)\}2 + \\
& y\{3v(2,4)\}2 + y\{3v(3,3)\}2 + y\{3v(3,4)\}2 = 1 \\
& x\{2v1, 3v4\} + x\{2v2, 3v4\} + x\{2v3, 3v4\} + x\{2v4, 3v4\} + x\{2v5, 3v4\} + y\{3v(1,4)\}2 + y\{3v(2,4)\}2 + y\{3v(3,4)\}2 + \\
& y\{3v(4,4)\}2 = 1
\end{aligned}$$

Appendix 5: The complete second type constraints for Example 2.1.

$$\begin{aligned}
 &x\{1v1, 2v2\} + x\{1v2, 2v1\} \leq 1 & x\{1v1, 3v2\} + x\{1v3, 3v1\} \leq 1 & x\{2v2, 3v2\} + x\{2v3, 3v1\} \leq 1 \\
 &x\{1v1, 2v3\} + x\{1v2, 2v1\} \leq 1 & x\{1v1, 3v3\} + x\{1v3, 3v1\} \leq 1 & x\{2v2, 3v3\} + x\{2v3, 3v1\} \leq 1 \\
 &x\{1v1, 2v4\} + x\{1v2, 2v1\} \leq 1 & x\{1v1, 3v4\} + x\{1v3, 3v1\} \leq 1 & x\{2v2, 3v4\} + x\{2v3, 3v1\} \leq 1 \\
 &x\{1v1, 2v5\} + x\{1v2, 2v1\} \leq 1 & x\{1v1, 3v3\} + x\{1v3, 3v2\} \leq 1 & x\{2v2, 3v3\} + x\{2v3, 3v2\} \leq 1 \\
 &x\{1v1, 2v3\} + x\{1v2, 2v2\} \leq 1 & x\{1v1, 3v4\} + x\{1v3, 3v2\} \leq 1 & x\{2v2, 3v4\} + x\{2v3, 3v2\} \leq 1 \\
 &x\{1v1, 2v4\} + x\{1v2, 2v2\} \leq 1 & x\{1v1, 3v4\} + x\{1v3, 3v3\} \leq 1 & x\{2v2, 3v4\} + x\{2v3, 3v3\} \leq 1 \\
 &x\{1v1, 2v5\} + x\{1v2, 2v2\} \leq 1 & x\{1v2, 3v2\} + x\{1v3, 3v1\} \leq 1 & x\{2v2, 3v2\} + x\{2v4, 3v1\} \leq 1 \\
 &x\{1v1, 2v4\} + x\{1v2, 2v3\} \leq 1 & x\{1v2, 3v3\} + x\{1v3, 3v1\} \leq 1 & x\{2v2, 3v3\} + x\{2v4, 3v1\} \leq 1 \\
 &x\{1v1, 2v5\} + x\{1v2, 2v3\} \leq 1 & x\{1v2, 3v4\} + x\{1v3, 3v1\} \leq 1 & x\{2v2, 3v4\} + x\{2v4, 3v1\} \leq 1 \\
 &x\{1v1, 2v5\} + x\{1v2, 2v4\} \leq 1 & x\{1v2, 3v3\} + x\{1v3, 3v2\} \leq 1 & x\{2v2, 3v3\} + x\{2v4, 3v2\} \leq 1 \\
 &x\{1v1, 2v2\} + x\{1v3, 2v1\} \leq 1 & x\{1v2, 3v4\} + x\{1v3, 3v2\} \leq 1 & x\{2v2, 3v4\} + x\{2v4, 3v2\} \leq 1 \\
 &x\{1v1, 2v3\} + x\{1v3, 2v1\} \leq 1 & x\{1v2, 3v4\} + x\{1v3, 3v3\} \leq 1 & x\{2v2, 3v4\} + x\{2v4, 3v3\} \leq 1 \\
 &x\{1v1, 2v4\} + x\{1v3, 2v1\} \leq 1 & x\{2v1, 3v2\} + x\{2v2, 3v1\} \leq 1 & x\{2v2, 3v2\} + x\{2v5, 3v1\} \leq 1 \\
 &x\{1v1, 2v5\} + x\{1v3, 2v1\} \leq 1 & x\{2v1, 3v3\} + x\{2v2, 3v1\} \leq 1 & x\{2v2, 3v3\} + x\{2v5, 3v1\} \leq 1 \\
 &x\{1v1, 2v3\} + x\{1v3, 2v2\} \leq 1 & x\{2v1, 3v4\} + x\{2v2, 3v1\} \leq 1 & x\{2v2, 3v4\} + x\{2v5, 3v1\} \leq 1 \\
 &x\{1v1, 2v4\} + x\{1v3, 2v2\} \leq 1 & x\{2v1, 3v3\} + x\{2v2, 3v2\} \leq 1 & x\{2v2, 3v3\} + x\{2v5, 3v2\} \leq 1 \\
 &x\{1v1, 2v5\} + x\{1v3, 2v2\} \leq 1 & x\{2v1, 3v4\} + x\{2v2, 3v2\} \leq 1 & x\{2v2, 3v4\} + x\{2v5, 3v2\} \leq 1 \\
 &x\{1v1, 2v4\} + x\{1v3, 2v3\} \leq 1 & x\{2v1, 3v4\} + x\{2v2, 3v3\} \leq 1 & x\{2v2, 3v4\} + x\{2v5, 3v3\} \leq 1 \\
 &x\{1v1, 2v5\} + x\{1v3, 2v3\} \leq 1 & x\{2v1, 3v2\} + x\{2v3, 3v1\} \leq 1 & x\{2v3, 3v2\} + x\{2v4, 3v1\} \leq 1 \\
 &x\{1v1, 2v5\} + x\{1v3, 2v4\} \leq 1 & x\{2v1, 3v3\} + x\{2v3, 3v1\} \leq 1 & x\{2v3, 3v3\} + x\{2v4, 3v1\} \leq 1 \\
 &x\{1v2, 2v2\} + x\{1v3, 2v1\} \leq 1 & x\{2v1, 3v4\} + x\{2v3, 3v1\} \leq 1 & x\{2v3, 3v4\} + x\{2v4, 3v1\} \leq 1 \\
 &x\{1v2, 2v3\} + x\{1v3, 2v1\} \leq 1 & x\{2v1, 3v3\} + x\{2v3, 3v2\} \leq 1 & x\{2v3, 3v3\} + x\{2v4, 3v2\} \leq 1 \\
 &x\{1v2, 2v4\} + x\{1v3, 2v1\} \leq 1 & x\{2v1, 3v4\} + x\{2v3, 3v2\} \leq 1 & x\{2v3, 3v4\} + x\{2v4, 3v2\} \leq 1 \\
 &x\{1v2, 2v5\} + x\{1v3, 2v1\} \leq 1 & x\{2v1, 3v4\} + x\{2v3, 3v3\} \leq 1 & x\{2v3, 3v4\} + x\{2v4, 3v3\} \leq 1 \\
 &x\{1v2, 2v3\} + x\{1v3, 2v2\} \leq 1 & x\{2v1, 3v2\} + x\{2v4, 3v1\} \leq 1 & x\{2v3, 3v2\} + x\{2v5, 3v1\} \leq 1 \\
 &x\{1v2, 2v4\} + x\{1v3, 2v2\} \leq 1 & x\{2v1, 3v3\} + x\{2v4, 3v1\} \leq 1 & x\{2v3, 3v3\} + x\{2v5, 3v1\} \leq 1 \\
 &x\{1v2, 2v5\} + x\{1v3, 2v2\} \leq 1 & x\{2v1, 3v4\} + x\{2v4, 3v1\} \leq 1 & x\{2v3, 3v4\} + x\{2v5, 3v1\} \leq 1
 \end{aligned}$$

