LAMPIRAN

LAMPIRAN 1

Metode Newton-Raphson

Misalkan $\boldsymbol{\theta}$ adalah vektor dari 2 buah parameter yaitu $\boldsymbol{\beta}$ dan $\boldsymbol{\gamma}$.

$\boldsymbol{\theta}$ dapat dituliskan sebagai berikut $\boldsymbol{\theta} = \begin{bmatrix} \boldsymbol{\beta} \\ \boldsymbol{\gamma} \end{bmatrix} = \begin{bmatrix} \beta_0 \\ \beta_1 \\ \vdots \\ \beta_p \\ \gamma_1 \\ \gamma_2 \\ \vdots \\ \gamma_{M-1} \end{bmatrix}$.

$L^*(\boldsymbol{\theta})$ adalah fungsi *log likelihood* dari vektor $\boldsymbol{\theta}$, sedangkan $\mathbf{f}'(\boldsymbol{\theta})$ merupakan vektor turunan parsial pertama dari $L^*(\boldsymbol{\theta})$ dapat ditulis sebagai berikut :

$$\mathbf{f}'(\boldsymbol{\theta}) = \begin{bmatrix} \dfrac{\partial L^*(\boldsymbol{\theta})}{\partial \beta_0} \\ \dfrac{\partial L^*(\boldsymbol{\theta})}{\partial \beta_1} \\ \dfrac{\partial L^*(\boldsymbol{\theta})}{\partial \beta_2} \\ \vdots \\ \dfrac{\partial L^*(\boldsymbol{\theta})}{\partial \beta_p} \\ \dfrac{\partial L^*(\boldsymbol{\theta})}{\partial \gamma_1} \\ \vdots \\ \dfrac{\partial L^*(\boldsymbol{\theta})}{\partial \gamma_{M-1}} \end{bmatrix}$$

dan $\mathbf{H}(\boldsymbol{\theta})$ adalah matriks turunan parsial kedua dari $L^*(\boldsymbol{\theta})$ yaitu :

$$\mathbf{H}(\boldsymbol{\theta}) = \begin{bmatrix} \dfrac{\partial \mathbf{f}'(\boldsymbol{\theta})}{\partial \beta_0} & \dfrac{\partial \mathbf{f}'(\boldsymbol{\theta})}{\partial \beta_1} & \dfrac{\partial \mathbf{f}'(\boldsymbol{\theta})}{\partial \beta_2} & \cdots & \dfrac{\partial \mathbf{f}'(\boldsymbol{\theta})}{\partial \beta_p} & \dfrac{\partial \mathbf{f}'(\boldsymbol{\theta})}{\partial \gamma_1} & \cdots & \dfrac{\partial \mathbf{f}'(\boldsymbol{\theta})}{\partial \gamma_{M-1}} \end{bmatrix}$$

$$= \begin{bmatrix}
\dfrac{\partial^2 L^*(\boldsymbol{\theta})}{\partial \beta_0 \beta_0} & \dfrac{\partial^2 L^*(\boldsymbol{\theta})}{\partial \beta_1 \beta_0} & \dfrac{\partial^2 L^*(\boldsymbol{\theta})}{\partial \beta_2 \beta_0} & \cdots & \dfrac{\partial^2 L^*(\boldsymbol{\theta})}{\partial \beta_p \beta_0} & \dfrac{\partial^2 L^*(\boldsymbol{\theta})}{\partial \gamma_1 \beta_0} & \cdots & \dfrac{\partial^2 L^*(\boldsymbol{\theta})}{\partial \gamma_{M-1} \beta_0} \\
\dfrac{\partial^2 L^*(\boldsymbol{\theta})}{\partial \beta_0 \beta_1} & \dfrac{\partial^2 L^*(\boldsymbol{\theta})}{\partial \beta_1 \beta_1} & \dfrac{\partial^2 L^*(\boldsymbol{\theta})}{\partial \beta_2 \beta_1} & \cdots & \dfrac{\partial^2 L^*(\boldsymbol{\theta})}{\partial \beta_p \beta_1} & \dfrac{\partial^2 L^*(\boldsymbol{\theta})}{\partial \gamma_1 \beta_1} & \cdots & \dfrac{\partial^2 L^*(\boldsymbol{\theta})}{\partial \gamma_{M-1} \beta_1} \\
\dfrac{\partial^2 L^*(\boldsymbol{\theta})}{\partial \beta_0 \beta_2} & \dfrac{\partial^2 L^*(\boldsymbol{\theta})}{\partial \beta_1 \beta_2} & \dfrac{\partial^2 L^*(\boldsymbol{\theta})}{\partial \beta_2 \beta_2} & \cdots & \dfrac{\partial^2 L^*(\boldsymbol{\theta})}{\partial \beta_p \beta_2} & \dfrac{\partial^2 L^*(\boldsymbol{\theta})}{\partial \gamma_1 \beta_2} & \cdots & \dfrac{\partial^2 L^*(\boldsymbol{\theta})}{\partial \gamma_{M-1} \beta_2} \\
\vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\
\dfrac{\partial^2 L^*(\boldsymbol{\theta})}{\partial \beta_0 \beta_p} & \dfrac{\partial^2 L^*(\boldsymbol{\theta})}{\partial \beta_1 \beta_p} & \dfrac{\partial^2 L^*(\boldsymbol{\theta})}{\partial \beta_2 \beta_p} & \cdots & \dfrac{\partial^2 L^*(\boldsymbol{\theta})}{\partial \beta_p \beta_p} & \dfrac{\partial^2 L^*(\boldsymbol{\theta})}{\partial \gamma_1 \beta_p} & \cdots & \dfrac{\partial^2 L^*(\boldsymbol{\theta})}{\partial \gamma_{m-1} \beta_p} \\
\dfrac{\partial^2 L^*(\boldsymbol{\theta})}{\partial \beta_0 \gamma_1} & \dfrac{\partial^2 L^*(\boldsymbol{\theta})}{\partial \beta_1 \gamma_1} & \dfrac{\partial^2 L^*(\boldsymbol{\theta})}{\partial \beta_2 \gamma_1} & \cdots & \dfrac{\partial^2 L^*(\boldsymbol{\theta})}{\partial \beta_p \gamma_1} & \dfrac{\partial^2 L^*(\boldsymbol{\theta})}{\partial \gamma_1 \gamma_1} & \cdots & \dfrac{\partial^2 L^*(\boldsymbol{\theta})}{\partial \gamma_{M-1} \gamma_1} \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
\dfrac{\partial^2 L^*(\boldsymbol{\theta})}{\partial \beta_0 \gamma_{M-1}} & \dfrac{\partial^2 L^*(\boldsymbol{\theta})}{\partial \beta_1 \gamma_{M-1}} & \dfrac{\partial^2 L^*(\boldsymbol{\theta})}{\partial \beta_2 \gamma_{M-1}} & \cdots & \dfrac{\partial^2 L^*(\boldsymbol{\theta})}{\partial \beta_p \gamma_{M-1}} & \dfrac{\partial^2 L^*(\boldsymbol{\theta})}{\partial \gamma_1 \gamma_{M-1}} & \cdots & \dfrac{\partial^2 L^*(\boldsymbol{\theta})}{\partial \gamma_{M-1} \gamma_{M-1}}
\end{bmatrix}$$

Dengan menggunakan metode Newton Raphson akan dicari taksiran dari $\boldsymbol{\theta}$ yang merupakan penyelesaian dari $\mathbf{f}'(\boldsymbol{\theta}) = \mathbf{0}$. Jika diberikan taksiran awal dari $\boldsymbol{\theta}$ yaitu $\hat{\boldsymbol{\theta}}^{(0)}$, maka dapat diperoleh pendekatan deret Taylor orde pertama dari vektor $\mathbf{f}'(\boldsymbol{\theta})$ di sekitar $\boldsymbol{\theta} = \hat{\boldsymbol{\theta}}^{(0)}$ yaitu :

$$\mathbf{f}'(\boldsymbol{\theta}) \approx \mathbf{f}'\left(\hat{\boldsymbol{\theta}}^{(0)}\right) + \left.\frac{\partial \mathbf{f}'(\boldsymbol{\theta})}{\partial \beta_0}\right|_{\boldsymbol{\theta}=\hat{\boldsymbol{\theta}}^{(0)}} \left(\beta_0 - \hat{\beta}_0^{(0)}\right) + \ldots + \left.\frac{\partial \mathbf{f}'(\boldsymbol{\theta})}{\partial \beta_p}\right|_{\boldsymbol{\theta}=\hat{\boldsymbol{\theta}}^{(0)}} \left(\beta_p - \hat{\beta}_p^{(0)}\right) + \left.\frac{\partial \mathbf{f}'(\boldsymbol{\theta})}{\partial \gamma_1}\right|_{\boldsymbol{\theta}=\hat{\boldsymbol{\theta}}^{(0)}} \left(\gamma_1 - \hat{\gamma}_1^{(0)}\right) + \ldots + \left.\frac{\partial \mathbf{f}'(\boldsymbol{\theta})}{\partial \gamma_{M-1}}\right|_{\boldsymbol{\theta}=\hat{\boldsymbol{\theta}}^{(0)}} \left(\gamma_{M-1} - \hat{\gamma}_{M-1}^{(0)}\right)$$

$$\approx \mathbf{f}'\left(\hat{\boldsymbol{\theta}}^{(0)}\right) + \begin{bmatrix} \left.\frac{\partial \mathbf{f}'(\boldsymbol{\theta})}{\partial \beta_0}\right|_{\boldsymbol{\theta}=\hat{\boldsymbol{\theta}}^{(0)}} & \cdots & \left.\frac{\partial \mathbf{f}'(\boldsymbol{\theta})}{\partial \beta_p}\right|_{\boldsymbol{\theta}=\hat{\boldsymbol{\theta}}^{(0)}} & \left.\frac{\partial \mathbf{f}'(\boldsymbol{\theta})}{\partial \gamma_1}\right|_{\boldsymbol{\theta}=\hat{\boldsymbol{\theta}}^{(0)}} & \cdots & \left.\frac{\partial \mathbf{f}'(\boldsymbol{\theta})}{\partial \gamma_{M-1}}\right|_{\boldsymbol{\theta}=\hat{\boldsymbol{\theta}}^{(0)}} \end{bmatrix} \begin{bmatrix} \left(\beta_0 - \hat{\beta}_0^{(0)}\right) \\ \vdots \\ \left(\beta_p - \hat{\beta}_p^{(0)}\right) \\ \left(\gamma_1 - \hat{\gamma}_1^{(0)}\right) \\ \vdots \\ \left(\gamma_{M-1} - \hat{\gamma}_{M-1}^{(0)}\right) \end{bmatrix}$$

