CHAPTER IV

CONCLUSION

Throughout the discussion in the previous chapters, we see that the Theorem 3.1.2, Classical Hardy-Littlewood-Polka Majorization inequality, is the special case of Theorem 3.2.1, Extended Hardy-Littlewood-Pólya Majorization Inequality, with g replaced by identity function. By choosing various g and the sequences $\{x_i\}$ and $\{y_i\}$ on Theorem 3.2.1, there are many other inequalities that can be concluded such as Power Mean Inequality , and other miscellaneous inequalities depended on the choice of f and g which satisfies $g \lhd f$.

Theorem 3.1.3, Riemann form of Hardy-Littlewood-Pólya Majorization inequality, and Theorem 3.2.2, The Riemann Form of Generalization of Hardy-Littlewood- Pólya Majorization Inequality, give the analog continuous version of Theorem 3.1.2 and 3.2.1; the hypothesis of Theorem 3.1.3 and Theorem 3.2.2 give an analog version of majorization through functions. Notice that Theorem 3.1.3 and Theorem 3.2.2 compare the values of the integrals $\int_a^b f(\varphi(t))dt$ and $\int_a^b f(\phi(t))dt$ without evaluate them, but we need to evaluate the integrals $\int_a^x \varphi(t)dt$ and $\int_a^x \phi(t)dt$ for $x \in [a,b]$. By this point of view it seems that the whole computation would be more complicated, but as we can see in Example 3.3.4 where we use $\varphi(t) = -S'(t)$ and $\phi(t) = -T'(t)$, the integrals $\int_a^x S'(t)dt$

and $\int_a^x T'(t)dt$ are easier to evaluate than integrals $\int_a^b \sqrt{1+\left(-S'(t)\right)^2}\,dt$

and $\int_a^b \sqrt{1+\left(-T'(t)\right)^2}\,dt$.