ON THE HARDY-LITTLEWOOD-PÓLYA MAJORIZATION INEQUALITY

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SKRIPSI : ON THE HARDY-LITTLEWOOD-PÓLYA MAJORIZATION

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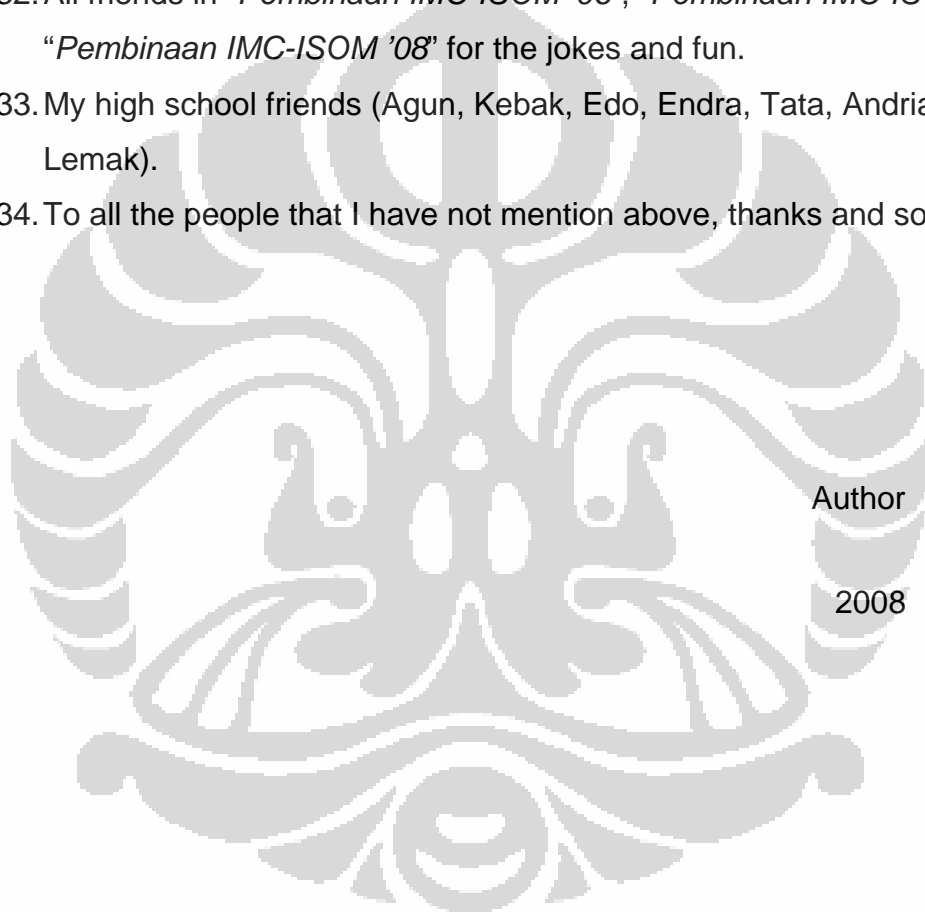
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"*The same thing we do every night, Pinky
—try to take over the world.*"

-The Brain
(From the Steven Spielberg's, Pinky and the Brain Cartoon Series)

ABSTRACT

This *skripsi* studies some variance of Majorization Inequalities such as the Classical Hardy-Littlewood-Pólya Majorization Inequality together with its Riemann integral form. The classical inequality is generalized by using the concept of relative convexity and as a result we have The Generalization of Hardy-Littlewood-Pólya Majorization Inequality together with its Riemann integral form. The proof of the generalized version given here is due to C.P Niculescu and F. Popovici. Some examples of the application of the resulting inequalities will also be given in this *skripsi* such as the proof of Jensen inequality and power mean inequality.

Keyword: majorization, convex function inequality, relative convexity.

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ABSTRAK

Skripsi ini mempelajari beberapa variasi dari Ketaksamaan Mayorisasi diantaranya Ketaksamaan Mayorisasi Klasik Hardy-Littlewood-Pólya dan bentuk Integral Riemann dari ketaksamaan tersebut. Ketaksamaan klasik tersebut digeneralisasi dengan menggunakan konsep relatif konveks dan sebagai hasilnya diperoleh Generalisasi Ketaksamaan Mayorisasi Hardy-Littlewood-Pólya dan bentuk Integral Riemann dari ketaksamaan tersebut. Bukti dari versi generalisasi yang diberikan disini ditemukan oleh C.P Niculescu and F. Popovici. Beberapa contoh pengaplikasian dari ketaksamaan-ketaksamaan yang dihasilkan juga akan diberikan di skripsi ini seperti bukti dari ketaksamaan Jensen dan ketaksamaaan rataan pangkat.

Kata kunci: mayorisasi, ketaksamaan fungsi konveks, relatif konveks.

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