$$\approx \mathbf{f}'\left(\hat{\boldsymbol{\theta}}^{(0)}\right) + \begin{bmatrix} \frac{\partial^2 L^*(\boldsymbol{\theta})}{\partial \beta_0 \beta_0} & \frac{\partial^2 L^*(\boldsymbol{\theta})}{\partial \beta_1 \beta_0} & \frac{\partial^2 L^*(\boldsymbol{\theta})}{\partial \beta_2 \beta_0} & \cdots & \frac{\partial^2 L^*(\boldsymbol{\theta})}{\partial \beta_p \beta_0} & \frac{\partial^2 L^*(\boldsymbol{\theta})}{\partial \gamma_1 \beta_0} & \cdots & \frac{\partial^2 L^*(\boldsymbol{\theta})}{\partial \gamma_{M-1} \beta_0} \\ \frac{\partial^2 L^*(\boldsymbol{\theta})}{\partial \beta_0 \beta_1} & \frac{\partial^2 L^*(\boldsymbol{\theta})}{\partial \beta_1 \beta_1} & \frac{\partial^2 L^*(\boldsymbol{\theta})}{\partial \beta_2 \beta_1} & \cdots & \frac{\partial^2 L^*(\boldsymbol{\theta})}{\partial \beta_p \beta_1} & \frac{\partial^2 L^*(\boldsymbol{\theta})}{\partial \gamma_1 \beta_1} & \cdots & \frac{\partial^2 L^*(\boldsymbol{\theta})}{\partial \gamma_{M-1} \beta_1} \\ \frac{\partial^2 L^*(\boldsymbol{\theta})}{\partial \beta_0 \beta_2} & \frac{\partial^2 L^*(\boldsymbol{\theta})}{\partial \beta_1 \beta_2} & \frac{\partial^2 L^*(\boldsymbol{\theta})}{\partial \beta_2 \beta_2} & \cdots & \frac{\partial^2 L^*(\boldsymbol{\theta})}{\partial \beta_p \beta_2} & \frac{\partial^2 L^*(\boldsymbol{\theta})}{\partial \gamma_1 \beta_2} & \cdots & \frac{\partial^2 L^*(\boldsymbol{\theta})}{\partial \gamma_{M-1} \beta_2} \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ \frac{\partial^2 L^*(\boldsymbol{\theta})}{\partial \beta_0 \beta_p} & \frac{\partial^2 L^*(\boldsymbol{\theta})}{\partial \beta_1 \beta_p} & \frac{\partial^2 L^*(\boldsymbol{\theta})}{\partial \beta_2 \beta_p} & \cdots & \frac{\partial^2 L^*(\boldsymbol{\theta})}{\partial \beta_p \beta_p} & \frac{\partial^2 L^*(\boldsymbol{\theta})}{\partial \gamma_1 \beta_p} & \cdots & \frac{\partial^2 L^*(\boldsymbol{\theta})}{\partial \gamma_{M-1} \beta_p} \\ \frac{\partial^2 L^*(\boldsymbol{\theta})}{\partial \beta_0 \gamma_1} & \frac{\partial^2 L^*(\boldsymbol{\theta})}{\partial \beta_1 \gamma_1} & \frac{\partial^2 L^*(\boldsymbol{\theta})}{\partial \beta_2 \gamma_1} & \cdots & \frac{\partial^2 L^*(\boldsymbol{\theta})}{\partial \beta_p \gamma_1} & \frac{\partial^2 L^*(\boldsymbol{\theta})}{\partial \gamma_1 \gamma_1} & \cdots & \frac{\partial^2 L^*(\boldsymbol{\theta})}{\partial \gamma_{M-1} \gamma_1} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \frac{\partial^2 L^*(\boldsymbol{\theta})}{\partial \beta_0 \gamma_{M-1}} & \frac{\partial^2 L^*(\boldsymbol{\theta})}{\partial \beta_1 \gamma_{M-1}} & \frac{\partial^2 L^*(\boldsymbol{\theta})}{\partial \beta_2 \gamma_{M-1}} & \cdots & \frac{\partial^2 L^*(\boldsymbol{\theta})}{\partial \beta_p \gamma_{M-1}} & \frac{\partial^2 L^*(\boldsymbol{\theta})}{\partial \gamma_1 \gamma_{M-1}} & \cdots & \frac{\partial^2 L^*(\boldsymbol{\theta})}{\partial \gamma_{M-1} \gamma_{M-1}} \end{bmatrix}_{\boldsymbol{\theta}=\hat{\boldsymbol{\theta}}^{(0)}} \left(\boldsymbol{\theta} - \hat{\boldsymbol{\theta}}^{(0)}\right)$$

$$\approx \mathbf{f}'\left(\hat{\boldsymbol{\theta}}^{(0)}\right) + \begin{bmatrix} H_{\beta_0\beta_0}(\boldsymbol{\theta}) & \cdots & H_{\beta_p\beta_0}(\boldsymbol{\theta}) & H_{\gamma_1\beta_0}(\boldsymbol{\theta}) & \cdots & H_{\gamma_{M-1}\beta_0}(\boldsymbol{\theta}) \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ H_{\beta_0\beta_p}(\boldsymbol{\theta}) & \cdots & H_{\beta_p\beta_p}(\boldsymbol{\theta}) & H_{\gamma_1\beta_p}(\boldsymbol{\theta}) & \cdots & H_{\gamma_{M-1}\beta_p}(\boldsymbol{\theta}) \\ H_{\beta_0\gamma_1}(\boldsymbol{\theta}) & \cdots & H_{\beta_p\gamma_1}(\boldsymbol{\theta}) & H_{\gamma_1\gamma_1}(\boldsymbol{\theta}) & \cdots & H_{\gamma_{M-1}\gamma_1}(\boldsymbol{\theta}) \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ H_{\beta_0\gamma_{M-1}}(\boldsymbol{\theta}) & \cdots & H_{\beta_p\gamma_{M-1}}(\boldsymbol{\theta}) & H_{\gamma_1\gamma_{M-1}}(\boldsymbol{\theta}) & \cdots & H_{\gamma_{M-1}\gamma_{M-1}}(\boldsymbol{\theta}) \end{bmatrix}_{\boldsymbol{\theta}=\hat{\boldsymbol{\theta}}^{(0)}} \left(\boldsymbol{\theta} - \hat{\boldsymbol{\theta}}^{(0)}\right)$$

Atau dapat ditulis juga

$$\mathbf{f}'(\boldsymbol{\theta}) \approx \mathbf{f}'\left(\hat{\boldsymbol{\theta}}^{(0)}\right) + \mathbf{H}\left(\hat{\boldsymbol{\theta}}^{(0)}\right)\left(\boldsymbol{\theta} - \hat{\boldsymbol{\theta}}^{(0)}\right)$$

dengan :

$\mathbf{f}'\left(\hat{\boldsymbol{\theta}}^{(0)}\right)$ adalah vektor turunan parsial pertama dari $L^*(\boldsymbol{\theta})$ dihitung pada $\boldsymbol{\theta} = \hat{\boldsymbol{\theta}}^{(0)}$.

$\mathbf{H}\left(\hat{\boldsymbol{\theta}}^{(0)}\right)$ adalah matriks turunan parsial kedua dari $L^*(\boldsymbol{\theta})$ dihitung pada $\boldsymbol{\theta} = \hat{\boldsymbol{\theta}}^{(0)}$.

Karena ingin mencari penyelesaian dari $\mathbf{f}'(\boldsymbol{\theta}) = \mathbf{0}$ maka :

$$\mathbf{0} = \mathbf{f}'(\boldsymbol{\theta}) \approx \mathbf{f}'\left(\hat{\boldsymbol{\theta}}^{(0)}\right) + \mathbf{H}\left(\hat{\boldsymbol{\theta}}^{(0)}\right)\left(\boldsymbol{\theta} - \hat{\boldsymbol{\theta}}^{(0)}\right)$$

$$\mathbf{0} = \mathbf{f}'\left(\hat{\boldsymbol{\theta}}^{(0)}\right) + \mathbf{H}\left(\hat{\boldsymbol{\theta}}^{(0)}\right)\left(\boldsymbol{\theta} - \hat{\boldsymbol{\theta}}^{(0)}\right)$$

$$-\mathbf{f}'\left(\hat{\boldsymbol{\theta}}^{(0)}\right) = \mathbf{H}\left(\hat{\boldsymbol{\theta}}^{(0)}\right)\left(\boldsymbol{\theta} - \hat{\boldsymbol{\theta}}^{(0)}\right)$$

Jika matriks $\mathbf{H}\left(\hat{\boldsymbol{\theta}}^{(0)}\right)$ tidak singular maka :

$$\mathbf{H}^{-1}\left(\hat{\boldsymbol{\theta}}^{(0)}\right).-\mathbf{f}'\left(\hat{\boldsymbol{\theta}}^{(0)}\right) = \mathbf{H}^{-1}\left(\hat{\boldsymbol{\theta}}^{(0)}\right)\mathbf{H}\left(\hat{\boldsymbol{\theta}}^{(0)}\right)\left(\boldsymbol{\theta} - \hat{\boldsymbol{\theta}}^{(0)}\right)$$

$$-\left(\mathbf{H}^{-1}\left(\hat{\boldsymbol{\theta}}^{(0)}\right)\mathbf{f}'\left(\hat{\boldsymbol{\theta}}^{(0)}\right)\right) = \mathbf{I}.\left(\boldsymbol{\theta} - \hat{\boldsymbol{\theta}}^{(0)}\right)$$

$$-\left(\mathbf{H}^{-1}\left(\hat{\boldsymbol{\theta}}^{(0)}\right)\mathbf{f}'\left(\hat{\boldsymbol{\theta}}^{(0)}\right)\right) = \boldsymbol{\theta} - \hat{\boldsymbol{\theta}}^{(0)}$$

$$\boldsymbol{\theta} = \hat{\boldsymbol{\theta}}^{(0)} - \left(\mathbf{H}^{-1}\left(\hat{\boldsymbol{\theta}}^{(0)}\right)\mathbf{f}'\left(\hat{\boldsymbol{\theta}}^{(0)}\right)\right)$$

Nilai $\boldsymbol{\theta}$ menjadi taksiran baru yang kemudian dinotasikan dengan $\hat{\boldsymbol{\theta}}^{(1)}$:

$$\hat{\boldsymbol{\theta}}^{(1)} = \hat{\boldsymbol{\theta}}^{(0)} - \left(\mathbf{H}^{-1}\left(\hat{\boldsymbol{\theta}}^{(0)}\right)\mathbf{f}'\left(\hat{\boldsymbol{\theta}}^{(0)}\right)\right)$$

Jika $\hat{\boldsymbol{\theta}}^{(1)} \approx \hat{\boldsymbol{\theta}}^{(0)}$ (misalkan $\left\|\hat{\boldsymbol{\theta}}^{(m+1)} - \hat{\boldsymbol{\theta}}^{(m)}\right\| < 10^{-5}$), maka hentikan proses iterasi dan kemudian ambil $\hat{\boldsymbol{\theta}}^{(1)}$ sebagai taksiran dari $\hat{\boldsymbol{\theta}}$. Apabila tidak maka lanjutkan iterasi. Jika diberikan taksiran $\hat{\boldsymbol{\theta}}^{(1)}$ maka dapat diperoleh pendekatan deret Taylor orde pertama dari vektor $f'(\boldsymbol{\theta})$ sekitar $\boldsymbol{\theta} = \hat{\boldsymbol{\theta}}^{(1)}$, yaitu :

$$\mathbf{f}'(\boldsymbol{\theta}) \approx \mathbf{f}'\left(\hat{\boldsymbol{\theta}}^{(1)}\right) + \mathbf{H}\left(\hat{\boldsymbol{\theta}}^{(1)}\right)\left(\boldsymbol{\theta} - \hat{\boldsymbol{\theta}}^{(1)}\right)$$

dengan :

$\mathbf{f}'\left(\hat{\boldsymbol{\theta}}^{(1)}\right)$ adalah vektor turunan parsial pertama dari $L^*(\boldsymbol{\theta})$ dihitung pada $\boldsymbol{\theta} = \hat{\boldsymbol{\theta}}^{(1)}$.