$$\begin{array}{lll}
x\{1v2, 2v4\} + x\{1v3, 2v3\} \leq 1 & x\{2v1, 3v3\} + x\{2v4, 3v2\} \leq 1 & x\{2v3, 3v3\} + x\{2v5, 3v2\} \leq 1 \\
x\{1v2, 2v5\} + x\{1v3, 2v3\} \leq 1 & x\{2v1, 3v4\} + x\{2v4, 3v2\} \leq 1 & x\{2v3, 3v4\} + x\{2v5, 3v2\} \leq 1 \\
x\{1v2, 2v5\} + x\{1v3, 2v4\} \leq 1 & x\{2v1, 3v4\} + x\{2v4, 3v3\} \leq 1 & x\{2v3, 3v4\} + x\{2v5, 3v3\} \leq 1 \\
x\{1v1, 3v2\} + x\{1v2, 3v1\} \leq 1 & x\{2v1, 3v2\} + x\{2v5, 3v1\} \leq 1 & x\{2v4, 3v2\} + x\{2v5, 3v1\} \leq 1 \\
x\{1v1, 3v3\} + x\{1v2, 3v1\} \leq 1 & x\{2v1, 3v3\} + x\{2v5, 3v1\} \leq 1 & x\{2v4, 3v3\} + x\{2v5, 3v1\} \leq 1 \\
x\{1v1, 3v4\} + x\{1v2, 3v1\} \leq 1 & x\{2v1, 3v4\} + x\{2v5, 3v1\} \leq 1 & x\{2v4, 3v4\} + x\{2v5, 3v1\} \leq 1 \\
x\{1v1, 3v3\} + x\{1v2, 3v2\} \leq 1 & x\{2v1, 3v3\} + x\{2v5, 3v2\} \leq 1 & x\{2v4, 3v3\} + x\{2v5, 3v2\} \leq 1 \\
x\{1v1, 3v4\} + x\{1v2, 3v2\} \leq 1 & x\{2v1, 3v4\} + x\{2v5, 3v2\} \leq 1 & x\{2v4, 3v4\} + x\{2v5, 3v2\} \leq 1 \\
x\{1v1, 3v4\} + x\{1v2, 3v3\} \leq 1 & x\{2v1, 3v4\} + x\{2v5, 3v3\} \leq 1 & x\{2v4, 3v4\} + x\{2v5, 3v3\} \leq 1 \\
\\
x\{1v1, 2v2\} + x\{1v2, 3v1\} + x\{2v1, 3v2\} \leq 2 & x\{1v2, 2v1\} + x\{1v1, 3v2\} + x\{2v2, 3v1\} \leq 2 & \\
x\{1v1, 2v3\} + x\{1v2, 3v1\} + x\{2v1, 3v2\} \leq 2 & x\{1v2, 2v1\} + x\{1v1, 3v3\} + x\{2v2, 3v1\} \leq 2 & \\
x\{1v1, 2v4\} + x\{1v2, 3v1\} + x\{2v1, 3v2\} \leq 2 & x\{1v2, 2v1\} + x\{1v1, 3v4\} + x\{2v2, 3v1\} \leq 2 & \\
x\{1v1, 2v5\} + x\{1v2, 3v1\} + x\{2v1, 3v2\} \leq 2 & x\{1v2, 2v1\} + x\{1v1, 3v3\} + x\{2v2, 3v2\} \leq 2 & \\
x\{1v1, 2v3\} + x\{1v2, 3v1\} + x\{2v2, 3v2\} \leq 2 & x\{1v2, 2v1\} + x\{1v1, 3v4\} + x\{2v2, 3v2\} \leq 2 & \\
x\{1v1, 2v4\} + x\{1v2, 3v1\} + x\{2v2, 3v2\} \leq 2 & x\{1v2, 2v1\} + x\{1v1, 3v4\} + x\{2v2, 3v3\} \leq 2 & \\
x\{1v1, 2v5\} + x\{1v2, 3v1\} + x\{2v2, 3v2\} \leq 2 & x\{1v2, 2v1\} + x\{1v1, 3v2\} + x\{2v3, 3v1\} \leq 2 & \\
x\{1v1, 2v4\} + x\{1v2, 3v1\} + x\{2v3, 3v2\} \leq 2 & x\{1v2, 2v1\} + x\{1v1, 3v3\} + x\{2v3, 3v1\} \leq 2 & \\
x\{1v1, 2v5\} + x\{1v2, 3v1\} + x\{2v3, 3v2\} \leq 2 & x\{1v2, 2v1\} + x\{1v1, 3v4\} + x\{2v3, 3v1\} \leq 2 & \\
x\{1v1, 2v5\} + x\{1v2, 3v1\} + x\{2v4, 3v2\} \leq 2 & x\{1v2, 2v1\} + x\{1v1, 3v3\} + x\{2v3, 3v2\} \leq 2 & \\
x\{1v1, 2v2\} + x\{1v2, 3v1\} + x\{2v1, 3v3\} \leq 2 & x\{1v2, 2v1\} + x\{1v1, 3v4\} + x\{2v3, 3v2\} \leq 2 & \\
x\{1v1, 2v3\} + x\{1v2, 3v1\} + x\{2v1, 3v3\} \leq 2 & x\{1v2, 2v1\} + x\{1v1, 3v4\} + x\{2v3, 3v3\} \leq 2 & \\
x\{1v1, 2v4\} + x\{1v2, 3v1\} + x\{2v1, 3v3\} \leq 2 & x\{1v2, 2v1\} + x\{1v1, 3v2\} + x\{2v4, 3v1\} \leq 2 & \\
x\{1v1, 2v5\} + x\{1v2, 3v1\} + x\{2v1, 3v3\} \leq 2 & x\{1v2, 2v1\} + x\{1v1, 3v3\} + x\{2v4, 3v1\} \leq 2 & \\
x\{1v1, 2v3\} + x\{1v2, 3v1\} + x\{2v2, 3v3\} \leq 2 & x\{1v2, 2v1\} + x\{1v1, 3v4\} + x\{2v4, 3v1\} \leq 2 & \\
x\{1v1, 2v4\} + x\{1v2, 3v1\} + x\{2v2, 3v3\} \leq 2 & x\{1v2, 2v1\} + x\{1v1, 3v3\} + x\{2v4, 3v2\} \leq 2 & \\
x\{1v1, 2v5\} + x\{1v2, 3v1\} + x\{2v2, 3v3\} \leq 2 & x\{1v2, 2v1\} + x\{1v1, 3v4\} + x\{2v4, 3v2\} \leq 2 & \\
x\{1v1, 2v4\} + x\{1v2, 3v1\} + x\{2v3, 3v3\} \leq 2 & x\{1v2, 2v1\} + x\{1v1, 3v4\} + x\{2v4, 3v3\} \leq 2 &
\end{array}$$

$$\begin{array}{ll}
 x\{1v1, 2v4\} + x\{1v2, 3v2\} + x\{2v3, 3v4\} \leq 2 & x\{1v2, 2v3\} + x\{1v1, 3v4\} + x\{2v4, 3v3\} \leq 2 \\
 x\{1v1, 2v5\} + x\{1v2, 3v2\} + x\{2v3, 3v4\} \leq 2 & x\{1v2, 2v3\} + x\{1v1, 3v2\} + x\{2v5, 3v1\} \leq 2 \\
 x\{1v1, 2v5\} + x\{1v2, 3v2\} + x\{2v4, 3v4\} \leq 2 & x\{1v2, 2v3\} + x\{1v1, 3v3\} + x\{2v5, 3v1\} \leq 2 \\
 x\{1v1, 2v2\} + x\{1v2, 3v3\} + x\{2v1, 3v4\} \leq 2 & x\{1v2, 2v3\} + x\{1v1, 3v4\} + x\{2v5, 3v1\} \leq 2 \\
 x\{1v1, 2v3\} + x\{1v2, 3v3\} + x\{2v1, 3v4\} \leq 2 & x\{1v2, 2v3\} + x\{1v1, 3v3\} + x\{2v5, 3v2\} \leq 2 \\
 x\{1v1, 2v4\} + x\{1v2, 3v3\} + x\{2v1, 3v4\} \leq 2 & x\{1v2, 2v3\} + x\{1v1, 3v4\} + x\{2v5, 3v2\} \leq 2 \\
 x\{1v1, 2v5\} + x\{1v2, 3v3\} + x\{2v1, 3v4\} \leq 2 & x\{1v2, 2v3\} + x\{1v1, 3v4\} + x\{2v5, 3v3\} \leq 2 \\
 x\{1v1, 2v3\} + x\{1v2, 3v3\} + x\{2v2, 3v4\} \leq 2 & x\{1v2, 2v4\} + x\{1v1, 3v2\} + x\{2v5, 3v1\} \leq 2 \\
 x\{1v1, 2v4\} + x\{1v2, 3v3\} + x\{2v2, 3v4\} \leq 2 & x\{1v2, 2v4\} + x\{1v1, 3v3\} + x\{2v5, 3v1\} \leq 2 \\
 x\{1v1, 2v5\} + x\{1v2, 3v3\} + x\{2v2, 3v4\} \leq 2 & x\{1v2, 2v4\} + x\{1v1, 3v4\} + x\{2v5, 3v1\} \leq 2 \\
 x\{1v1, 2v4\} + x\{1v2, 3v3\} + x\{2v3, 3v4\} \leq 2 & x\{1v2, 2v4\} + x\{1v1, 3v3\} + x\{2v5, 3v2\} \leq 2 \\
 x\{1v1, 2v5\} + x\{1v2, 3v3\} + x\{2v3, 3v4\} \leq 2 & x\{1v2, 2v4\} + x\{1v1, 3v4\} + x\{2v5, 3v2\} \leq 2 \\
 x\{1v1, 2v5\} + x\{1v2, 3v3\} + x\{2v4, 3v4\} \leq 2 & x\{1v2, 2v4\} + x\{1v1, 3v4\} + x\{2v5, 3v3\} \leq 2 \\
 x\{1v1, 2v2\} + x\{1v3, 3v1\} + x\{2v1, 3v2\} \leq 2 & x\{1v3, 2v1\} + x\{1v1, 3v2\} + x\{2v2, 3v1\} \leq 2 \\
 x\{1v1, 2v3\} + x\{1v3, 3v1\} + x\{2v1, 3v2\} \leq 2 & x\{1v3, 2v1\} + x\{1v1, 3v3\} + x\{2v2, 3v1\} \leq 2 \\
 x\{1v1, 2v4\} + x\{1v3, 3v1\} + x\{2v1, 3v2\} \leq 2 & x\{1v3, 2v1\} + x\{1v1, 3v4\} + x\{2v2, 3v1\} \leq 2 \\
 x\{1v1, 2v5\} + x\{1v3, 3v1\} + x\{2v1, 3v2\} \leq 2 & x\{1v3, 2v1\} + x\{1v1, 3v3\} + x\{2v2, 3v2\} \leq 2 \\
 x\{1v1, 2v3\} + x\{1v3, 3v1\} + x\{2v2, 3v2\} \leq 2 & x\{1v3, 2v1\} + x\{1v1, 3v4\} + x\{2v2, 3v2\} \leq 2 \\
 x\{1v1, 2v4\} + x\{1v3, 3v1\} + x\{2v2, 3v2\} \leq 2 & x\{1v3, 2v1\} + x\{1v1, 3v4\} + x\{2v2, 3v3\} \leq 2 \\
 x\{1v1, 2v5\} + x\{1v3, 3v1\} + x\{2v2, 3v2\} \leq 2 & x\{1v3, 2v1\} + x\{1v1, 3v2\} + x\{2v3, 3v1\} \leq 2 \\
 x\{1v1, 2v4\} + x\{1v3, 3v1\} + x\{2v3, 3v2\} \leq 2 & x\{1v3, 2v1\} + x\{1v1, 3v3\} + x\{2v3, 3v1\} \leq 2 \\
 x\{1v1, 2v5\} + x\{1v3, 3v1\} + x\{2v3, 3v2\} \leq 2 & x\{1v3, 2v1\} + x\{1v1, 3v4\} + x\{2v3, 3v1\} \leq 2 \\
 x\{1v1, 2v2\} + x\{1v3, 3v1\} + x\{2v1, 3v3\} \leq 2 & x\{1v3, 2v1\} + x\{1v1, 3v4\} + x\{2v3, 3v2\} \leq 2 \\
 x\{1v1, 2v3\} + x\{1v3, 3v1\} + x\{2v1, 3v3\} \leq 2 & x\{1v3, 2v1\} + x\{1v1, 3v4\} + x\{2v3, 3v3\} \leq 2 \\
 x\{1v1, 2v4\} + x\{1v3, 3v1\} + x\{2v1, 3v3\} \leq 2 & x\{1v3, 2v1\} + x\{1v1, 3v2\} + x\{2v4, 3v1\} \leq 2 \\
 x\{1v1, 2v5\} + x\{1v3, 3v1\} + x\{2v1, 3v3\} \leq 2 & x\{1v3, 2v1\} + x\{1v1, 3v3\} + x\{2v4, 3v1\} \leq 2 \\
 x\{1v1, 2v3\} + x\{1v3, 3v1\} + x\{2v2, 3v3\} \leq 2 & x\{1v3, 2v1\} + x\{1v1, 3v4\} + x\{2v4, 3v1\} \leq 2
 \end{array}$$