$\mathbf{H}\left(\hat{\boldsymbol{\theta}}^{(1)}\right)$ adalah matriks turunan parsial kedua dari $L^*(\boldsymbol{\theta})$ dihitung pada $\boldsymbol{\theta} = \hat{\boldsymbol{\theta}}^{(1)}$.

Dengan menyelesaikan :

$$\mathbf{0} = \mathbf{f}'(\boldsymbol{\theta}) \approx \mathbf{f}'\left(\hat{\boldsymbol{\theta}}^{(1)}\right) + \mathbf{H}\left(\hat{\boldsymbol{\theta}}^{(1)}\right)\left(\boldsymbol{\theta} - \hat{\boldsymbol{\theta}}^{(1)}\right)$$

diperoleh :

$$-\mathbf{f}'\left(\hat{\boldsymbol{\theta}}^{(1)}\right) = \mathbf{H}\left(\hat{\boldsymbol{\theta}}^{(1)}\right)\left(\boldsymbol{\theta} - \hat{\boldsymbol{\theta}}^{(1)}\right)$$

Jika matriks $\mathbf{H}\left(\hat{\boldsymbol{\theta}}^{(1)}\right)$ tidak singular maka :

$$\mathbf{H}^{-1}\left(\hat{\boldsymbol{\theta}}^{(1)}\right). -\mathbf{f}'\left(\hat{\boldsymbol{\theta}}^{(1)}\right) = \mathbf{H}^{-1}\left(\hat{\boldsymbol{\theta}}^{(1)}\right)\mathbf{H}\left(\hat{\boldsymbol{\theta}}^{(1)}\right)\left(\boldsymbol{\theta} - \hat{\boldsymbol{\theta}}^{(1)}\right)$$

$$-\left(\mathbf{H}^{-1}\left(\hat{\boldsymbol{\theta}}^{(1)}\right)\mathbf{f}'\left(\hat{\boldsymbol{\theta}}^{(1)}\right)\right) = \mathbf{I}.\left(\boldsymbol{\theta} - \hat{\boldsymbol{\theta}}^{(1)}\right)$$

$$-\left(\mathbf{H}^{-1}\left(\hat{\boldsymbol{\theta}}^{(1)}\right)\mathbf{f}'\left(\hat{\boldsymbol{\theta}}^{(1)}\right)\right) = \boldsymbol{\theta} - \hat{\boldsymbol{\theta}}^{(1)}$$

$$\boldsymbol{\theta} = \hat{\boldsymbol{\theta}}^{(1)} - \left(\mathbf{H}^{-1}\left(\hat{\boldsymbol{\theta}}^{(1)}\right)\mathbf{f}'\left(\hat{\boldsymbol{\theta}}^{(1)}\right)\right)$$

Nilai $\boldsymbol{\theta}$ menjadi taksiran baru yang kemudian dinotasikan dengan $\hat{\boldsymbol{\theta}}^{(2)}$:

$$\hat{\boldsymbol{\theta}}^{(2)} = \hat{\boldsymbol{\theta}}^{(1)} - \left(\mathbf{H}^{-1}\left(\hat{\boldsymbol{\theta}}^{(1)}\right)\mathbf{f}'\left(\hat{\boldsymbol{\theta}}^{(1)}\right)\right)$$

Jika $\hat{\boldsymbol{\theta}}^{(2)} \approx \hat{\boldsymbol{\theta}}^{(1)}$ (misalkan $\left\|\hat{\boldsymbol{\theta}}^{(m+1)} - \hat{\boldsymbol{\theta}}^{(m)}\right\| < 10^{-5}$), maka hentikan proses iterasi dan kemudian ambil $\hat{\boldsymbol{\theta}}^{(2)}$ sebagai taksiran dari $\hat{\boldsymbol{\theta}}$. Apabila tidak maka lanjutkan iterasi.

Dengan mengulang langkah yang sama dapat diperoleh taksiran $\hat{\boldsymbol{\theta}}^{(3)}$, $\hat{\boldsymbol{\theta}}^{(4)}$, dan seterusnya. Secara umum dapat ditentukan taksiran dari $\boldsymbol{\theta}$ pada iterasi ke-m+1, yaitu $\hat{\boldsymbol{\theta}}^{(m+1)}$, secara iteratif menggunakan formula :

$$\hat{\boldsymbol{\theta}}^{(m+1)} = \hat{\boldsymbol{\theta}}^{(m)} - \left(\mathbf{H}^{-1}\left(\hat{\boldsymbol{\theta}}^{(m)}\right)\mathbf{f}'\left(\hat{\boldsymbol{\theta}}^{(m)}\right)\right) \text{, m=0,1,2,3,...}$$

dengan :

$\mathbf{f}'\left(\hat{\boldsymbol{\theta}}^{(m)}\right)$ adalah vektor turunan parsial pertama dari $L^*(\boldsymbol{\theta})$ dihitung pada $\boldsymbol{\theta} = \hat{\boldsymbol{\theta}}^{(m)}$.

$\mathbf{H}\left(\hat{\boldsymbol{\theta}}^{(m)}\right)$ adalah matriks turunan parsial kedua dari $L^*(\boldsymbol{\theta})$ dihitung pada $\boldsymbol{\theta} = \hat{\boldsymbol{\theta}}^{(m)}$.

Hentikan proses iterasi apabila $\hat{\boldsymbol{\theta}}^{(m+1)} \approx \hat{\boldsymbol{\theta}}^{(m)}$ (misalkan $\left\|\hat{\boldsymbol{\theta}}^{(m+1)} - \hat{\boldsymbol{\theta}}^{(m)}\right\| < 10^{-5}$), kemudian ambil $\hat{\boldsymbol{\theta}}^{(m+1)}$ sebagai taksiran dari $\hat{\boldsymbol{\theta}}$.

LAMPIRAN 2

Mencari Matriks Variansi – Kovariansi Dari $\hat{\boldsymbol{\beta}}$

Untuk Model Regresi Linear

Akan dicari matriks variansi – kovariansi dari $\hat{\boldsymbol{\beta}}$ yang didefinisikan sebagai berikut :

$$V\left(\hat{\boldsymbol{\beta}}\right) = E\left(\left(\hat{\boldsymbol{\beta}} - E(\hat{\boldsymbol{\beta}})\right)\left(\hat{\boldsymbol{\beta}} - E(\hat{\boldsymbol{\beta}})\right)^T\right). \qquad (1)$$

Untuk tujuan tersebut mula-mula dimisalkan terdapat *n* observasi yang saling bebas dan Y_j merupakan variabel *dependent* untuk observasi ke-j; $j = 1,2,...,n$. Misalkan $f\left(y_j \mid \mathbf{x}_j; \boldsymbol{\beta}\right)$ adalah pdf bersyarat dari Y_j diberikan nilai $\mathbf{x}_j$, dimana $\mathbf{x}_j = \left(x_{1j}, x_{2j}, ..., x_{pj}\right)$ adalah vektor dari nilai variabel penjelas untuk observasi ke-j; $j = 1,2,...,n$, dan $\boldsymbol{\beta} = \left(\beta_0, \beta_1, \beta_2, ..., \beta_p\right)^T$.

Fungsi log-likelihood dapat dituliskan sebagai berikut :

$$\begin{aligned} L^*(\boldsymbol{\beta}) &= \log \prod_{j=1}^{n} f\left(y_j \mid \mathbf{x}_j, \beta_0, \beta_1, ..., \beta_p\right) \\ &= \sum_{j=1}^{n} \log f\left(y_j \mid \mathbf{x}_j, \beta_0, \beta_1, ..., \beta_p\right) \qquad (2) \\ &= \sum_{j=1}^{n} \log f\left(y_j \mid \mathbf{x}_j, \boldsymbol{\beta}\right) \end{aligned}$$

Turunan parsial pertama dari $L^*(\boldsymbol{\beta})$ terhadap $\boldsymbol{\beta}$ dinyatakan oleh :

$$\begin{aligned} f'_u(\boldsymbol{\beta}) &= \frac{\partial L^*(\boldsymbol{\beta})}{\partial \beta_u} \\ &= \frac{\partial}{\partial \beta_u} \sum_{j=1}^{n} \log f\left(y_j \mid \mathbf{x}_j, \boldsymbol{\beta}\right) \\ &= \sum_{j=1}^{n} \frac{\partial \log f\left(y_j \mid \mathbf{x}_j, \boldsymbol{\beta}\right)}{\partial \beta_u} \qquad ; u = 0,1,2,\ldots,p \end{aligned} \tag{3}$$

Misalkan $\mathbf{f}'(\boldsymbol{\beta})$ adalah vektor dengan elemen $f'_0(\boldsymbol{\beta})$, $f'_1(\boldsymbol{\beta})$, …, $f'_p(\boldsymbol{\beta})$, yaitu :

$\mathbf{f}'(\boldsymbol{\beta}) = \begin{bmatrix} f'_0(\boldsymbol{\beta}) \\ f'_1(\boldsymbol{\beta}) \\ \vdots \\ f'_p(\boldsymbol{\beta}) \end{bmatrix}$ dan misalkan $L^*(\boldsymbol{\beta})$ memiliki maksimum yang unik pada $\hat{\boldsymbol{\beta}}$;

$\hat{\boldsymbol{\beta}} = \left(\hat{\beta}_0, \hat{\beta}_1, \ldots, \hat{\beta}_p\right)^T$.

Dengan menggunakan pendekatan deret Taylor orde pertama dari matriks $\mathbf{f}'(\boldsymbol{\beta})$ di sekitar $\boldsymbol{\beta} = \hat{\boldsymbol{\beta}}$ diperoleh :

$$\begin{aligned} \mathbf{f}'(\boldsymbol{\beta}) &\approx \mathbf{f}'\left(\hat{\boldsymbol{\beta}}\right) + \left.\frac{\partial \mathbf{f}'(\boldsymbol{\beta})}{\partial \beta_0}\right|_{\boldsymbol{\beta}=\hat{\boldsymbol{\beta}}} \left(\beta_0 - \hat{\beta}_0\right) + \left.\frac{\partial \mathbf{f}'(\boldsymbol{\beta})}{\partial \beta_1}\right|_{\boldsymbol{\beta}=\hat{\boldsymbol{\beta}}} \left(\beta_1 - \hat{\beta}_1\right) + \ldots + \left.\frac{\partial \mathbf{f}'(\boldsymbol{\beta})}{\partial \beta_p}\right|_{\boldsymbol{\beta}=\hat{\boldsymbol{\beta}}} \left(\beta_p - \hat{\beta}_p\right) \\ &\approx \mathbf{f}'\left(\hat{\boldsymbol{\beta}}\right) + \begin{bmatrix} \left.\frac{\partial \mathbf{f}'(\boldsymbol{\beta})}{\partial \beta_0}\right|_{\boldsymbol{\beta}=\hat{\boldsymbol{\beta}}} & \left.\frac{\partial \mathbf{f}'(\boldsymbol{\beta})}{\partial \beta_1}\right|_{\boldsymbol{\beta}=\hat{\boldsymbol{\beta}}} & \cdots & \left.\frac{\partial \mathbf{f}'(\boldsymbol{\beta})}{\partial \beta_p}\right|_{\boldsymbol{\beta}=\hat{\boldsymbol{\beta}}} \end{bmatrix} \begin{bmatrix} \beta_0 - \hat{\beta}_0 \\ \beta_1 - \hat{\beta}_1 \\ \vdots \\ \beta_p - \hat{\beta}_p \end{bmatrix} \\ &\approx \mathbf{f}'\left(\hat{\boldsymbol{\beta}}\right) + \left.\begin{bmatrix} \frac{\partial^2 L^*(\boldsymbol{\theta})}{\partial \beta_0 \beta_0} & \frac{\partial^2 L^*(\boldsymbol{\theta})}{\partial \beta_1 \beta_0} & \cdots & \frac{\partial^2 L^*(\boldsymbol{\theta})}{\partial \beta_p \beta_0} \\ \frac{\partial^2 L^*(\boldsymbol{\theta})}{\partial \beta_0 \beta_1} & \frac{\partial^2 L^*(\boldsymbol{\theta})}{\partial \beta_1 \beta_1} & \cdots & \frac{\partial^2 L^*(\boldsymbol{\theta})}{\partial \beta_p \beta_1} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{\partial^2 L^*(\boldsymbol{\theta})}{\partial \beta_0 \beta_p} & \frac{\partial^2 L^*(\boldsymbol{\theta})}{\partial \beta_1 \beta_p} & \cdots & \frac{\partial^2 L^*(\boldsymbol{\theta})}{\partial \beta_p \beta_p} \end{bmatrix}\right|_{\boldsymbol{\beta}=\hat{\boldsymbol{\beta}}} \left[\boldsymbol{\beta} - \hat{\boldsymbol{\beta}}\right] \end{aligned}$$

atau

$$\mathbf{f}'(\boldsymbol{\beta}) \approx \mathbf{f}'(\hat{\boldsymbol{\beta}}) + \mathbf{H}(\hat{\boldsymbol{\beta}})(\boldsymbol{\beta} - \hat{\boldsymbol{\beta}}) \quad (4)$$

dengan

$\mathbf{f}'(\hat{\boldsymbol{\beta}})$ adalah vektor turunan parsial pertama dari $L^*(\boldsymbol{\beta})$ dihitung pada $\boldsymbol{\beta} = \hat{\boldsymbol{\beta}}$.

$\mathbf{H}(\hat{\boldsymbol{\beta}})$ adalah matriks turunan parsial kedua dari $L^*(\boldsymbol{\beta})$ dihitung pada $\boldsymbol{\beta} = \hat{\boldsymbol{\beta}}$.

Karena $\hat{\beta}$ adalah taksiran maksimum likelihood, maka $\mathbf{f}'(\hat{\boldsymbol{\beta}}) = \mathbf{0}$. Oleh karena itu (4) menjadi :

$$\mathbf{f}'(\boldsymbol{\beta}) \approx \mathbf{H}(\hat{\boldsymbol{\beta}})(\boldsymbol{\beta} - \hat{\boldsymbol{\beta}}) \quad (5)$$

Jika diasumsikan $\left[\mathbf{H}(\hat{\boldsymbol{\beta}})\right]$ tidak singular, maka :

$$\begin{aligned} \mathbf{H}^{-1}(\hat{\boldsymbol{\beta}}).\mathbf{f}'(\boldsymbol{\beta}) &\approx \mathbf{H}^{-1}(\hat{\boldsymbol{\beta}})\mathbf{H}(\hat{\boldsymbol{\beta}})(\boldsymbol{\beta} - \hat{\boldsymbol{\beta}}) \\ &\approx \mathbf{I}.(\boldsymbol{\beta} - \hat{\boldsymbol{\beta}}) \\ &\approx \boldsymbol{\beta} - \hat{\boldsymbol{\beta}} \\ \hat{\boldsymbol{\beta}} - \boldsymbol{\beta} &\approx -\left[\mathbf{H}(\hat{\boldsymbol{\beta}})\right]^{-1} \mathbf{f}'(\boldsymbol{\beta}) \\ E(\hat{\boldsymbol{\beta}} - \boldsymbol{\beta}) &\approx E\left(-\left[\mathbf{H}(\hat{\boldsymbol{\beta}})\right]^{-1} \mathbf{f}'(\boldsymbol{\beta})\right) \end{aligned} \quad (6)$$

$$E(\hat{\boldsymbol{\beta}} - \boldsymbol{\beta}) \approx -E\left(\left[\mathbf{H}(\hat{\boldsymbol{\beta}})\right]^{-1} \mathbf{f}'(\boldsymbol{\beta})\right) \quad (7)$$

Karena elemen $\left[\mathbf{H}(\hat{\boldsymbol{\beta}})\right]^{-1}$ merupakan konstanta, maka

$E(\hat{\boldsymbol{\beta}} - \boldsymbol{\beta}) \approx -E\left(\left[\mathbf{H}(\hat{\boldsymbol{\beta}})\right]^{-1} \mathbf{f}'(\boldsymbol{\beta})\right)$ menjadi :

$$E(\hat{\boldsymbol{\beta}} - \boldsymbol{\beta}) \approx \left[\mathbf{H}(\hat{\boldsymbol{\beta}})\right]^{-1} E(\mathbf{f}'(\boldsymbol{\beta})) \quad (8)$$

Untuk itu akan dicari :

$$E\left(\mathbf{f}'(\boldsymbol{\beta})\right) = E\left(\begin{bmatrix} f'_0(\boldsymbol{\beta}) \\ f'_1(\boldsymbol{\beta}) \\ \vdots \\ f'_p(\boldsymbol{\beta}) \end{bmatrix}\right) = \begin{bmatrix} E\left(f'_0(\boldsymbol{\beta})\right) \\ E\left(f'_1(\boldsymbol{\beta})\right) \\ \vdots \\ E\left(f'_p(\boldsymbol{\beta})\right) \end{bmatrix} \quad (9)$$

Jika persamaan

$$\begin{aligned} f'_u\left(\boldsymbol{\beta}\right) &= \frac{\partial L^*\left(\boldsymbol{\beta}\right)}{\partial \beta_u} \\ &= \frac{\partial}{\partial \beta_u}\sum_{j=1}^{n} \log f\left(y_j \mid \mathbf{x}_j, \boldsymbol{\beta}\right) \\ &= \sum_{j=1}^{n} \frac{\partial \log f\left(y_j \mid \mathbf{x}_j, \boldsymbol{\beta}\right)}{\partial \beta_u} \quad ; u = 0,1,2,\dots,p \end{aligned}$$

diekspektasikan, maka diperoleh :

$$\begin{aligned} E\left(f'_u(\boldsymbol{\beta})\right) &= E\left(\sum_{j=1}^{n} \frac{\partial \log f\left(y_j \mid \mathbf{x}_j, \boldsymbol{\beta}\right)}{\partial \beta_u} \quad ; u = 0,1,2,\dots,p\right) \\ &= E\left(\frac{\partial \log f\left(y_1 \mid \mathbf{x}_1, \boldsymbol{\beta}\right)}{\partial \beta_u} + \frac{\partial \log f\left(y_2 \mid \mathbf{x}_2, \boldsymbol{\beta}\right)}{\partial \beta_u} + \dots + \frac{\partial \log f\left(y_n \mid \mathbf{x}_n, \boldsymbol{\beta}\right)}{\partial \beta_u}\right) \\ &= E\left(\frac{\partial \log f\left(y_1 \mid \mathbf{x}_1, \boldsymbol{\beta}\right)}{\partial \beta_u}\right) + E\left(\frac{\partial \log f\left(y_2 \mid \mathbf{x}_2, \boldsymbol{\beta}\right)}{\partial \beta_u}\right) + \dots + E\left(\frac{\partial \log f\left(y_n \mid \mathbf{x}_n, \boldsymbol{\beta}\right)}{\partial \beta_u}\right) \end{aligned}$$

Dengan mencari :

$$E\left(\frac{\partial \log f\left(y_j \mid \mathbf{x}_j, \boldsymbol{\beta}\right)}{\partial \beta_u}\right) = \sum_{y_j} \frac{\partial \log f\left(y_j \mid \mathbf{x}_j, \boldsymbol{\beta}\right)}{\partial \beta_u} f\left(y_j \mid \mathbf{x}_j, \boldsymbol{\beta}\right) \; : j = 1,2,\dots,n \;\; ; u = 0,1,2,\dots,p$$

$$= \sum_{y_j} \frac{1}{f(y_j \mid \mathbf{x}_j, \boldsymbol{\beta})} \frac{\partial f(y_j \mid \mathbf{x}_j, \boldsymbol{\beta})}{\partial \beta_u} f(y_j \mid \mathbf{x}_j, \boldsymbol{\beta})$$

$$= \sum_{y_j} \frac{\partial f(y_j \mid \mathbf{x}_j, \boldsymbol{\beta})}{\partial \beta_u}$$

$$= \frac{\partial}{\partial \beta_u} \sum_{y_j} f(y_j \mid \mathbf{x}_j, \boldsymbol{\beta})$$

$$= \frac{\partial}{\partial \beta_u} 1$$

$$= 0$$

diperoleh :

$$E\left(f'_u(\boldsymbol{\beta})\right) = 0 + 0 + \ldots + 0 = 0 \text{ untuk setiap } u = 0,1,2,\ldots,p \qquad (10)$$

Jadi :

$$E\left(\mathbf{f}'(\boldsymbol{\beta})\right) = E\left(\begin{bmatrix} f'_0(\boldsymbol{\beta}) \\ f'_1(\boldsymbol{\beta}) \\ \vdots \\ f'_p(\boldsymbol{\beta}) \end{bmatrix}\right) = \begin{bmatrix} E\left(f'_0(\boldsymbol{\beta})\right) \\ E\left(f'_1(\boldsymbol{\beta})\right) \\ \vdots \\ E\left(f'_p(\boldsymbol{\beta})\right) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix} = \mathbf{0} \qquad (11)$$

Dengan mensubstitusikan persamaan di atas ke dalam

$E\left(\hat{\boldsymbol{\beta}} - \boldsymbol{\beta}\right) \approx \left[\mathbf{H}\left(\hat{\boldsymbol{\beta}}\right)\right]^{-1} E\left(\mathbf{f}'(\boldsymbol{\beta})\right)$, diperoleh :

$$E\left(\hat{\boldsymbol{\beta}} - \boldsymbol{\beta}\right) \approx \mathbf{0}$$

$$E\left(\hat{\boldsymbol{\beta}}\right) \approx \boldsymbol{\beta} \qquad (12)$$

Oleh karena itu $Var\left(\hat{\boldsymbol{\beta}}\right)$ pada (1) menjadi :

$$Var\left(\hat{\boldsymbol{\beta}}\right) = E\left[\left(\hat{\boldsymbol{\beta}} - \boldsymbol{\beta}\right)\left(\hat{\boldsymbol{\beta}} - \boldsymbol{\beta}\right)^T\right] \qquad (13)$$