$$\begin{array}{ll}
x\{1v1, 2v4\} + x\{1v3, 3v1\} + x\{2v2, 3v3\} \leq 2 & x\{1v3, 2v1\} + x\{1v1, 3v3\} + x\{2v4, 3v2\} \leq 2 \\
x\{1v1, 2v5\} + x\{1v3, 3v1\} + x\{2v2, 3v3\} \leq 2 & x\{1v3, 2v1\} + x\{1v1, 3v4\} + x\{2v4, 3v2\} \leq 2 \\
x\{1v1, 2v4\} + x\{1v3, 3v1\} + x\{2v3, 3v3\} \leq 2 & x\{1v3, 2v1\} + x\{1v1, 3v4\} + x\{2v4, 3v3\} \leq 2 \\
x\{1v1, 2v5\} + x\{1v3, 3v1\} + x\{2v3, 3v3\} \leq 2 & x\{1v3, 2v1\} + x\{1v1, 3v2\} + x\{2v5, 3v1\} \leq 2 \\
x\{1v1, 2v5\} + x\{1v3, 3v1\} + x\{2v4, 3v3\} \leq 2 & x\{1v3, 2v1\} + x\{1v1, 3v3\} + x\{2v5, 3v1\} \leq 2 \\
x\{1v1, 2v2\} + x\{1v3, 3v1\} + x\{2v1, 3v4\} \leq 2 & x\{1v3, 2v1\} + x\{1v1, 3v4\} + x\{2v5, 3v1\} \leq 2 \\
x\{1v1, 2v3\} + x\{1v3, 3v1\} + x\{2v1, 3v4\} \leq 2 & x\{1v3, 2v1\} + x\{1v1, 3v3\} + x\{2v5, 3v2\} \leq 2 \\
x\{1v1, 2v4\} + x\{1v3, 3v1\} + x\{2v1, 3v4\} \leq 2 & x\{1v3, 2v1\} + x\{1v1, 3v4\} + x\{2v5, 3v2\} \leq 2 \\
x\{1v1, 2v5\} + x\{1v3, 3v1\} + x\{2v1, 3v4\} \leq 2 & x\{1v3, 2v1\} + x\{1v1, 3v4\} + x\{2v5, 3v3\} \leq 2 \\
x\{1v1, 2v3\} + x\{1v3, 3v1\} + x\{2v2, 3v4\} \leq 2 & x\{1v3, 2v2\} + x\{1v1, 3v2\} + x\{2v3, 3v1\} \leq 2 \\
x\{1v1, 2v4\} + x\{1v3, 3v1\} + x\{2v2, 3v4\} \leq 2 & x\{1v3, 2v2\} + x\{1v1, 3v3\} + x\{2v3, 3v1\} \leq 2 \\
x\{1v1, 2v5\} + x\{1v3, 3v1\} + x\{2v2, 3v4\} \leq 2 & x\{1v3, 2v2\} + x\{1v1, 3v4\} + x\{2v3, 3v1\} \leq 2 \\
x\{1v1, 2v4\} + x\{1v3, 3v1\} + x\{2v3, 3v4\} \leq 2 & x\{1v3, 2v2\} + x\{1v1, 3v3\} + x\{2v3, 3v2\} \leq 2 \\
x\{1v1, 2v5\} + x\{1v3, 3v1\} + x\{2v3, 3v4\} \leq 2 & x\{1v3, 2v2\} + x\{1v1, 3v4\} + x\{2v3, 3v2\} \leq 2 \\
x\{1v1, 2v5\} + x\{1v3, 3v1\} + x\{2v4, 3v4\} \leq 2 & x\{1v3, 2v2\} + x\{1v1, 3v4\} + x\{2v3, 3v3\} \leq 2 \\
x\{1v1, 2v2\} + x\{1v3, 3v2\} + x\{2v1, 3v3\} \leq 2 & x\{1v3, 2v2\} + x\{1v1, 3v2\} + x\{2v4, 3v1\} \leq 2 \\
x\{1v1, 2v3\} + x\{1v3, 3v2\} + x\{2v1, 3v3\} \leq 2 & x\{1v3, 2v2\} + x\{1v1, 3v3\} + x\{2v4, 3v1\} \leq 2 \\
x\{1v1, 2v4\} + x\{1v3, 3v2\} + x\{2v1, 3v3\} \leq 2 & x\{1v3, 2v2\} + x\{1v1, 3v4\} + x\{2v4, 3v1\} \leq 2 \\
x\{1v1, 2v5\} + x\{1v3, 3v2\} + x\{2v1, 3v3\} \leq 2 & x\{1v3, 2v2\} + x\{1v1, 3v3\} + x\{2v4, 3v2\} \leq 2 \\
x\{1v1, 2v3\} + x\{1v3, 3v2\} + x\{2v2, 3v3\} \leq 2 & x\{1v3, 2v2\} + x\{1v1, 3v4\} + x\{2v4, 3v2\} \leq 2 \\
x\{1v1, 2v4\} + x\{1v3, 3v2\} + x\{2v2, 3v3\} \leq 2 & x\{1v3, 2v2\} + x\{1v1, 3v4\} + x\{2v4, 3v3\} \leq 2 \\
x\{1v1, 2v5\} + x\{1v3, 3v2\} + x\{2v2, 3v3\} \leq 2 & x\{1v3, 2v2\} + x\{1v1, 3v2\} + x\{2v5, 3v1\} \leq 2 \\
x\{1v1, 2v4\} + x\{1v3, 3v2\} + x\{2v3, 3v3\} \leq 2 & x\{1v3, 2v2\} + x\{1v1, 3v3\} + x\{2v5, 3v1\} \leq 2 \\
x\{1v1, 2v5\} + x\{1v3, 3v2\} + x\{2v3, 3v3\} \leq 2 & x\{1v3, 2v2\} + x\{1v1, 3v4\} + x\{2v5, 3v1\} \leq 2 \\
x\{1v1, 2v5\} + x\{1v3, 3v2\} + x\{2v4, 3v3\} \leq 2 & x\{1v3, 2v2\} + x\{1v1, 3v3\} + x\{2v5, 3v2\} \leq 2 \\
x\{1v1, 2v2\} + x\{1v3, 3v2\} + x\{2v1, 3v4\} \leq 2 & x\{1v3, 2v2\} + x\{1v1, 3v4\} + x\{2v5, 3v2\} \leq 2 \\
x\{1v1, 2v3\} + x\{1v3, 3v2\} + x\{2v1, 3v4\} \leq 2 & x\{1v3, 2v2\} + x\{1v1, 3v4\} + x\{2v5, 3v3\} \leq 2 \\
x\{1v1, 2v4\} + x\{1v3, 3v2\} + x\{2v1, 3v4\} \leq 2 & x\{1v3, 2v3\} + x\{1v1, 3v2\} + x\{2v4, 3v1\} \leq 2 \\
x\{1v1, 2v5\} + x\{1v3, 3v2\} + x\{2v1, 3v4\} \leq 2 & x\{1v3, 2v3\} + x\{1v1, 3v3\} + x\{2v4, 3v1\} \leq 2
\end{array}$$

$$\begin{array}{ll}
x\{1v1, 2v3\} + x\{1v3, 3v2\} + x\{2v2, 3v4\} \leq 2 & x\{1v3, 2v3\} + x\{1v1, 3v4\} + x\{2v4, 3v1\} \leq 2 \\
x\{1v1, 2v4\} + x\{1v3, 3v2\} + x\{2v2, 3v4\} \leq 2 & x\{1v3, 2v3\} + x\{1v1, 3v3\} + x\{2v4, 3v2\} \leq 2 \\
x\{1v1, 2v5\} + x\{1v3, 3v2\} + x\{2v2, 3v4\} \leq 2 & x\{1v3, 2v3\} + x\{1v1, 3v4\} + x\{2v4, 3v2\} \leq 2 \\
x\{1v1, 2v4\} + x\{1v3, 3v2\} + x\{2v3, 3v4\} \leq 2 & x\{1v3, 2v3\} + x\{1v1, 3v4\} + x\{2v4, 3v3\} \leq 2 \\
x\{1v1, 2v5\} + x\{1v3, 3v2\} + x\{2v3, 3v4\} \leq 2 & x\{1v3, 2v3\} + x\{1v1, 3v2\} + x\{2v5, 3v1\} \leq 2 \\
x\{1v1, 2v5\} + x\{1v3, 3v2\} + x\{2v4, 3v4\} \leq 2 & x\{1v3, 2v3\} + x\{1v1, 3v3\} + x\{2v5, 3v1\} \leq 2 \\
x\{1v1, 2v2\} + x\{1v3, 3v3\} + x\{2v1, 3v4\} \leq 2 & x\{1v3, 2v3\} + x\{1v1, 3v4\} + x\{2v5, 3v1\} \leq 2 \\
x\{1v1, 2v3\} + x\{1v3, 3v3\} + x\{2v1, 3v4\} \leq 2 & x\{1v3, 2v3\} + x\{1v1, 3v3\} + x\{2v5, 3v2\} \leq 2 \\
x\{1v1, 2v4\} + x\{1v3, 3v3\} + x\{2v1, 3v4\} \leq 2 & x\{1v3, 2v3\} + x\{1v1, 3v4\} + x\{2v5, 3v2\} \leq 2 \\
x\{1v1, 2v5\} + x\{1v3, 3v3\} + x\{2v1, 3v4\} \leq 2 & x\{1v3, 2v3\} + x\{1v1, 3v4\} + x\{2v5, 3v3\} \leq 2 \\
x\{1v1, 2v3\} + x\{1v3, 3v3\} + x\{2v2, 3v4\} \leq 2 & x\{1v3, 2v4\} + x\{1v1, 3v2\} + x\{2v5, 3v1\} \leq 2 \\
x\{1v1, 2v4\} + x\{1v3, 3v3\} + x\{2v2, 3v4\} \leq 2 & x\{1v3, 2v4\} + x\{1v1, 3v3\} + x\{2v5, 3v1\} \leq 2 \\
x\{1v1, 2v5\} + x\{1v3, 3v3\} + x\{2v2, 3v4\} \leq 2 & x\{1v3, 2v4\} + x\{1v1, 3v4\} + x\{2v5, 3v1\} \leq 2 \\
x\{1v1, 2v4\} + x\{1v3, 3v3\} + x\{2v3, 3v4\} \leq 2 & x\{1v3, 2v4\} + x\{1v1, 3v3\} + x\{2v5, 3v2\} \leq 2 \\
x\{1v1, 2v5\} + x\{1v3, 3v3\} + x\{2v3, 3v4\} \leq 2 & x\{1v3, 2v4\} + x\{1v1, 3v4\} + x\{2v5, 3v2\} \leq 2 \\
x\{1v1, 2v5\} + x\{1v3, 3v3\} + x\{2v4, 3v4\} \leq 2 & x\{1v3, 2v4\} + x\{1v1, 3v4\} + x\{2v5, 3v3\} \leq 2 \\
x\{1v2, 2v2\} + x\{1v3, 3v1\} + x\{2v1, 3v2\} \leq 2 & x\{1v3, 2v1\} + x\{1v2, 3v2\} + x\{2v2, 3v1\} \leq 2 \\
x\{1v2, 2v3\} + x\{1v3, 3v1\} + x\{2v1, 3v2\} \leq 2 & x\{1v3, 2v1\} + x\{1v2, 3v3\} + x\{2v2, 3v1\} \leq 2 \\
x\{1v2, 2v4\} + x\{1v3, 3v1\} + x\{2v1, 3v2\} \leq 2 & x\{1v3, 2v1\} + x\{1v2, 3v4\} + x\{2v2, 3v1\} \leq 2 \\
x\{1v2, 2v5\} + x\{1v3, 3v1\} + x\{2v1, 3v2\} \leq 2 & x\{1v3, 2v1\} + x\{1v2, 3v3\} + x\{2v2, 3v2\} \leq 2 \\
x\{1v2, 2v3\} + x\{1v3, 3v1\} + x\{2v2, 3v2\} \leq 2 & x\{1v3, 2v1\} + x\{1v2, 3v4\} + x\{2v2, 3v2\} \leq 2 \\
x\{1v2, 2v4\} + x\{1v3, 3v1\} + x\{2v2, 3v2\} \leq 2 & x\{1v3, 2v1\} + x\{1v2, 3v4\} + x\{2v2, 3v3\} \leq 2 \\
x\{1v2, 2v5\} + x\{1v3, 3v1\} + x\{2v2, 3v2\} \leq 2 & x\{1v3, 2v1\} + x\{1v2, 3v2\} + x\{2v3, 3v1\} \leq 2 \\
x\{1v2, 2v4\} + x\{1v3, 3v1\} + x\{2v3, 3v2\} \leq 2 & x\{1v3, 2v1\} + x\{1v2, 3v3\} + x\{2v3, 3v1\} \leq 2 \\
x\{1v2, 2v5\} + x\{1v3, 3v1\} + x\{2v3, 3v2\} \leq 2 & x\{1v3, 2v1\} + x\{1v2, 3v4\} + x\{2v3, 3v1\} \leq 2 \\
x\{1v2, 2v5\} + x\{1v3, 3v1\} + x\{2v4, 3v2\} \leq 2 & x\{1v3, 2v1\} + x\{1v2, 3v3\} + x\{2v3, 3v2\} \leq 2 \\
x\{1v2, 2v2\} + x\{1v3, 3v1\} + x\{2v1, 3v3\} \leq 2 & x\{1v3, 2v1\} + x\{1v2, 3v4\} + x\{2v3, 3v2\} \leq 2 \\
x\{1v2, 2v3\} + x\{1v3, 3v1\} + x\{2v1, 3v3\} \leq 2 & x\{1v3, 2v1\} + x\{1v2, 3v4\} + x\{2v3, 3v3\} \leq 2
\end{array}$$