Dari (6) diperoleh $\hat{\boldsymbol{\beta}} - \boldsymbol{\beta} \approx -\left[\mathbf{H}\left(\hat{\boldsymbol{\beta}}\right)\right]^{-1} \mathbf{f}'(\boldsymbol{\beta})$, oleh karena itu (13) menjadi :

$$Var\left(\hat{\boldsymbol{\beta}}\right) = E\left[\left(-\left[\mathbf{H}\left(\hat{\boldsymbol{\beta}}\right)\right]^{-1}\mathbf{f}'(\boldsymbol{\beta})\right)\left(-\left[\mathbf{H}\left(\hat{\boldsymbol{\beta}}\right)\right]^{-1}\mathbf{f}'(\boldsymbol{\beta})\right)^{T}\right]$$

$$= E\left[\left(\left[\mathbf{H}\left(\hat{\boldsymbol{\beta}}\right)\right]^{-1}\right)\mathbf{f}'(\boldsymbol{\beta})\mathbf{f}'(\boldsymbol{\beta})^{T}\left(\left[\mathbf{H}\left(\hat{\boldsymbol{\beta}}\right)\right]^{-1}\right)^{T}\right] \quad (14)$$

$$= \left[\mathbf{H}\left(\hat{\boldsymbol{\beta}}\right)\right]^{-1}E\left(\mathbf{f}'(\boldsymbol{\beta})\mathbf{f}'(\boldsymbol{\beta})^{T}\right)\left[\mathbf{H}\left(\hat{\boldsymbol{\beta}}\right)\right]^{-1}$$

karena matriks $\left[\mathbf{H}\left(\hat{\boldsymbol{\beta}}\right)\right]$ simetris dengan sifat $\left(\left[\mathbf{H}\left(\hat{\boldsymbol{\beta}}\right)\right]^{-1}\right)^{T} = \left(\left[\mathbf{H}\left(\hat{\boldsymbol{\beta}}\right)\right]^{-1}\right)$.

$$E\left(\mathbf{f}'(\boldsymbol{\beta})\mathbf{f}'(\boldsymbol{\beta})^{T}\right) = E\left(\begin{bmatrix} f'_0(\boldsymbol{\beta}) \\ f'_1(\boldsymbol{\beta}) \\ \vdots \\ f'_p(\boldsymbol{\beta}) \end{bmatrix}\begin{bmatrix} f'_0(\boldsymbol{\beta}) & f'_1(\boldsymbol{\beta}) & \cdots & f'_p(\boldsymbol{\beta}) \end{bmatrix}\right)$$

$$= E\begin{pmatrix} f'_0(\boldsymbol{\beta})f'_0(\boldsymbol{\beta}) & f'_0(\boldsymbol{\beta})f'_1(\boldsymbol{\beta}) & \cdots & f'_0(\boldsymbol{\beta})f'_p(\boldsymbol{\beta}) \\ f'_1(\boldsymbol{\beta})f'_0(\boldsymbol{\beta}) & f'_1(\boldsymbol{\beta})f'_1(\boldsymbol{\beta}) & \cdots & f'_1(\boldsymbol{\beta})f'_p(\boldsymbol{\beta}) \\ \vdots & \vdots & \ddots & \vdots \\ f'_p(\boldsymbol{\beta})f'_0(\boldsymbol{\beta}) & f'_p(\boldsymbol{\beta})f'_1(\boldsymbol{\beta}) & \cdots & f'_p(\boldsymbol{\beta})f'_p(\boldsymbol{\beta}) \end{pmatrix} \quad (15)$$

$$= \begin{pmatrix} E\left(f'_0(\boldsymbol{\beta})f'_0(\boldsymbol{\beta})\right) & E\left(f'_0(\boldsymbol{\beta})f'_1(\boldsymbol{\beta})\right) & \cdots & E\left(f'_0(\boldsymbol{\beta})f'_p(\boldsymbol{\beta})\right) \\ E\left(f'_1(\boldsymbol{\beta})f'_0(\boldsymbol{\beta})\right) & E\left(f'_1(\boldsymbol{\beta})f'_1(\boldsymbol{\beta})\right) & \cdots & E\left(f'_1(\boldsymbol{\beta})f'_p(\boldsymbol{\beta})\right) \\ \vdots & \vdots & \ddots & \vdots \\ E\left(f'_p(\boldsymbol{\beta})f'_0(\boldsymbol{\beta})\right) & E\left(f'_p(\boldsymbol{\beta})f'_1(\boldsymbol{\beta})\right) & \cdots & E\left(f'_p(\boldsymbol{\beta})f'_p(\boldsymbol{\beta})\right) \end{pmatrix}$$

Untuk u,v=0,1,…,p diperoleh :

$$
\begin{aligned}
E\left(f'_u(\boldsymbol{\beta})f'_v(\boldsymbol{\beta})\right) &= E\left(\frac{\partial L^*(\boldsymbol{\beta})}{\partial\beta_u}\frac{\partial L^*(\boldsymbol{\beta})}{\partial\beta_v}\right)\\
&= E\left[\left(\sum_{j=1}^{n}\frac{\partial\log f\left(y_j\,|\,\mathbf{x}_j,\boldsymbol{\beta}\right)}{\partial\beta_u}\right)\left(\sum_{j=1}^{n}\frac{\partial\log f\left(y_j\,|\,\mathbf{x}_j,\boldsymbol{\beta}\right)}{\partial\beta_v}\right)\right]\\
&= E\left[\left(\sum_{j=1}^{n}\frac{\partial\log f\left(y_j\,|\,\mathbf{x}_j,\boldsymbol{\beta}\right)}{\partial\beta_u}\right)\left(\sum_{j=1}^{n}\frac{\partial\log f\left(y_j\,|\,\mathbf{x}_j,\boldsymbol{\beta}\right)}{\partial\beta_v}\right)+\sum_{u\neq v}\sum\frac{\partial\log f\left(y_j\,|\,\mathbf{x}_j,\boldsymbol{\beta}\right)}{\partial\beta_u}\frac{\partial\log f\left(y_k\,|\,\mathbf{x}_k,\boldsymbol{\beta}\right)}{\partial\beta_v}\right];\ j,k=1,2,\ldots,n\\
&= \left(\sum_{j=1}^{n}E\left(\frac{\partial\log f\left(y_j\,|\,\mathbf{x}_j,\boldsymbol{\beta}\right)}{\partial\beta_u}\frac{\partial\log f\left(y_j\,|\,\mathbf{x}_j,\boldsymbol{\beta}\right)}{\partial\beta_v}\right)\right)+\sum_{u\neq v}\sum E\left(\frac{\partial\log f\left(y_j\,|\,\mathbf{x}_j,\boldsymbol{\beta}\right)}{\partial\beta_u}\frac{\partial\log f\left(y_k\,|\,\mathbf{x}_k,\boldsymbol{\beta}\right)}{\partial\beta_v}\right)\\
&= \sum_{j=1}^{n}\left(\sum_{y_j}\frac{\partial\log f\left(y_j\,|\,\mathbf{x}_j,\boldsymbol{\beta}\right)}{\partial\beta_v}\frac{\partial\log f\left(y_j\,|\,\mathbf{x}_j,\boldsymbol{\beta}\right)}{\partial\beta_v}f\left(y_j\,|\,\mathbf{x}_j,\boldsymbol{\beta}\right)\right)+\sum_{u\neq v}\sum E\left(\frac{\partial\log f\left(y_j\,|\,\mathbf{x}_j,\boldsymbol{\beta}\right)}{\partial\beta_u}\frac{\partial\log f\left(y_k\,|\,\mathbf{x}_k,\boldsymbol{\beta}\right)}{\partial\beta_v}\right)\\
&= \sum_{j=1}^{n}\left(\sum_{y_j}\left(\frac{1}{f\left(y_j\,|\,\mathbf{x}_j,\boldsymbol{\beta}\right)}\frac{\partial f\left(y_j\,|\,\mathbf{x}_j,\boldsymbol{\beta}\right)}{\partial\beta_u}\right)\left(\frac{1}{f\left(y_j\,|\,\mathbf{x}_j,\boldsymbol{\beta}\right)}\frac{\partial f\left(y_j\,|\,\mathbf{x}_j,\boldsymbol{\beta}\right)}{\partial\beta_v}\right)f\left(y_j\,|\,\mathbf{x}_j,\boldsymbol{\beta}\right)\right)+\sum_{u\neq v}\sum E\left(\frac{\partial\log f\left(y_j\,|\,\mathbf{x}_j,\boldsymbol{\beta}\right)}{\partial\beta_u}\frac{\partial\log f\left(y_k\,|\,\mathbf{x}_k,\boldsymbol{\beta}\right)}{\partial\beta_v}\right)\\
&= \sum_{j=1}^{n}\left(\sum_{y_j}\left(\frac{1}{f\left(y_j\,|\,\mathbf{x}_j,\boldsymbol{\beta}\right)}\frac{\partial f\left(y_j\,|\,\mathbf{x}_j,\boldsymbol{\beta}\right)}{\partial\beta_u}\frac{\partial f\left(y_j\,|\,\mathbf{x}_j,\boldsymbol{\beta}\right)}{\partial\beta_v}\right)+0\right)+\sum_{u\neq v}\sum E\left(\frac{\partial\log f\left(y_j\,|\,\mathbf{x}_j,\boldsymbol{\beta}\right)}{\partial\beta_u}\frac{\partial\log f\left(y_k\,|\,\mathbf{x}_k,\boldsymbol{\beta}\right)}{\partial\beta_v}\right)\\
&= \sum_{j=1}^{n}\left(\sum_{y_j}\left(\frac{1}{f\left(y_j\,|\,\mathbf{x}_j,\boldsymbol{\beta}\right)}\frac{\partial f\left(y_j\,|\,\mathbf{x}_j,\boldsymbol{\beta}\right)}{\partial\beta_u}\frac{\partial f\left(y_j\,|\,\mathbf{x}_j,\boldsymbol{\beta}\right)}{\partial\beta_v}\right)+\frac{\partial^2}{\partial\beta_u\partial\beta_v}1\right)+\sum_{u\neq v}\sum E\left(\frac{\partial\log f\left(y_j\,|\,\mathbf{x}_j,\boldsymbol{\beta}\right)}{\partial\beta_u}\frac{\partial\log f\left(y_k\,|\,\mathbf{x}_k,\boldsymbol{\beta}\right)}{\partial\beta_v}\right)\\
&= \sum_{j=1}^{n}\left(\sum_{y_j}\left(\frac{1}{f\left(y_j\,|\,\mathbf{x}_j,\boldsymbol{\beta}\right)}{}^{2}\frac{\partial f\left(y_j\,|\,\mathbf{x}_j,\boldsymbol{\beta}\right)}{\partial\beta_u}\frac{\partial f\left(y_j\,|\,\mathbf{x}_j,\boldsymbol{\beta}\right)}{\partial\beta_v}\right)+\frac{\partial^2}{\partial\beta_u\partial\beta_v}\sum_{y_j}f\left(y_j\,|\,\mathbf{x}_j,\boldsymbol{\beta}\right)\right)+\sum_{u\neq v}\sum E\left(\frac{\partial\log f\left(y_j\,|\,\mathbf{x}_j,\boldsymbol{\beta}\right)}{\partial\beta_u}\frac{\partial\log f\left(y_k\,|\,\mathbf{x}_k,\boldsymbol{\beta}\right)}{\partial\beta_v}\right)\\
&= \sum_{j=1}^{n}\left(\sum_{y_j}\left(\frac{1}{f\left(y_j\,|\,\mathbf{x}_j,\boldsymbol{\beta}\right)}\frac{\partial f\left(y_j\,|\,\mathbf{x}_j,\boldsymbol{\beta}\right)}{\partial\beta_u}\frac{\partial f\left(y_j\,|\,\mathbf{x}_j,\boldsymbol{\beta}\right)}{\partial\beta_v}\right)+\sum_{y_j}\left(\frac{\partial^2 f\left(y_j\,|\,\mathbf{x}_j,\boldsymbol{\beta}\right)}{\partial\beta_u\partial\beta_v}\right)\right)+\sum_{u\neq v}\sum E\left(\frac{\partial\log f\left(y_j\,|\,\mathbf{x}_j,\boldsymbol{\beta}\right)}{\partial\beta_u}\frac{\partial\log f\left(y_k\,|\,\mathbf{x}_k,\boldsymbol{\beta}\right)}{\partial\beta_v}\right)\\
&= \sum_{j=1}^{n}\left(\sum_{y_j}\left(\frac{1}{f\left(y_j\,|\,\mathbf{x}_j,\boldsymbol{\beta}\right)}\frac{\partial f\left(y_j\,|\,\mathbf{x}_j,\boldsymbol{\beta}\right)}{\partial\beta_u}\frac{\partial f\left(y_j\,|\,\mathbf{x}_j,\boldsymbol{\beta}\right)}{\partial\beta_v}f\left(y_j\,|\,\mathbf{x}_j,\boldsymbol{\beta}\right)+\frac{1}{f\left(y_j\,|\,\mathbf{x}_j,\boldsymbol{\beta}\right)}\frac{\partial^2 f\left(y_j\,|\,\mathbf{x}_j,\boldsymbol{\beta}\right)}{\partial\beta_u\partial\beta_v}f\left(y_j\,|\,\mathbf{x}_j,\boldsymbol{\beta}\right)\right)\right)+\sum_{u\neq v}\sum E\left(\frac{\partial\log f\left(y_j\,|\,\mathbf{x}_j,\boldsymbol{\beta}\right)}{\partial\beta_u}\frac{\partial\log f\left(y_k\,|\,\mathbf{x}_k,\boldsymbol{\beta}\right)}{\partial\beta_v}\right)\\
&= \sum_{j=1}^{n}\left(\sum_{y_j}\left(\left(\left(-\frac{\partial}{\partial\beta_v}\frac{1}{f\left(y_j\,|\,\mathbf{x}_j,\boldsymbol{\beta}\right)}\right)\left(\frac{\partial f\left(y_j\,|\,\mathbf{x}_j,\boldsymbol{\beta}\right)}{\partial\beta_u}\right)+\frac{1}{f\left(y_j\,|\,\mathbf{x}_j,\boldsymbol{\beta}\right)}\left(\frac{\partial^2 f\left(y_j\,|\,\mathbf{x}_j,\boldsymbol{\beta}\right)}{\partial\beta_u\partial\beta_v}\right)\right)f\left(y_j\,|\,\mathbf{x}_j,\boldsymbol{\beta}\right)\right)\right)+\sum_{u\neq v}\sum E\left(\frac{\partial\log f\left(y_j\,|\,\mathbf{x}_j,\boldsymbol{\beta}\right)}{\partial\beta_u}\frac{\partial\log f\left(y_k\,|\,\mathbf{x}_k,\boldsymbol{\beta}\right)}{\partial\beta_v}\right)
\end{aligned}
$$

$$=\sum_{j=1}^{n}\left(\sum_{y_j}\left(\left(\left(-\frac{\partial}{\partial\beta_v}\frac{1}{f(y_j\,|\,\mathbf{x}_j,\boldsymbol{\beta})}\right)\left(\frac{\partial f(y_j\,|\,\mathbf{x}_j,\boldsymbol{\beta})}{\partial\beta_u}\right)+\frac{1}{f(y_j\,|\,\mathbf{x}_j,\boldsymbol{\beta})}\left(\frac{\partial^2 f(y_j\,|\,\mathbf{x}_j,\boldsymbol{\beta})}{\partial\beta_u\partial\beta_v}\right)\right)f(y_j\,|\,\mathbf{x}_j,\boldsymbol{\beta})\right)\right)+\sum_{u\neq v}\sum E\left(\frac{\partial\log f(y_j\,|\,\mathbf{x}_j,\boldsymbol{\beta})}{\partial\beta_u}\frac{\partial\log f(y_k\,|\,\mathbf{x}_k,\boldsymbol{\beta})}{\partial\beta_v}\right)$$

$$=\sum_{j=1}^{n}\left(-\sum_{y_j}\frac{\partial}{\partial\beta_v}\left(\frac{1}{f(y_j\,|\,\mathbf{x}_j,\boldsymbol{\beta})}\frac{\partial f(y_j\,|\,\mathbf{x}_j,\boldsymbol{\beta})}{\partial\beta_u}\right)f(y_j\,|\,\mathbf{x}_j,\boldsymbol{\beta})\right)+\sum_{u\neq v}\sum E\left(\frac{\partial\log f(y_j\,|\,\mathbf{x}_j,\boldsymbol{\beta})}{\partial\beta_u}\frac{\partial\log f(y_k\,|\,\mathbf{x}_k,\boldsymbol{\beta})}{\partial\beta_v}\right)$$

$$=\sum_{j=1}^{n}\left(-\sum_{y_j}\frac{\partial}{\partial\beta_v}\left(\frac{\partial\log f(y_j\,|\,\mathbf{x}_j,\boldsymbol{\beta})}{\partial\beta_u}\right)f(y_j\,|\,\mathbf{x}_j,\boldsymbol{\beta})\right)+\sum_{u\neq v}\sum E\left(\frac{\partial\log f(y_j\,|\,\mathbf{x}_j,\boldsymbol{\beta})}{\partial\beta_u}\frac{\partial\log f(y_k\,|\,\mathbf{x}_k,\boldsymbol{\beta})}{\partial\beta_v}\right)$$

$$=\sum_{j=1}^{n}\left(-\sum_{y_j}\left(\frac{\partial^2\log f(y_j\,|\,\mathbf{x}_j,\boldsymbol{\beta})}{\partial\beta_v\partial\beta_u}\right)f(y_j\,|\,\mathbf{x}_j,\boldsymbol{\beta})\right)+\sum_{u\neq v}\sum E\left(\frac{\partial\log f(y_j\,|\,\mathbf{x}_j,\boldsymbol{\beta})}{\partial\beta_u}\frac{\partial\log f(y_k\,|\,\mathbf{x}_k,\boldsymbol{\beta})}{\partial\beta_v}\right)$$

$$=-\sum_{j}\left(E\left(\frac{\partial^2\log f(y_j\,|\,\mathbf{x}_j,\boldsymbol{\beta})}{\partial\beta_v\partial\beta_u}\right)\right)f(y_j\,|\,\mathbf{x}_j,\boldsymbol{\beta})+\sum_{u\neq v}\sum E\left(\frac{\partial\log f(y_j\,|\,\mathbf{x}_j,\boldsymbol{\beta})}{\partial\beta_u}\frac{\partial\log f(y_k\,|\,\mathbf{x}_k,\boldsymbol{\beta})}{\partial\beta_v}\right)$$

karena y_j dan y_k saling bebas, maka :

$$=-\sum_{j}\left(E\left(\frac{\partial^2\log f(y_j\,|\,\mathbf{x}_j,\boldsymbol{\beta})}{\partial\beta_v\partial\beta_u}\right)\right)f(y_j\,|\,\mathbf{x}_j,\boldsymbol{\beta})+0$$

$$=-E\left(\frac{\partial^2\sum_{j=1}^{n}\log f(y_j\,|\,\mathbf{x}_j,\boldsymbol{\beta})}{\partial\beta_v\partial\beta_u}\right)\qquad(16)$$

$$=-E\left(\frac{\partial^2 L^*}{\partial\beta_v\partial\beta_u}\right)$$

Oleh karena itu (15) menjadi :

$$E\left(\mathbf{f}'(\boldsymbol{\beta})\mathbf{f}'(\boldsymbol{\beta})^T\right)=-\begin{pmatrix}E\left(\frac{\partial^2 L^*}{\partial\beta_0\partial\beta_0}\right) & E\left(\frac{\partial^2 L^*}{\partial\beta_0\partial\beta_1}\right) & \cdots & E\left(\frac{\partial^2 L^*}{\partial\beta_0\partial\beta_p}\right)\\ E\left(\frac{\partial^2 L^*}{\partial\beta_1\partial\beta_0}\right) & E\left(\frac{\partial^2 L^*}{\partial\beta_1\partial\beta_1}\right) & \cdots & E\left(\frac{\partial^2 L^*}{\partial\beta_1\partial\beta_p}\right)\\ \vdots & \vdots & \ddots & \vdots\\ E\left(\frac{\partial^2 L^*}{\partial\beta_p\partial\beta_0}\right) & E\left(\frac{\partial^2 L^*}{\partial\beta_p\partial\beta_1}\right) & \cdots & E\left(\frac{\partial^2 L^*}{\partial\beta_p\partial\beta_p}\right)\end{pmatrix}=-\left[\mathbf{H}(\boldsymbol{\beta})\right]$$

Jadi matriks variansi – kovariansi dari $(\hat{\boldsymbol{\beta}})$ menjadi :

$$Var(\hat{\boldsymbol{\beta}}) = -\left[\mathbf{H}(\hat{\boldsymbol{\beta}})\right]^{-1}\mathbf{H}(\hat{\boldsymbol{\beta}})\left[\mathbf{H}(\hat{\boldsymbol{\beta}})\right]^{-1}.$$

Taksiran matriks variansi – kovariansi dari $(\hat{\boldsymbol{\beta}})$ diberikan oleh :

$$Var(\hat{\boldsymbol{\beta}}) = -\left[\mathbf{H}(\hat{\boldsymbol{\beta}})\right]^{-1}\mathbf{H}(\hat{\boldsymbol{\beta}})\left[\mathbf{H}(\hat{\boldsymbol{\beta}})\right]^{-1}.$$
$$Var(\hat{\boldsymbol{\beta}}) = -\left[\mathbf{H}(\hat{\boldsymbol{\beta}})\right]^{-1}$$