$$\begin{aligned}
& x\{1v2, 2v3\} + x\{1v3, 3v2\} + x\{2v1, 3v4\} \leq 2 & x\{1v3, 2v2\} + x\{1v2, 3v4\} + x\{2v5, 3v3\} \leq 2 \\
& x\{1v2, 2v4\} + x\{1v3, 3v2\} + x\{2v1, 3v4\} \leq 2 & x\{1v3, 2v3\} + x\{1v2, 3v2\} + x\{2v4, 3v1\} \leq 2 \\
& x\{1v2, 2v5\} + x\{1v3, 3v2\} + x\{2v1, 3v4\} \leq 2 & x\{1v3, 2v3\} + x\{1v2, 3v3\} + x\{2v4, 3v1\} \leq 2 \\
& x\{1v2, 2v3\} + x\{1v3, 3v2\} + x\{2v2, 3v4\} \leq 2 & x\{1v3, 2v3\} + x\{1v2, 3v4\} + x\{2v4, 3v1\} \leq 2 \\
& x\{1v2, 2v4\} + x\{1v3, 3v2\} + x\{2v2, 3v4\} \leq 2 & x\{1v3, 2v3\} + x\{1v2, 3v3\} + x\{2v4, 3v2\} \leq 2 \\
& x\{1v2, 2v5\} + x\{1v3, 3v2\} + x\{2v2, 3v4\} \leq 2 & x\{1v3, 2v3\} + x\{1v2, 3v4\} + x\{2v4, 3v2\} \leq 2 \\
& x\{1v2, 2v4\} + x\{1v3, 3v2\} + x\{2v3, 3v4\} \leq 2 & x\{1v3, 2v3\} + x\{1v2, 3v4\} + x\{2v4, 3v3\} \leq 2 \\
& x\{1v2, 2v5\} + x\{1v3, 3v2\} + x\{2v3, 3v4\} \leq 2 & x\{1v3, 2v3\} + x\{1v2, 3v2\} + x\{2v5, 3v1\} \leq 2 \\
& x\{1v2, 2v5\} + x\{1v3, 3v2\} + x\{2v4, 3v4\} \leq 2 & x\{1v3, 2v3\} + x\{1v2, 3v3\} + x\{2v5, 3v1\} \leq 2 \\
& x\{1v2, 2v2\} + x\{1v3, 3v3\} + x\{2v1, 3v4\} \leq 2 & x\{1v3, 2v3\} + x\{1v2, 3v4\} + x\{2v5, 3v1\} \leq 2 \\
& x\{1v2, 2v3\} + x\{1v3, 3v3\} + x\{2v1, 3v4\} \leq 2 & x\{1v3, 2v3\} + x\{1v2, 3v3\} + x\{2v5, 3v2\} \leq 2 \\
& x\{1v2, 2v4\} + x\{1v3, 3v3\} + x\{2v1, 3v4\} \leq 2 & x\{1v3, 2v3\} + x\{1v2, 3v4\} + x\{2v5, 3v2\} \leq 2 \\
& x\{1v2, 2v5\} + x\{1v3, 3v3\} + x\{2v1, 3v4\} \leq 2 & x\{1v3, 2v3\} + x\{1v2, 3v4\} + x\{2v5, 3v3\} \leq 2 \\
& x\{1v2, 2v3\} + x\{1v3, 3v3\} + x\{2v2, 3v4\} \leq 2 & x\{1v3, 2v4\} + x\{1v2, 3v2\} + x\{2v5, 3v1\} \leq 2 \\
& x\{1v2, 2v4\} + x\{1v3, 3v3\} + x\{2v2, 3v4\} \leq 2 & x\{1v3, 2v4\} + x\{1v2, 3v3\} + x\{2v5, 3v1\} \leq 2 \\
& x\{1v2, 2v5\} + x\{1v3, 3v3\} + x\{2v2, 3v4\} \leq 2 & x\{1v3, 2v4\} + x\{1v2, 3v4\} + x\{2v5, 3v1\} \leq 2 \\
& x\{1v2, 2v4\} + x\{1v3, 3v3\} + x\{2v3, 3v4\} \leq 2 & x\{1v3, 2v4\} + x\{1v2, 3v3\} + x\{2v5, 3v2\} \leq 2 \\
& x\{1v2, 2v5\} + x\{1v3, 3v3\} + x\{2v3, 3v4\} \leq 2 & x\{1v3, 2v4\} + x\{1v2, 3v4\} + x\{2v5, 3v2\} \leq 2 \\
& x\{1v2, 2v5\} + x\{1v3, 3v3\} + x\{2v4, 3v4\} \leq 2 & x\{1v3, 2v4\} + x\{1v2, 3v4\} + x\{2v5, 3v3\} \leq 2
\end{aligned}$$

Appendix 6: The complete third type constraints for Example 2.1.

$$\begin{aligned}
& y(1v(1,1))^2 + y(1v(1,2))^2 + y(1v(1,3))^2 + y(1v(2,2))^2 + y(1v(2,3))^2 \leq 1 \\
& y(1v(1,2))^2 + y(1v(1,3))^2 + y(1v(2,2))^2 + y(1v(2,3))^2 + y(1v(3,3))^2 \leq 1 \\
& y(1v(1,1))^3 + y(1v(1,2))^3 + y(1v(1,3))^3 + y(1v(2,2))^3 + y(1v(2,3))^3 \leq 1 \\
& y(1v(1,2))^3 + y(1v(1,3))^3 + y(1v(2,2))^3 + y(1v(2,3))^3 + y(1v(3,3))^3 \leq 1 \\
& y(2v(1,1))^1 + y(2v(1,2))^1 + y(2v(1,3))^1 + y(2v(1,4))^1 + y(2v(1,5))^1 + y(2v(2,2))^1 + y(2v(2,3))^1 + y(2v(2,4))^1 + \\
& y(2v(2,5))^1 \leq 1 \\
& y(2v(1,2))^1 + y(2v(1,3))^1 + y(2v(1,4))^1 + y(2v(1,5))^1 + y(2v(2,2))^1 + y(2v(2,3))^1 + y(2v(2,4))^1 + y(2v(2,5))^1 + \\
& y(2v(3,3))^1 + y(2v(3,4))^1 + y(2v(3,5))^1 \leq 1
\end{aligned}$$

$$y(2v(1,3))1 + y(2v(1,4))1 + y(2v(1,5))1 + y(2v(2,3))1 + y(2v(2,4))1 + y(2v(2,5))1 + y(2v(3,3))1 + y(2v(3,4))1 + y(2v(3,5))1 + y(2v(4,4))1 + y(2v(4,5))1 \leq 1$$

$$y(2v(1,4))1 + y(2v(1,5))1 + y(2v(2,4))1 + y(2v(2,5))1 + y(2v(3,4))1 + y(2v(3,5))1 + y(2v(4,4))1 + y(2v(4,5))1 + y(2v(5,5))1 \leq 1$$

$$y(2v(1,1))3 + y(2v(1,2))3 + y(2v(1,3))3 + y(2v(1,4))3 + y(2v(1,5))3 + y(2v(2,2))3 + y(2v(2,3))3 + y(2v(2,4))3 + y(2v(2,5))3 \leq 1$$

$$y(2v(1,2))3 + y(2v(1,3))3 + y(2v(1,4))3 + y(2v(1,5))3 + y(2v(2,2))3 + y(2v(2,3))3 + y(2v(2,4))3 + y(2v(2,5))3 + y(2v(3,3))3 + y(2v(3,4))3 + y(2v(3,5))3 \leq 1$$