Dalam tugas akhir ini, misal $\boldsymbol{\theta} = \begin{bmatrix}\boldsymbol{\beta}\\ \boldsymbol{\gamma}\end{bmatrix} = \begin{bmatrix}\beta_0\\ \beta_1\\ \vdots\\ \beta_p\\ \gamma_2\\ \gamma_3\\ \vdots\\ \gamma_{M-1}\end{bmatrix}$ maka matriks variansi-kovariansi

dari $\boldsymbol{\theta}$ memiliki bentuk sebagai berikut :

$$\hat{V}(\hat{\boldsymbol{\theta}}) \approx -\left[\mathbf{H}(\hat{\boldsymbol{\theta}})\right]^{-1}$$

$$\approx -\begin{bmatrix}
\frac{\partial^2 L^*(\hat{\boldsymbol{\theta}})}{\partial\beta_0\beta_0} & \frac{\partial^2 L^*(\hat{\boldsymbol{\theta}})}{\partial\beta_1\beta_0} & \frac{\partial^2 L^*(\hat{\boldsymbol{\theta}})}{\partial\beta_2\beta_0} & \cdots & \frac{\partial^2 L^*(\hat{\boldsymbol{\theta}})}{\partial\beta_p\beta_0} & \frac{\partial^2 L^*(\hat{\boldsymbol{\theta}})}{\partial\gamma_1\beta_0} & \cdots & \frac{\partial^2 L^*(\hat{\boldsymbol{\theta}})}{\partial\gamma_{M-1}\beta_0}\\
\frac{\partial^2 L^*(\hat{\boldsymbol{\theta}})}{\partial\beta_0\beta_1} & \frac{\partial^2 L^*(\hat{\boldsymbol{\theta}})}{\partial\beta_1\beta_1} & \frac{\partial^2 L^*(\hat{\boldsymbol{\theta}})}{\partial\beta_2\beta_1} & \cdots & \frac{\partial^2 L^*(\hat{\boldsymbol{\theta}})}{\partial\beta_p\beta_1} & \frac{\partial^2 L^*(\hat{\boldsymbol{\theta}})}{\partial\gamma_1\beta_1} & \cdots & \frac{\partial^2 L^*(\hat{\boldsymbol{\theta}})}{\partial\gamma_{M-1}\beta_1}\\
\frac{\partial^2 L^*(\hat{\boldsymbol{\theta}})}{\partial\beta_0\beta_2} & \frac{\partial^2 L^*(\hat{\boldsymbol{\theta}})}{\partial\beta_1\beta_2} & \frac{\partial^2 L^*(\hat{\boldsymbol{\theta}})}{\partial\beta_2\beta_2} & \cdots & \frac{\partial^2 L^*(\hat{\boldsymbol{\theta}})}{\partial\beta_p\beta_2} & \frac{\partial^2 L^*(\hat{\boldsymbol{\theta}})}{\partial\gamma_1\beta_2} & \cdots & \frac{\partial^2 L^*(\hat{\boldsymbol{\theta}})}{\partial\gamma_{M-1}\beta_2}\\
\vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots\\
\frac{\partial^2 L^*(\hat{\boldsymbol{\theta}})}{\partial\beta_0\beta_p} & \frac{\partial^2 L^*(\hat{\boldsymbol{\theta}})}{\partial\beta_1\beta_p} & \frac{\partial^2 L^*(\hat{\boldsymbol{\theta}})}{\partial\beta_2\beta_p} & \cdots & \frac{\partial^2 L^*(\hat{\boldsymbol{\theta}})}{\partial\beta_p\beta_p} & \frac{\partial^2 L^*(\hat{\boldsymbol{\theta}})}{\partial\gamma_1\beta_p} & \cdots & \frac{\partial^2 L^*(\hat{\boldsymbol{\theta}})}{\partial\gamma_{m-1}\beta_p}\\
\frac{\partial^2 L^*(\hat{\boldsymbol{\theta}})}{\partial\beta_0\gamma_1} & \frac{\partial^2 L^*(\hat{\boldsymbol{\theta}})}{\partial\beta_1\gamma_1} & \frac{\partial^2 L^*(\hat{\boldsymbol{\theta}})}{\partial\beta_2\gamma_1} & \cdots & \frac{\partial^2 L^*(\hat{\boldsymbol{\theta}})}{\partial\beta_p\gamma_1} & \frac{\partial^2 L^*(\hat{\boldsymbol{\theta}})}{\partial\gamma_1\gamma_1} & \cdots & \frac{\partial^2 L^*(\hat{\boldsymbol{\theta}})}{\partial\gamma_{M-1}\gamma_1}\\
\vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots\\
\frac{\partial^2 L^*(\hat{\boldsymbol{\theta}})}{\partial\beta_0\gamma_{M-1}} & \frac{\partial^2 L^*(\hat{\boldsymbol{\theta}})}{\partial\beta_1\gamma_{M-1}} & \frac{\partial^2 L^*(\hat{\boldsymbol{\theta}})}{\partial\beta_2\gamma_{M-1}} & \cdots & \frac{\partial^2 L^*(\hat{\boldsymbol{\theta}})}{\partial\beta_p\gamma_{M-1}} & \frac{\partial^2 L^*(\hat{\boldsymbol{\theta}})}{\partial\gamma_1\gamma_{M-1}} & \cdots & \frac{\partial^2 L^*(\hat{\boldsymbol{\theta}})}{\partial\gamma_{M-1}\gamma_{M-1}}
\end{bmatrix}^{-1}$$

dengan :

$$\frac{\partial^2 L^*}{\partial\beta_u\beta_v}=\sum_{j=1}^{n}\sum_{k=1}^{M}Y_{j,k}\left[\frac{\left(\Phi_{j,k}-\Phi_{j,k-1}\right)\left(N_{j,k-1}Z_{j,k-1}-N_{j,k}Z_{j,k}\right)}{\left(\Phi_{j,k}-\Phi_{j,k-1}\right)^2}-\frac{\left(N_{j,k-1}-N_{j,k}\right)^2}{\left(\Phi_{j,k}-\Phi_{j,k-1}\right)^2}\right]X_{u,j}X_{v,j}$$

$$\frac{\partial^2 L^*}{\partial\beta_u\gamma_v}=\sum_{j=1}^{n}\sum_{k=1}^{M}Y_{j,k}\left[\begin{array}{l}\dfrac{\left(\Phi_{j,k}-\Phi_{j,k-1}\right)\left(N_{j,k}Z_{j,k}\delta_{k,v}-N_{j,k-1}Z_{j,k-1}\delta_{k-1,v}\right)}{\left(\Phi_{j,k}-\Phi_{j,k-1}\right)^2}\\ -\dfrac{\left(N_{j,k-1}-N_{j,k}\right)\left(N_{j,k}\delta_{k,v}-N_{j,k-1}\delta_{k-1,v}\right)}{\left(\Phi_{j,k}-\Phi_{j,k-1}\right)^2}\end{array}\right]X_{u,j}$$

$$\frac{\partial^2 L^*}{\partial\gamma_u\gamma_v}=\sum_{j=1}^{n}\sum_{k=1}^{M}Y_{j,k}\left[\begin{array}{l}\dfrac{\left(\Phi_{j,k}-\Phi_{j,k-1}\right)\left(N_{j,k-1}Z_{j,k-1}\delta_{u,k-1}\delta_{v,k-1}-N_{j,k}Z_{j,k}\delta_{u,k}\delta_{v,k}\right)}{\left(\Phi_{j,k}-\Phi_{j,k-1}\right)^2}\\ -\dfrac{\left(N_{j,k}\delta_{u,k}-N_{j,k-1}\delta_{u,k-1}\right)\left(N_{j,k}\delta_{v,k}-N_{j,k-1}\delta_{v,k-1}\right)}{\left(\Phi_{j,k}-\Phi_{j,k-1}\right)^2}\end{array}\right]$$

LAMPIRAN 3

Mencari Distribusi Uji Rasio Likelihood

Uji rasio likelihood digunakan dalam pengujian kegunaan model, dengan hipotesis sebagai berikut :

$H_0 : \beta_1 = \beta_2 = ... = \beta_p = 0$

$H_1 : \exists \beta_i \neq 0$, *untuk* $i = 1, 2, ..., p$

Definisikan statistik uji Deviance (Hosmer & Lemeshow, 2000 : 13), sebagai berikut :

$$D = -2 \ln \left[\frac{L(\hat{\boldsymbol{\beta}})}{L(\hat{\boldsymbol{\beta}}_{\max})} \right]$$
$$= 2 \left[L^*(\hat{\boldsymbol{\beta}}_{\max}) - L^*(\hat{\boldsymbol{\beta}}) \right]$$

dengan $L^*(\hat{\boldsymbol{\beta}}_{\max})$ adalah nilai dari fungsi log-likelihood untuk model yang jumlah parameternya sama dengan jumlah observasi, yaitu n (*saturated model*), yang dihitung pada $\boldsymbol{\beta}_{\max} = \hat{\boldsymbol{\beta}}_{\max}$ dan $L^*(\hat{\boldsymbol{\beta}})$ adalah nilai fungsi log-likelihood untuk model yang dibentuk yang terdiri dari *p+1* parameter (*fitted model*), yang dihitung pada $\boldsymbol{\beta} = \hat{\boldsymbol{\beta}}$, dimana $\boldsymbol{\beta} = (\beta_0, \beta_1, \beta_2, ..., \beta_p)^T$.

Pendekatan deret Taylor orde kedua untuk mencari nilai $L^*(\boldsymbol{\beta})$ di $\boldsymbol{\beta} = \hat{\boldsymbol{\beta}}$

adalah : $L^*(\boldsymbol{\beta}) \approx L^*(\hat{\boldsymbol{\beta}}) + (\boldsymbol{\beta} - \hat{\boldsymbol{\beta}})^T \left.\frac{\partial L^*(\boldsymbol{\beta})}{\partial \boldsymbol{\beta}}\right|_{\boldsymbol{\beta}=\hat{\boldsymbol{\beta}}} + \frac{1}{2}(\boldsymbol{\beta} - \hat{\boldsymbol{\beta}})^T \left.\frac{\partial^2 L^*(\boldsymbol{\beta})}{\partial \boldsymbol{\beta} \partial \boldsymbol{\beta}^T}\right|_{\boldsymbol{\beta}=\hat{\boldsymbol{\beta}}} (\boldsymbol{\beta} - \hat{\boldsymbol{\beta}})$

Diketahui bahwa $-\mathbf{H}(\boldsymbol{\beta}) = -\frac{\partial^2 L^*(\boldsymbol{\beta})}{\partial \boldsymbol{\beta} \partial \boldsymbol{\beta}^T}$ dan karena $\hat{\boldsymbol{\beta}}$ merupakan taksiran

maximum likelihood maka $\left.\frac{\partial L^*(\boldsymbol{\beta})}{\partial \boldsymbol{\beta}}\right|_{\boldsymbol{\beta}=\hat{\boldsymbol{\beta}}} = \mathbf{0}$. Dengan demikian diperoleh :

$$L^*(\boldsymbol{\beta}) \approx L^*(\hat{\boldsymbol{\beta}}) - \frac{1}{2}(\boldsymbol{\beta} - \hat{\boldsymbol{\beta}})^T \left[-\mathbf{H}(\hat{\boldsymbol{\beta}})\right](\boldsymbol{\beta} - \hat{\boldsymbol{\beta}})$$
$$L^*(\boldsymbol{\beta}) \approx L^*(\hat{\boldsymbol{\beta}}) - \frac{1}{2}(\hat{\boldsymbol{\beta}} - \boldsymbol{\beta})^T \left[-\mathbf{H}(\hat{\boldsymbol{\beta}})\right](\hat{\boldsymbol{\beta}} - \boldsymbol{\beta})$$
$$\frac{1}{2}(\hat{\boldsymbol{\beta}} - \boldsymbol{\beta})^T \left[-\mathbf{H}(\hat{\boldsymbol{\beta}})\right](\hat{\boldsymbol{\beta}} - \boldsymbol{\beta}) \approx L^*(\hat{\boldsymbol{\beta}}) - L^*(\boldsymbol{\beta})$$

Jika *n* besar maka dengan menggunakan *Central Limit Theorem* diperoleh

bahwa $\frac{(\hat{\boldsymbol{\beta}} - \boldsymbol{\beta})}{\sqrt{\left[-H(\hat{\boldsymbol{\beta}})\right]^{-1}}} \sim N(\mathbf{0},\mathbf{1})$, dan juga diperoleh

$(\hat{\boldsymbol{\beta}} - \boldsymbol{\beta})^T \left[-\mathbf{H}(\hat{\boldsymbol{\beta}})\right](\hat{\boldsymbol{\beta}} - \boldsymbol{\beta}) \sim \lambda^2_{p+1}$, dengan demikian didapatkan :

$2\left[L^*(\hat{\boldsymbol{\beta}}) - L^*(\boldsymbol{\beta})\right] \sim \lambda^2_{p+1}$ dimana *p+1* merupakan jumlah parameter dalam

model.

Dari statistik uji Deviance:

$$D = 2\left[L^*(\hat{\boldsymbol{\beta}}_{\max}) - L^*(\hat{\boldsymbol{\beta}})\right]$$

dapat diperoleh :

$$G = 2\left(L^*\left(\hat{\boldsymbol{\beta}}_{\max}\right) - L^*\left(\hat{\boldsymbol{\beta}}\right)\right)$$

$$= 2\left\{\left[\left(L^*\left(\hat{\boldsymbol{\beta}}_{\max}\right) - L^*\left(\boldsymbol{\beta}_{\max}\right)\right)\right] - \left[\left(L^*\left(\hat{\boldsymbol{\beta}}\right) - L^*\left(\boldsymbol{\beta}\right)\right)\right] + \left[\left(L^*\left(\boldsymbol{\beta}_{\max}\right) - L^*\left(\boldsymbol{\beta}\right)\right)\right]\right\}$$

$$= 2\left[\left(L^*\left(\hat{\boldsymbol{\beta}}_{\max}\right) - L^*\left(\boldsymbol{\beta}_{\max}\right)\right)\right] - 2\left[\left(L^*\left(\hat{\boldsymbol{\beta}}\right) - L^*\left(\boldsymbol{\beta}\right)\right)\right] + 2\left[\left(L^*\left(\boldsymbol{\beta}_{\max}\right) - L^*\left(\boldsymbol{\beta}\right)\right)\right]$$

Berdasarkan definisi model lengkap dan berdasarkan persamaan :

$$2\left[L^*(\hat{\boldsymbol{\beta}}) - L^*(\boldsymbol{\beta})\right] \sim \lambda^2_{p+1}$$

maka dari persamaan berikut :

$$2\left[\left(L^*\left(\hat{\boldsymbol{\beta}}_{\max}\right) - L^*\left(\boldsymbol{\beta}_{\max}\right)\right)\right] - 2\left[\left(L^*\left(\hat{\boldsymbol{\beta}}\right) - L^*\left(\boldsymbol{\beta}\right)\right)\right] + 2\left[\left(L^*\left(\boldsymbol{\beta}_{\max}\right) - L^*\left(\boldsymbol{\beta}\right)\right)\right]$$

suku pertama pada persamaan diatas mendekati distribusi Chi-Kuadrat dengan derajat bebas *n*, sedangkan suku kedua mendekati distribusi Chi-Kuadrat dengan derajat bebas *p+1*, dan suku ketiga merupakan suatu konstanta. Konstanta ini akan mendekati nol apabila model model dengan *p+1* parameter dapat menggambarkan data seperti pada model lengkap. Dengan demikian :

$$D = 2\left[L^*(\hat{\boldsymbol{\beta}}_{\max}) - L^*(\hat{\boldsymbol{\beta}})\right] \sim \lambda^2_{n-(p+1)}$$

Untuk menguji apakah variabel penjelas secara signifikan mempengaruhi variabel *dependent*, statistik uji Deviance dari model dengan variabel bebas dan statistik uji dari model tanpa menggunakan variabel bebas akan dibandingkan.

Hipotesis sebagai berikut :

$H_0 : \beta_1 = \beta_2 = ... = \beta_p = 0$

$H_1 : \exists \beta_i \neq 0$, *untuk* $i = 1, 2, ..., p$

Statistik uji :

$$\begin{aligned} G &= D_0 - D_1 \\ &= 2\left[L^*(\hat{\boldsymbol{\beta}}_{\max}) - L^*(\hat{\beta}_0) \right] - 2\left[L^*(\hat{\boldsymbol{\beta}}_{\max}) - L^*(\hat{\boldsymbol{\beta}}) \right] \\ &= 2\left[L^*(\hat{\boldsymbol{\beta}}) - L^*(\beta_0) \right] \end{aligned}$$

dengan D_0 merupakan statistik uji Deviance untuk model tanpa menggunakan variabel penjelas sedangkan D_0 merupakan statistik uji Deviance untuk model dengan menggunakan variabel penjelas.

Dari persamaan :

$$D = 2\left[L^*(\hat{\boldsymbol{\beta}}_{\max}) - L^*(\hat{\boldsymbol{\beta}}) \right] \sim \lambda^2_{n-(p+1)}$$

diperoleh $D_0 \sim \lambda^2_{n-(1)}$ dan $D_1 \sim \lambda^2_{n-(p+1)}$, dengan demikian :

$$G = 2\left[L^*(\hat{\boldsymbol{\beta}}) - L^*(\beta_0) \right] \sim \chi^2_p$$

Statistik uji di atas yang akan digunakan untuk pengujian kegunaan model.

LAMPIRAN 4

Program Matlab Metode Newton Raphson Untuk Menaksir Parameter

taksir.m

```
function out = taksir(x, y, b, g, tol)
clc;
g = [0 g 100];
lb = length(b);
lg = length(g);
lg1 = lg-2;
theta = [b g];
cond = true;

while(cond)
    H=calcH(x,y,theta(1:lb),theta(lb+1:lb+lg))
    iH=inv(H)
    siH=sqrt(-iH)

    F=zeros(length(theta),1);
    for m=1:length(theta)-1
```

```
    F(m)=calcF(x,y,theta(1:lb),theta(lb+1:lb+lg),m);
  end
  F = F';
  F = [F(1:lb) F(lb+2:(lb+lg)-1)];
  F = F';
  theta1 = [theta(1:lb) theta(lb+2:(lb+lg)-1)];
  theta2=(theta1'-(iH*F))';
  cond = norm(abs(theta1 - theta2)) > tol;
  theta =[theta2(1:lb) 0 theta2(lb+1:(lb+lg1)) 100];
  disp(theta2);
end
out = theta2;
```

---

calcH.m

```
function H = calcH(x,y,b,g)

[j1 k1] = size(y);

H = zeros(length([b g])-2);
for j=1:j1
  for k=1:k1
```

```
for m=1:length([b g])-2
   for n=m:length([b g])-2
      if m<=length(b) & n<=length(b)
         %beta beta
         u=m-1;
         v=n-1;
         %disp([j k m n]);
         H(m,n) = H(m,n) + (y(j,k)*((((psi1(x,j,k,b,g)-psi1(x,j,k-1,b,g)) *
(en(x,j,k-1,b,g)*zet(x,j,k-1,b,g)- en(x,j,k,b,g)*zet(x,j,k,b,g)) / ( ( psi1(x,j,k,b,g)-
psi1(x,j,k-1,b,g) )^2 ))-(( ( en(x,j,k-1,b,g)-en(x,j,k,b,g))^2 )/( ( psi1(x,j,k,b,g)-
psi1(x,j,k-1,b,g) )^2 ))) * x(u+1,j) * x(v+1,j)));
      elseif m>length(b) & n>length(b)
         %gamma gamma
         u=m-length(b);
         v=n-length(b);
         H(m,n) = H(m,n) + (y(j,k) * (((psi1(x,j,k,b,g)-psi1(x,j,k-
1,b,g))*(en(x,j,k-1,b,g)*zet(x,j,k-1,b,g)*delta(u,k-1)*delta(v,k-1) -
en(x,j,k,b,g)*zet(x,j,k,b,g)*delta(u,k)*delta(v,k))/((psi1(x,j,k,b,g)-psi1(x,j,k-
1,b,g))^2))-(((en(x,j,k,b,g)*delta(u,k)-en(x,j,k-1,b,g)*delta(u,k-
1))*(en(x,j,k,b,g)*delta(v,k)-en(x,j,k-1,b,g)*delta(v,k-1)))/((psi1(x,j,k,b,g)-
psi1(x,j,k-1,b,g))^2))));
      else
```

```
                    %beta gamma / gamma beta
                    u=m-1;
                    v=n-length(b);
                    H(m,n) = H(m,n) +(y(j,k) * (((psi1(x,j,k,b,g)-psi1(x,j,k-1,b,g))*(en(x,j,k,b,g)*zet(x,j,k,b,g)*delta(v,k)-en(x,j,k-1,b,g)*zet(x,j,k-1,b,g)*delta(v,k-1))/((psi1(x,j,k,b,g)-psi1(x,j,k-1,b,g))^2))-(((en(x,j,k-1,b,g)-en(x,j,k,b,g))*(en(x,j,k,b,g)*delta(v,k)-en(x,j,k-1,b,g)*delta(v,k-1)))/((psi1(x,j,k,b,g)-psi1(x,j,k-1,b,g))^2)))* x(u+1,j));
                end
            end
        end
    end
end

for m=1:length([b g])-2
    for n=m+1:length([b g])-2
        H(n,m)=H(m,n);
    end
end
```

calcF.m

```
function out = calcF(x,y,b,g,m)


[j1 k1] = size(y);
out=0;
for j=1:j1
   %disp(j);
   for k=1:k1
      %disp(k);
      if m<=length(b)
         %beta
         u=m-1;
         out = out + (y(j,k) * ((en(x,j,k-1,b,g)-en(x,j,k,b,g))/(psi1(x,j,k,b,g)-
psi1(x,j,k-1,b,g))) * x(u+1,j));
      else
         %gamma
         u=m-length(b)-1;
         out = out + (y(j,k) * ((en(x,j,k,b,g)*delta(u,k)-en(x,j,k-1,b,g)*delta(u,k-
1))/(psi1(x,j,k,b,g)-psi1(x,j,k-1,b,g))));
      end
   end
end
```

psi1.m

```
function out = psi1(x, j, k, b, g)

out = ((1/2) * (1+erf(zet(x, j, k, b, g)/sqrt(2))));
```

---

en.m

```
function out = en(x, j, k, b, g)

out = ((1/sqrt(2*pi)) * exp((-(zet(x, j, k, b, g)^2))/2));
```

---

delta.m

```
function out = delta(i, j)

if(i==j)
    out = 1;
else
    out = 0;
end
```

---

zet.m

```
function out = zet(x, j, k, b, g);


out = g(k+1) - (b(1)*x(1,j) + b(2)*x(2,j) + b(3)*x(3,j));
```