$$y(2v(1,3))3 + y(2v(1,4))3 + y(2v(1,5))3 + y(2v(2,3))3 + y(2v(2,4))3 + y(2v(2,5))3 + y(2v(3,3))3 + y(2v(3,4))3 + y(2v(3,5))3 + y(2v(4,4))3 + y(2v(4,5))3 \leq 1$$

$$y(2v(1,4))3 + y(2v(1,5))3 + y(2v(2,4))3 + y(2v(2,5))3 + y(2v(3,4))3 + y(2v(3,5))3 + y(2v(4,4))3 + y(2v(4,5))3 + y(2v(5,5))3 \leq 1$$

$$y(3v(1,1))1 + y(3v(1,2))1 + y(3v(1,3))1 + y(3v(1,4))1 + y(3v(2,2))1 + y(3v(2,3))1 + y(3v(2,4))1 \leq 1$$

$$y(3v(1,2))1 + y(3v(1,3))1 + y(3v(1,4))1 + y(3v(2,2))1 + y(3v(2,3))1 + y(3v(2,4))1 + y(3v(3,3))1 + y(3v(3,4))1 \leq 1$$

$$y(3v(1,3))1 + y(3v(1,4))1 + y(3v(2,3))1 + y(3v(2,4))1 + y(3v(3,3))1 + y(3v(3,4))1 + y(3v(4,4))1 \leq 1$$

$$y(3v(1,1))2 + y(3v(1,2))2 + y(3v(1,3))2 + y(3v(1,4))2 + y(3v(2,2))2 + y(3v(2,3))2 + y(3v(2,4))2 \leq 1$$

$$y(3v(1,2))2 + y(3v(1,3))2 + y(3v(1,4))2 + y(3v(2,2))2 + y(3v(2,3))2 + y(3v(2,4))2 + y(3v(3,3))2 + y(3v(3,4))2 \leq 1$$

$$y(3v(1,3))2 + y(3v(1,4))2 + y(3v(2,3))2 + y(3v(2,4))2 + y(3v(3,3))2 + y(3v(3,4))2 + y(3v(4,4))2 \leq 1$$

Appendix 7: The complete fourth type constraints for Example 2.1.

$$x\{1v1, 2v1\} - x\{1v1, 3v1\} + x\{2v1, 3v1\} \leq 1 \quad -x\{1v1, 2v3\} + x\{1v1, 3v3\} + x\{2v3, 3v3\} \leq 1$$

$$x\{1v1, 2v1\} - x\{1v1, 3v2\} + x\{2v1, 3v2\} \leq 1 \quad -x\{1v2, 2v3\} + x\{1v2, 3v3\} + x\{2v3, 3v3\} \leq 1$$

$$x\{1v1, 2v1\} - x\{1v1, 3v3\} + x\{2v1, 3v3\} \leq 1 \quad -x\{1v3, 2v3\} + x\{1v3, 3v3\} + x\{2v3, 3v3\} \leq 1$$

$$x\{1v1, 2v1\} - x\{1v1, 3v4\} + x\{2v1, 3v4\} \leq 1 \quad -x\{1v1, 2v3\} + x\{1v1, 3v4\} + x\{2v3, 3v4\} \leq 1$$

$$x\{1v1, 2v2\} - x\{1v1, 3v1\} + x\{2v2, 3v1\} \leq 1 \quad -x\{1v2, 2v3\} + x\{1v2, 3v4\} + x\{2v3, 3v4\} \leq 1$$

$$x\{1v1, 2v2\} - x\{1v1, 3v2\} + x\{2v2, 3v2\} \leq 1 \quad -x\{1v3, 2v3\} + x\{1v3, 3v4\} + x\{2v3, 3v4\} \leq 1$$

$$x\{1v1, 2v2\} - x\{1v1, 3v3\} + x\{2v2, 3v3\} \leq 1 \quad -x\{1v1, 2v4\} + x\{1v1, 3v1\} + x\{2v4, 3v1\} \leq 1$$

$$x\{1v1, 2v2\} - x\{1v1, 3v4\} + x\{2v2, 3v4\} \leq 1 \quad -x\{1v2, 2v4\} + x\{1v2, 3v1\} + x\{2v4, 3v1\} \leq 1$$

$$x\{1v1, 2v3\} - x\{1v1, 3v1\} + x\{2v3, 3v1\} \leq 1 \quad -x\{1v3, 2v4\} + x\{1v3, 3v1\} + x\{2v4, 3v1\} \leq 1$$

$$x\{1v1, 2v3\} - x\{1v1, 3v2\} + x\{2v3, 3v2\} \leq 1 \quad -x\{1v1, 2v4\} + x\{1v1, 3v2\} + x\{2v4, 3v2\} \leq 1$$

$$x\{1v1, 2v3\} - x\{1v1, 3v3\} + x\{2v3, 3v3\} \leq 1 \quad -x\{1v2, 2v4\} + x\{1v2, 3v2\} + x\{2v4, 3v2\} \leq 1$$

$$\begin{array}{ll}
x\{1v1, 2v3\} - x\{1v1, 3v4\} + x\{2v3, 3v4\} \leq 1 & -x\{1v3, 2v4\} + x\{1v3, 3v2\} + x\{2v4, 3v2\} \leq 1 \\
x\{1v1, 2v4\} - x\{1v1, 3v1\} + x\{2v4, 3v1\} \leq 1 & -x\{1v1, 2v4\} + x\{1v1, 3v3\} + x\{2v4, 3v3\} \leq 1 \\
x\{1v1, 2v4\} - x\{1v1, 3v2\} + x\{2v4, 3v2\} \leq 1 & -x\{1v2, 2v4\} + x\{1v2, 3v3\} + x\{2v4, 3v3\} \leq 1 \\
x\{1v1, 2v4\} - x\{1v1, 3v3\} + x\{2v4, 3v3\} \leq 1 & -x\{1v3, 2v4\} + x\{1v3, 3v3\} + x\{2v4, 3v3\} \leq 1 \\
x\{1v1, 2v4\} - x\{1v1, 3v4\} + x\{2v4, 3v4\} \leq 1 & -x\{1v1, 2v4\} + x\{1v1, 3v4\} + x\{2v4, 3v4\} \leq 1 \\
x\{1v1, 2v5\} - x\{1v1, 3v1\} + x\{2v5, 3v1\} \leq 1 & -x\{1v2, 2v4\} + x\{1v2, 3v4\} + x\{2v4, 3v4\} \leq 1 \\
x\{1v1, 2v5\} - x\{1v1, 3v2\} + x\{2v5, 3v2\} \leq 1 & -x\{1v3, 2v4\} + x\{1v3, 3v4\} + x\{2v4, 3v4\} \leq 1 \\
x\{1v1, 2v5\} - x\{1v1, 3v3\} + x\{2v5, 3v3\} \leq 1 & -x\{1v1, 2v5\} + x\{1v1, 3v1\} + x\{2v5, 3v1\} \leq 1 \\
x\{1v1, 2v5\} - x\{1v1, 3v4\} + x\{2v5, 3v4\} \leq 1 & -x\{1v2, 2v5\} + x\{1v2, 3v1\} + x\{2v5, 3v1\} \leq 1 \\
x\{1v2, 2v1\} - x\{1v2, 3v1\} + x\{2v1, 3v1\} \leq 1 & -x\{1v3, 2v5\} + x\{1v3, 3v1\} + x\{2v5, 3v1\} \leq 1 \\
x\{1v2, 2v1\} - x\{1v2, 3v2\} + x\{2v1, 3v2\} \leq 1 & -x\{1v1, 2v5\} + x\{1v1, 3v2\} + x\{2v5, 3v2\} \leq 1 \\
x\{1v2, 2v1\} - x\{1v2, 3v3\} + x\{2v1, 3v3\} \leq 1 & -x\{1v2, 2v5\} + x\{1v2, 3v2\} + x\{2v5, 3v2\} \leq 1 \\
x\{1v2, 2v1\} - x\{1v2, 3v4\} + x\{2v1, 3v4\} \leq 1 & -x\{1v3, 2v5\} + x\{1v3, 3v2\} + x\{2v5, 3v2\} \leq 1 \\
x\{1v2, 2v2\} - x\{1v2, 3v1\} + x\{2v2, 3v1\} \leq 1 & -x\{1v1, 2v5\} + x\{1v1, 3v3\} + x\{2v5, 3v3\} \leq 1 \\
x\{1v2, 2v2\} - x\{1v2, 3v2\} + x\{2v2, 3v2\} \leq 1 & -x\{1v2, 2v5\} + x\{1v2, 3v3\} + x\{2v5, 3v3\} \leq 1 \\
x\{1v2, 2v2\} - x\{1v2, 3v3\} + x\{2v2, 3v3\} \leq 1 & -x\{1v3, 2v5\} + x\{1v3, 3v3\} + x\{2v5, 3v3\} \leq 1 \\
x\{1v2, 2v2\} - x\{1v2, 3v4\} + x\{2v2, 3v4\} \leq 1 & -x\{1v1, 2v5\} + x\{1v1, 3v4\} + x\{2v5, 3v4\} \leq 1 \\
x\{1v2, 2v3\} - x\{1v2, 3v1\} + x\{2v3, 3v1\} \leq 1 & -x\{1v2, 2v5\} + x\{1v2, 3v4\} + x\{2v5, 3v4\} \leq 1 \\
x\{1v2, 2v3\} - x\{1v2, 3v2\} + x\{2v3, 3v2\} \leq 1 & -x\{1v3, 2v5\} + x\{1v3, 3v4\} + x\{2v5, 3v4\} \leq 1 \\
x\{1v2, 2v3\} - x\{1v2, 3v3\} + x\{2v3, 3v3\} \leq 1 & x\{1v1, 2v1\} + x\{1v1, 3v1\} - x\{2v1, 3v1\} \leq 1 \\
x\{1v2, 2v3\} - x\{1v2, 3v4\} + x\{2v3, 3v4\} \leq 1 & x\{1v1, 2v2\} + x\{1v1, 3v1\} - x\{2v2, 3v1\} \leq 1 \\
x\{1v2, 2v4\} - x\{1v2, 3v1\} + x\{2v4, 3v1\} \leq 1 & x\{1v1, 2v3\} + x\{1v1, 3v1\} - x\{2v3, 3v1\} \leq 1 \\
x\{1v2, 2v4\} - x\{1v2, 3v2\} + x\{2v4, 3v2\} \leq 1 & x\{1v1, 2v4\} + x\{1v1, 3v1\} - x\{2v4, 3v1\} \leq 1 \\
x\{1v2, 2v4\} - x\{1v2, 3v3\} + x\{2v4, 3v3\} \leq 1 & x\{1v1, 2v5\} + x\{1v1, 3v1\} - x\{2v5, 3v1\} \leq 1 \\
x\{1v2, 2v4\} - x\{1v2, 3v4\} + x\{2v4, 3v4\} \leq 1 & x\{1v2, 2v1\} + x\{1v2, 3v1\} - x\{2v1, 3v1\} \leq 1 \\
x\{1v2, 2v5\} - x\{1v2, 3v1\} + x\{2v5, 3v1\} \leq 1 & x\{1v2, 2v2\} + x\{1v2, 3v1\} - x\{2v2, 3v1\} \leq 1 \\
x\{1v2, 2v5\} - x\{1v2, 3v2\} + x\{2v5, 3v2\} \leq 1 & x\{1v2, 2v3\} + x\{1v2, 3v1\} - x\{2v3, 3v1\} \leq 1 \\
x\{1v2, 2v5\} - x\{1v2, 3v3\} + x\{2v5, 3v3\} \leq 1 & x\{1v2, 2v4\} + x\{1v2, 3v1\} - x\{2v4, 3v1\} \leq 1
\end{array}$$

$$\begin{array}{ll}
x\{1v2, 2v5\} - x\{1v2, 3v4\} + x\{2v5, 3v4\} \leq 1 & x\{1v2, 2v5\} + x\{1v2, 3v1\} - x\{2v5, 3v1\} \leq 1 \\
x\{1v3, 2v1\} - x\{1v3, 3v1\} + x\{2v1, 3v1\} \leq 1 & x\{1v3, 2v1\} + x\{1v3, 3v1\} - x\{2v1, 3v1\} \leq 1 \\
x\{1v3, 2v1\} - x\{1v3, 3v2\} + x\{2v1, 3v2\} \leq 1 & x\{1v3, 2v2\} + x\{1v3, 3v1\} - x\{2v2, 3v1\} \leq 1 \\
x\{1v3, 2v1\} - x\{1v3, 3v3\} + x\{2v1, 3v3\} \leq 1 & x\{1v3, 2v3\} + x\{1v3, 3v1\} - x\{2v3, 3v1\} \leq 1 \\
x\{1v3, 2v1\} - x\{1v3, 3v4\} + x\{2v1, 3v4\} \leq 1 & x\{1v3, 2v4\} + x\{1v3, 3v1\} - x\{2v4, 3v1\} \leq 1 \\
x\{1v3, 2v2\} - x\{1v3, 3v1\} + x\{2v2, 3v1\} \leq 1 & x\{1v3, 2v5\} + x\{1v3, 3v1\} - x\{2v5, 3v1\} \leq 1 \\
x\{1v3, 2v2\} - x\{1v3, 3v2\} + x\{2v2, 3v2\} \leq 1 & x\{1v1, 2v1\} + x\{1v1, 3v2\} - x\{2v1, 3v2\} \leq 1 \\
x\{1v3, 2v2\} - x\{1v3, 3v3\} + x\{2v2, 3v3\} \leq 1 & x\{1v1, 2v2\} + x\{1v1, 3v2\} - x\{2v2, 3v2\} \leq 1 \\
x\{1v3, 2v2\} - x\{1v3, 3v4\} + x\{2v2, 3v4\} \leq 1 & x\{1v1, 2v3\} + x\{1v1, 3v2\} - x\{2v3, 3v2\} \leq 1 \\
x\{1v3, 2v3\} - x\{1v3, 3v1\} + x\{2v3, 3v1\} \leq 1 & x\{1v1, 2v4\} + x\{1v1, 3v2\} - x\{2v4, 3v2\} \leq 1 \\
x\{1v3, 2v3\} - x\{1v3, 3v2\} + x\{2v3, 3v2\} \leq 1 & x\{1v1, 2v5\} + x\{1v1, 3v2\} - x\{2v5, 3v2\} \leq 1 \\
x\{1v3, 2v3\} - x\{1v3, 3v3\} + x\{2v3, 3v3\} \leq 1 & x\{1v2, 2v1\} + x\{1v2, 3v2\} - x\{2v1, 3v2\} \leq 1 \\
x\{1v3, 2v3\} - x\{1v3, 3v4\} + x\{2v3, 3v4\} \leq 1 & x\{1v2, 2v2\} + x\{1v2, 3v2\} - x\{2v2, 3v2\} \leq 1 \\
x\{1v3, 2v4\} - x\{1v3, 3v1\} + x\{2v4, 3v1\} \leq 1 & x\{1v2, 2v3\} + x\{1v2, 3v2\} - x\{2v3, 3v2\} \leq 1 \\
x\{1v3, 2v4\} - x\{1v3, 3v2\} + x\{2v4, 3v2\} \leq 1 & x\{1v2, 2v4\} + x\{1v2, 3v2\} - x\{2v4, 3v2\} \leq 1 \\
x\{1v3, 2v4\} - x\{1v3, 3v3\} + x\{2v4, 3v3\} \leq 1 & x\{1v2, 2v5\} + x\{1v2, 3v2\} - x\{2v5, 3v2\} \leq 1 \\
x\{1v3, 2v4\} - x\{1v3, 3v4\} + x\{2v4, 3v4\} \leq 1 & x\{1v3, 2v1\} + x\{1v3, 3v2\} - x\{2v1, 3v2\} \leq 1 \\
x\{1v3, 2v5\} - x\{1v3, 3v1\} + x\{2v5, 3v1\} \leq 1 & x\{1v3, 2v2\} + x\{1v3, 3v2\} - x\{2v2, 3v2\} \leq 1 \\
x\{1v3, 2v5\} - x\{1v3, 3v2\} + x\{2v5, 3v2\} \leq 1 & x\{1v3, 2v3\} + x\{1v3, 3v2\} - x\{2v3, 3v2\} \leq 1 \\
x\{1v3, 2v5\} - x\{1v3, 3v3\} + x\{2v5, 3v3\} \leq 1 & x\{1v3, 2v4\} + x\{1v3, 3v2\} - x\{2v4, 3v2\} \leq 1 \\
x\{1v3, 2v5\} - x\{1v3, 3v4\} + x\{2v5, 3v4\} \leq 1 & x\{1v3, 2v5\} + x\{1v3, 3v2\} - x\{2v5, 3v2\} \leq 1 \\
-x\{1v1, 2v1\} + x\{1v1, 3v1\} + x\{2v1, 3v1\} \leq 1 & x\{1v1, 2v1\} + x\{1v1, 3v3\} - x\{2v1, 3v3\} \leq 1 \\
-x\{1v2, 2v1\} + x\{1v2, 3v1\} + x\{2v1, 3v1\} \leq 1 & x\{1v1, 2v2\} + x\{1v1, 3v3\} - x\{2v2, 3v3\} \leq 1 \\
-x\{1v3, 2v1\} + x\{1v3, 3v1\} + x\{2v1, 3v1\} \leq 1 & x\{1v1, 2v3\} + x\{1v1, 3v3\} - x\{2v3, 3v3\} \leq 1 \\
-x\{1v1, 2v1\} + x\{1v1, 3v2\} + x\{2v1, 3v2\} \leq 1 & x\{1v1, 2v4\} + x\{1v1, 3v3\} - x\{2v4, 3v3\} \leq 1 \\
-x\{1v2, 2v1\} + x\{1v2, 3v2\} + x\{2v1, 3v2\} \leq 1 & x\{1v1, 2v5\} + x\{1v1, 3v3\} - x\{2v5, 3v3\} \leq 1 \\
-x\{1v3, 2v1\} + x\{1v3, 3v2\} + x\{2v1, 3v2\} \leq 1 & x\{1v2, 2v1\} + x\{1v2, 3v3\} - x\{2v1, 3v3\} \leq 1 \\
-x\{1v1, 2v1\} + x\{1v1, 3v3\} + x\{2v1, 3v3\} \leq 1 & x\{1v2, 2v2\} + x\{1v2, 3v3\} - x\{2v2, 3v3\} \leq 1 \\
-x\{1v2, 2v1\} + x\{1v2, 3v3\} + x\{2v1, 3v3\} \leq 1 & x\{1v2, 2v3\} + x\{1v2, 3v3\} - x\{2v3, 3v3\} \leq 1
\end{array}$$

$$\begin{array}{ll}
-x\{1v3, 2v1\} + x\{1v3, 3v3\} + x\{2v1, 3v3\} \leq 1 & x\{1v2, 2v4\} + x\{1v2, 3v3\} - x\{2v4, 3v3\} \leq 1 \\
-x\{1v1, 2v1\} + x\{1v1, 3v4\} + x\{2v1, 3v4\} \leq 1 & x\{1v2, 2v5\} + x\{1v2, 3v3\} - x\{2v5, 3v3\} \leq 1 \\
-x\{1v2, 2v1\} + x\{1v2, 3v4\} + x\{2v1, 3v4\} \leq 1 & x\{1v3, 2v1\} + x\{1v3, 3v3\} - x\{2v1, 3v3\} \leq 1 \\
-x\{1v3, 2v1\} + x\{1v3, 3v4\} + x\{2v1, 3v4\} \leq 1 & x\{1v3, 2v2\} + x\{1v3, 3v3\} - x\{2v2, 3v3\} \leq 1 \\
-x\{1v1, 2v2\} + x\{1v1, 3v1\} + x\{2v2, 3v1\} \leq 1 & x\{1v3, 2v3\} + x\{1v3, 3v3\} - x\{2v3, 3v3\} \leq 1 \\
-x\{1v2, 2v2\} + x\{1v2, 3v1\} + x\{2v2, 3v1\} \leq 1 & x\{1v3, 2v4\} + x\{1v3, 3v3\} - x\{2v4, 3v3\} \leq 1 \\
-x\{1v3, 2v2\} + x\{1v3, 3v1\} + x\{2v2, 3v1\} \leq 1 & x\{1v3, 2v5\} + x\{1v3, 3v3\} - x\{2v5, 3v3\} \leq 1 \\
-x\{1v1, 2v2\} + x\{1v1, 3v2\} + x\{2v2, 3v2\} \leq 1 & x\{1v1, 2v1\} + x\{1v1, 3v4\} - x\{2v1, 3v4\} \leq 1 \\
-x\{1v2, 2v2\} + x\{1v2, 3v2\} + x\{2v2, 3v2\} \leq 1 & x\{1v1, 2v2\} + x\{1v1, 3v4\} - x\{2v2, 3v4\} \leq 1 \\
-x\{1v3, 2v2\} + x\{1v3, 3v2\} + x\{2v2, 3v2\} \leq 1 & x\{1v1, 2v3\} + x\{1v1, 3v4\} - x\{2v3, 3v4\} \leq 1 \\
-x\{1v1, 2v2\} + x\{1v1, 3v3\} + x\{2v2, 3v3\} \leq 1 & x\{1v1, 2v4\} + x\{1v1, 3v4\} - x\{2v4, 3v4\} \leq 1 \\
-x\{1v2, 2v2\} + x\{1v2, 3v3\} + x\{2v2, 3v3\} \leq 1 & x\{1v1, 2v5\} + x\{1v1, 3v4\} - x\{2v5, 3v4\} \leq 1 \\
-x\{1v3, 2v2\} + x\{1v3, 3v3\} + x\{2v2, 3v3\} \leq 1 & x\{1v2, 2v1\} + x\{1v2, 3v4\} - x\{2v1, 3v4\} \leq 1 \\
-x\{1v1, 2v2\} + x\{1v1, 3v4\} + x\{2v2, 3v4\} \leq 1 & x\{1v2, 2v2\} + x\{1v2, 3v4\} - x\{2v2, 3v4\} \leq 1 \\
-x\{1v2, 2v2\} + x\{1v2, 3v4\} + x\{2v2, 3v4\} \leq 1 & x\{1v2, 2v3\} + x\{1v2, 3v4\} - x\{2v3, 3v4\} \leq 1 \\
-x\{1v3, 2v2\} + x\{1v3, 3v4\} + x\{2v2, 3v4\} \leq 1 & x\{1v2, 2v4\} + x\{1v2, 3v4\} - x\{2v4, 3v4\} \leq 1 \\
-x\{1v1, 2v3\} + x\{1v1, 3v1\} + x\{2v3, 3v1\} \leq 1 & x\{1v2, 2v5\} + x\{1v2, 3v4\} - x\{2v5, 3v4\} \leq 1 \\
-x\{1v2, 2v3\} + x\{1v2, 3v1\} + x\{2v3, 3v1\} \leq 1 & x\{1v3, 2v1\} + x\{1v3, 3v4\} - x\{2v1, 3v4\} \leq 1 \\
-x\{1v3, 2v3\} + x\{1v3, 3v1\} + x\{2v3, 3v1\} \leq 1 & x\{1v3, 2v2\} + x\{1v3, 3v4\} - x\{2v2, 3v4\} \leq 1 \\
-x\{1v1, 2v3\} + x\{1v1, 3v2\} + x\{2v3, 3v2\} \leq 1 & x\{1v3, 2v3\} + x\{1v3, 3v4\} - x\{2v3, 3v4\} \leq 1 \\
-x\{1v2, 2v3\} + x\{1v2, 3v2\} + x\{2v3, 3v2\} \leq 1 & x\{1v3, 2v4\} + x\{1v3, 3v4\} - x\{2v4, 3v4\} \leq 1 \\
-x\{1v3, 2v3\} + x\{1v3, 3v2\} + x\{2v3, 3v2\} \leq 1 & x\{1v3, 2v5\} + x\{1v3, 3v4\} - x\{2v5, 3v4\} \leq 1
\end{array}$$

Appendix 8: ILP solution of Example 2.1.

The optimal solution of Example 2.1:

$x =$

Columns 1 through 18

0 0 0 0 0 0 1 0 0 0 0 0 1 0 0 1 0 0


```

end
c1 = 1;
case '2'
    path = input('Input the path of the file : ','s');
    seq = textread(path, '%s');
    c1 = 1;
case '3'
    fprintf('\nproblem 1\n');
    temp = textread('Sequence Database\Seq1.txt', '%s');
    for i = 1:size(temp,1)
        fprintf(' %s\n',temp{i});
    end
    fprintf('\nproblem 2\n');
    temp = textread('Sequence Database\Seq2.txt', '%s');
    for i = 1:size(temp,1)
        fprintf(' %s\n',temp{i});
    end
    fprintf('\nproblem 3\n');
    temp = textread('Sequence Database\Seq3.txt', '%s');
    for i = 1:size(temp,1)
        fprintf(' %s\n',temp{i});
    end
    c2 = 0;
    while c2 == 0;
        choice2 = input('Which problem do you want to solve? ','s');
        if isempty(choice2)
            choice = '1';
        end
        switch choice2
            case '1'
                seq = textread('Sequence Database\Seq1.txt', '%s'); c2 = 1;
            case '2'
                seq = textread('Sequence Database\Seq2.txt', '%s'); c2 = 1;
            case '3'
                seq = textread('Sequence Database\Seq3.txt', '%s'); c2 = 1;
            otherwise
                fprintf('Input choice not recognized.\n\n');
            end
        end
    end
    c1 = 1;
otherwise
    fprintf('Input choice not recognized.\n\n');
end
end

k = length(seq);

TAE = sum(arrayfun(@(x) length(seq{x})*sum(cellfun(@length,seq(x + 1:k))),1:k -
1));
TGA = (length(seq) - 1)*sum(cellfun(@(y) sum(1:length(y)),seq));

[dictx dicty] = transdict(seq,k);

f = obj_func(seq,TAE,TGA,dictx,dicty);
Aeq = first_con(seq,TAE,TGA,dictx,dicty);
beq = ones(size(Aeq,1),1);
A1 = second_con(seq,TAE,TGA,dictx);
b1 = zeros(size(A1,1),1); c = 1;
for i = 1:size(A1,1)
    e = 0;
    for j = 1:TAE
        if A1(i,j) == 1
            e = e + 1;
        end
    end
    b1(c) = (e - 1);
    c = c + 1;
end
end

```

```

A2 = third_con(seq,TAE,TGA,dicty);
b2 = ones(size(A2,1),1);
if size(seq,1) >=3
    A3 = fourth_con(seq,TAE,TGA,dictx);
    b3 = ones(size(A3,1),1);
    A = [A1; A2; A3];
    b = [b1; b2; b3];
else
    A = [A1; A2];
    b = [b1; b2];
end

clear functions
options = optimset('MaxTime',12000000);
[x,fval,exitflag,output] = bintprog(-f,A,b,Aeq,beq,[],options);
fval = -fval;

clc;
fprintf('MSA - ILP PROGRAM\n\n');
disp(output);
for i = 1:(TAE + TGA)
    if i == 1
        fprintf('Realized alignment edges:\n');
    elseif i == TAE + 1
        fprintf('\nRealized gap arcs:\n');
    end
    if x(i) == 1
        if i <= TAE
            fprintf('    {%dv%d, %dv%d}\n', translator(i,'x',dictx,seq,TAE));
        else
            fprintf('    (%dv(%d,%d), %d)\n', translator(i,'y',dicty,seq,TAE));
        end
    end
end

[result1 result2] = interpreter(x, seq, TAE, dictx, dicty);

fprintf('\nThe DNA MSA sequences :\n');
for i = 1:size(seq,1)
    fprintf(' %s\n',seq{i});
end

fprintf('\nThe aligned sequences :\n');
for i = 1:size(result1,1)
    fprintf(' %s\n',result1(i,:));
end

r = input('Do you want to try another alignment? y/n [y]: ', 's');
if isempty(r)
    r = 'y';
end

end
clc;
fprintf('MSA - ILP PROGRAM\n\nby Pudiahwai Anton Wibowo (0303010281)\n');

```

transdict.m:

```

function [result1 result2] = transdict(seq, k)

% transdict builds two helpers for the translator function
result1 = nchoosek(1:k,2);
result1 = [result1 zeros(size(result1,1),1)];
for i = 2:size(result1,1)
    temp = result1(i - 1,3) + length(seq{result1(i - 1,1)})*length(seq{result1(i - 1,2)});
end

```

```

    result1(i,3) = temp;
end

result2 = zeros(k*(k - 1),2); c = 1;
for i = 1:k
    seq_array = 1:k;
    seq_array(i) = [];
    for j = 1:k - 1
        result2(c,:) = [i seq_array(j)];
        c = c + 1;
    end
end
result2 = [result2 zeros(size(result2,1),1)];
for i = 2:size(result2,1)
    temp = result2(i - 1,3) + sum(1:length(seq{result2(i - 1,1)}));
    result2(i,3) = temp;
end

```

translator.m:

```

function result = translator(input, mode, dict, seq, TAE)
% translator translates the input both array index and variable index
if length(input) == 1
    if mode == 'x'
        for i = size(dict,1):-1:1
            if input > dict(i,3)
                input = input - dict(i,3);
                break
            end
        end
        if mod(input,length(seq{dict(i,2)})) == 0
            m = length(seq{dict(i,2)});
        else
            m = mod(input,length(seq{dict(i,2)}));
        end
        result = [dict(i,1) ceil(input/length(seq{dict(i,2)})) dict(i,2) m];
    elseif mode == 'y'
        input = input - TAE;
        for i = size(dict,1):-1:1
            if input > dict(i,3)
                input = input - dict(i,3);
                break
            end
        end
        l = 1;
        for j = length(seq{dict(i,1)}):-1:1
            if input > j
                input = input - j;
                l = l + 1;
            else
                break
            end
        end
        array = 1:length(seq{dict(i,1)});
        m = array(input);
        result = [dict(i,1) l m dict(i,2)];
    else
        fprintf('Input mode not recognized.\n')
    end
elseif length(input) == 4

```

```

if mode == 'x'
    for i = 1:size(dict,1)
        if input(1) == dict(i,1) && input(3) == dict(i,2)
            result = dict(i,3);
            break
        end
    end
    result = result + (input(2) - 1)*length(seq{input(3)}) + input(4);

elseif mode == 'y'
    for i = 1:size(dict,1)
        if input(1) == dict(i,1) && input(4) == dict(i,2)
            result = dict(i,3);
            break
        end
    end
    array = input(2):length(seq{dict(i,1)});
    for m = 1:length(array)
        if input(3) == array(m)
            break
        end
    end
    result = result + sum(length(seq{input(1)}):-1:length(seq{input(1)}) -
input(2) + 2) + m + TAE;

else
    fprintf('Input mode not recognized.\n')
end

else
    fprintf('Input not recognized.\n')
end
end

```

o.m:

```

function result = obj_func(seq, TAE, TGA, dictx, dicty)

% obj_func builds the objective function
f = inline('a + b*log(1)'); a = 0; b = 1;
result = zeros(TAE + TGA,1);
l = max(arrayfun(@(x) length(seq{x}),1:size(seq,1)));

for i = 1:TAE + TGA
    if i <= TAE
        var = translator(i,'x',dictx,seq,TAE);
        if seq{var(1)}(var(2)) == seq{var(3)}(var(4))
            result(i) = f(a,b,(l + 1));
        else
            result(i) = -1;
        end
    else
        var = translator(i,'y',dicty,seq,TAE);
        result(i) = f(a,b,(var(3) - var(2) + 1));
    end
end
end

```

first_con.m:

```

function result = first_con(seq, TAE, TGA, dictx, dicty)

% first_con builds the left side of the first constraint type

```

```

result = zeros(sum(arrayfun(@(x) (size(seq,1) - 1)*length(seq{x}),1:size(seq,1))),TAE
+ TGA);
c = 1;
for a = 1:size(dicty,1)
    i = dicty(a,1);
    j = dicty(a,2);
    for l = 1:length(seq{i})
        array = zeros(1,TAE + TGA);
        for m = 1:length(seq{j})
            if i < j
                temp = translator([i l j m], 'x', dictx, seq, TAE);
            else
                temp = translator([j m i l], 'x', dictx, seq, TAE);
            end
            array(temp) = 1;
        end
        for p = 1:l
            for q = 1:length(seq{i})
                temp = translator([i p q j], 'y', dicty, seq, TAE);
                array(temp) = 1;
            end
        end
        result(c,:) = array;
        c = c + 1;
    end
end

```

second_con.m:

```

function result = second_con(seq, TAE, TGA, dictx)

% second_con builds the left side of the second constraint type
seq_com = cell(size(seq,2) - 1,1);
seq_com{1,1} = nchoosek(1:size(seq,1),2);
for i = 3:size(seq,1)
    temp1 = nchoosek(1:size(seq,1),i);
    temp3 = zeros(factorial(size(temp1,2) - 1),size(temp1,2)); c = 1;
    for j = 1:size(temp1,1)
        temp2 = perms(temp1(j,2:size(temp1,2)));
        temp3((c:c + size(temp2,1) - 1),:) = [temp1(j,1)*ones(size(temp2,1),1) temp2];
        c = c + size(temp2,1);
    end
    seq_com{i - 1,1} = temp3;
end

position_arc_set = cell(1,size(seq,1));
for n = 1:size(seq,1)
    temp = zeros(sum(1:(length(seq{n}) - 1)),2); c = 1;
    for i = 1:length(seq{n})
        for j = i + 1:length(seq{n})
            temp(c,:) = [i j];
            c = c + 1;
        end
    end
    position_arc_set{1,n} = temp;
end

n = 0;
for i = 1:size(seq_com,1)
    for j = 1:size(seq_com{i})
        n = n + prod(arrayfun(@(x) sum(1:(length(seq{x}) - 1)),seq_com{i}(j,:)));
    end
end
result = zeros(n,TAE + TGA); c = 1;
for i = 1:size(seq_com,1)

```

```

for j = 1:size(seq_com{i},1)
    %seq_com{i}(j,:)
    temp1 = cycle_build(seq_com{i}(j,:),position_arc_set);
    for k = 1:size(temp1,1)
        temp2 = cycle_read(seq_com{i}(j,:), temp1(k,:), seq, TAE, TGA, dictx);
        result((c:c + size(temp2,1) - 1),:) = temp2;
        c = c + size(temp2,1);
    end
end
end

function result = cycle_build(seq_set, position_arc_set)

new_p_arc_set = cell(1,size(seq_set,2));
for i = 1:size(seq_set,2)
    new_p_arc_set{1,i} = position_arc_set{1,seq_set(i)};
end

result = zeros(1,2*size(seq_set,2));
for i = 1:size(seq_set,2)
    c = 1; c1 = 1; c2 = 1;
    for i1 = 1:i - 1
        c1 = c1*size(new_p_arc_set{i1},1);
    end
    for i2 = i + 1:size(seq_set,2)
        c2 = c2*size(new_p_arc_set{i2},1);
    end
    for j = 1:c1
        for k = 1:size(new_p_arc_set{i},1)
            for l = 1:c2
                result(c,[(2*i - 1) 2*i]) = new_p_arc_set{i}(k,1:2);
                c = c + 1;
            end
        end
    end
end
end

function result = cycle_read(seq_set, cyclearray, seq, TAE, TGA, dictx)

result = zeros(1,TAE + TGA);
for k = 1:size(seq_set,2)
    if k == size(seq_set,2)
        i = seq_set(k);
        l = cyclearray(2*k);
        j = seq_set(1);
        m = cyclearray(1);
    else
        i = seq_set(k);
        l = cyclearray(2*k);
        j = seq_set(k + 1);
        m = cyclearray(2*k + 1);
    end
    if i < j
        temp = translator([i l j m], 'x',dictx,seq,TAE);
    else
        temp = translator([j m i l], 'x',dictx,seq,TAE);
    end
    result(1,temp) = 1;
end
end

```

third_con.m:

```

function result = third_con(seq, TAE, TGA, dicty)

% third_con builds the left side of the third constraint type

```

```

result = zeros(sum((size(seq,1) - 1)*arrayfun(@(x) length(seq{x}),1:size(seq,1))),TAE
+ TGA);
c = 1;
for a = 1:size(dicty,1)
    i = dicty(a,1);
    j = dicty(a,2);
    for l = 1:length(seq{i})
        array = zeros(1,TAE + TGA);
        if l < length(seq{i})
            n = 1;
        else
            n = 0;
        end
        for p = 1:(1 + n)
            for q = 1:length(seq{i})
                if p <= q
                    temp = translator([i p q j], 'y', dicty, seq, TAE);
                else
                    temp = translator([i q p j], 'y', dicty, seq, TAE);
                end
                array(temp) = 1;
            end
        end
        result(c,:) = array;
        c = c + 1;
    end
end

```

fourth_con.m:

```

function result = fourth_con(seq, TAE, TGA, dictx)

% fourth_con builds the left side of the fourth constraint type
seq_com = nchoosek(1:size(seq,1),3);
all_seq_perm = zeros(6*size(seq_com,1),3); c = 1;
for i = 1:size(seq_com,1)
    all_seq_perm((c:c + 5),:) = perms(seq_com(i,:));
    c = c + 6;
end

n = 0;
for i = 1:size(all_seq_perm,1)
    n = n + prod(arrayfun(@(x) length(seq{x}),all_seq_perm(i,:)));
end
result = zeros(n,TAE + TGA);
c = 1;
for i = 1:size(all_seq_perm,1)
    seq_set = all_seq_perm(i,:);
    for j1 = 1:length(seq{seq_set(1)})
        for j2 = 1:length(seq{seq_set(2)})
            for j3 = 1:length(seq{seq_set(3)})
                array = zeros(1,TAE + TGA);
                if seq_set(1) < seq_set(2)
                    temp = translator([seq_set(1) j1 seq_set(2) j2],
'x',dictx,seq,TAE);
                else
                    temp = translator([seq_set(2) j2 seq_set(1) j1],
'x',dictx,seq,TAE);
                end
                array(temp) = 1;
                if seq_set(2) < seq_set(3)
                    temp = translator([seq_set(2) j2 seq_set(3) j3],
'x',dictx,seq,TAE);
                else

```



```

        temp = translator([seq_set(3) j3 seq_set(2) j2],
        'x',dictx,seq,TAE);
        end
        array(temp) = 1;
        if seq_set(1) < seq_set(3)
            temp = translator([seq_set(1) j1 seq_set(3) j3],
        'x',dictx,seq,TAE);
        else
            temp = translator([seq_set(3) j3 seq_set(1) j1],
        'x',dictx,seq,TAE);
        end
        array(temp) = -1;
        result(c,:) = array;
        c = c + 1;
    end
end
end
end
end

```

interpreter.m:

```

function [result1 result2] = interpreter(x, seq, TAE, dictx, dicty)

% interpreter interprets the ILP solution to an aligned sequence
templ = find(x == 1);
edge = zeros(size(find(templ <= TAE),1),4);
arc = zeros(size(find(templ > TAE),1),4);
c1 = 1; c2 = 1;
for i = 1:size(templ,1)
    if templ(i) <= TAE
        edge(c1,:) = translator(templ(i),'x',dictx,seq,TAE);
        c1 = c1 + 1;
    else
        arc(c2,:) = translator(templ(i),'y',dicty,seq,TAE);
        c2 = c2 + 1;
    end
end
end

result1 = []; result2 = [];
temp_seq = seq; column = '-';
for n1 = 2:size(seq,1)
    column = [column; '-'];
end
for i = 1:size(seq,1)
    for j = 1:length(seq{i})
        if temp_seq{i}(j) ~= '-'
            temp = search_node([i j], edge);
            in_col = column; info = Inf(size(seq,1),1);
            in_col(i) = seq{i}(j);
            info(i) = j;
            temp_seq{i}(j) = '-';
            if ~isempty(temp) == 1
                for k = 1:size(temp,1)
                    in_col(temp(k,1)) = seq{temp(k,1)}(temp(k,2));
                    info(temp(k,1)) = temp(k,2);
                    temp_seq{temp(k,1)}(temp(k,2)) = '-';
                end
            end
            [result1 result2] = insert_column(result1,result2,in_col,info);
        end
    end
end
end

function result = search_node(node, edge)

```

```

result = [];
for i = 1:size(edge,1)
    for j = 1:2
        if edge(i,(2*j - 1)) == node(1) && edge(i,2*j) == node(2)
            if j == 1
                result = [result; edge(i,(3:4))];
            else
                result = [result; edge(i,(1:2))];
            end
        end
    end
end

function [result1 result2] = insert_column(aligned_seq, position_info, in_col, info)

if isempty(aligned_seq) == 1
    result1 = in_col;
    result2 = info;
else
    seq = find(info < Inf,1,'first');
    column = find(position_info(seq,:) < info(seq),1,'last');
    if size(find(info < Inf),1) > 1
        while isempty(column) == 1 && seq < size(info,1)
            for i = 1:size(info)
                if info(i) > seq && info(i) < Inf
                    break
                end
            end
            seq = i;
            column = find(position_info(seq,:) < info(seq),1,'last');
            if seq == size(info,1) && isempty(column) == 1
                break
            end
        end
    end
    if isempty(column) == 1
        result1 = [in_col aligned_seq];
        result2 = [info position_info];
    elseif column == size(position_info,2)
        result1 = [aligned_seq in_col];
        result2 = [position_info info];
    else
        result1 = [aligned_seq(:,(1:column)) in_col aligned_seq(:,((column + 1):size(aligned_seq,2)))];
        result2 = [position_info(:,(1:column)) info position_info(:,((column + 1):size(position_info,2)))];
    end
end
end

```