LAMPIRAN A

A1. Daftar Singkatan

GEVP	Generalized Eigen Value Minimization Problem
GUI	Graphical User Interface
LQG	Linear Quadratic Gaussian
T-S	Takagi Sugeno
PDC	Parallel Distributed Compensation
LMI	Linear Matrix Inequality
VAF	Variance Accounted For

A2. Daftar Simbol

$\equiv$	identik sama dengan
$\neq$	tidak sama dengan
$\triangleq$	didefinisikan sebagai
$>$	lebih besar dari
$\geq$	lebih besar atau sama dengan
$<$	lebih kecil dari
$\leq$	lebih kecil atau sama dengan
$\in$	elemen dari
$\Rightarrow$	implikasi ('maka')
Π	produk (perkalian)
Σ	penjumlahan
$\lvert x \rvert$	nilai mutlak dari *x*
$\lvert\lvert x \rvert\rvert$	norm vector *x*
$\lvert\lvert x \rvert\rvert_2$	Euclidean norm vector *x*
max	maksimum

min	minimum
R^k , $R^{m\times n}$	Real vector *k*, Real matriks *m×n*
$P > 0$	matriks definit positif *P*
$P \geq 0$	matriks semidefinit positif *P*
A^T	*transpose* dari matriks *A*
A^{-1}	invers dari matriks *A*
I , I_n	matriks identitas, matriks identitas *n×n*
var(*y*)	varians dari kelompok data *y*
$\lim_{x\to 0}$	limit *x* menuju nol
$\dot{x}$	turunan pertama dari *x*
x(0)	kondisi awal *x*, nilai *x* pada saat *t* = 0.

LAMPIRAN B

B1. Pembuktian Persamaan 2.21

Persamaan konstrain untuk truk menurut (2.18) adalah

$$\dot{x}_{p0} \sin\theta_0 - \dot{y}_{p0} \cos\theta = 0 \quad \text{(B.1)}$$

Diferensiasi persamaan (2.19) menghasilkan

$$\dot{x}_f = \dot{x}_{p0} - l\dot{\theta}_0 \sin\theta_0 \qquad \dot{y}_f = \dot{y}_{p0} + l\dot{\theta}_0 \cos\theta_0 \quad \text{(B.2)}$$

Substitusi (B.1) dan (B.2) pada persamaan *constrain*

$$\dot{x}_f \ \sin(\theta_0 + \phi) = \dot{y}_f \ \cos(\theta_0 + \phi)$$

menghasilkan:

$$(\dot{x}_{p0} - l\dot{\theta}_0 \sin\theta_0)\sin(\theta_0 + \Phi) = (\dot{y}_{p0} + l\dot{\theta}_0 \cos\theta_0)\cos(\theta_0 + \Phi) \quad \text{(B.3)}$$

Ruas kiri persamaan (B.3) menghasilkan

$$= \dot{x}_{p0} \sin(\theta_0 + \Phi) - l\dot{\theta}_0 \sin\theta_0 \sin(\theta_0 + \Phi)$$

$$= \dot{x}_{p0} \sin(\theta_0 + \Phi) - l\dot{\theta}_0 \sin^2\theta_0 \cos\Phi - l\dot{\theta}_0 \sin\theta_0 \cos\theta_0 \sin\Phi \quad \text{(B.4)}$$

Dengan cara yang sama untuk ruas kanan, menghasilkan

$$= \dot{y}_{p0} \cos(\theta_0 + \Phi) + l\dot{\theta}_0 \cos\theta_0 \cos(\theta_0 + \Phi)$$

$$= \dot{y}_{p0} \cos(\theta_0 + \Phi) + l\dot{\theta}_0 \cos^2\theta_0 \cos\Phi - l\dot{\theta}_0 \cos\theta_0 \sin\theta_0 \sin\Phi \quad \text{(B.5)}$$

Menggabungkan (B.4) dan (B.5) menghasilkan

$$\dot{x}_{p0} \sin(\theta_0 + \Phi) - l\dot{\theta}_0 \sin^2\theta_0 \cos\Phi - l\dot{\theta}_0 \sin\theta_0 \cos\theta_0 \sin\Phi$$

$$- \dot{y}_{p0} \cos(\theta_0 + \Phi) - l\dot{\theta}_0 \cos^2\theta_0 \cos\Phi + l\dot{\theta}_0 \cos\theta_0 \sin\theta_0 \sin\Phi = 0$$

$$\dot{x}_{p0} \sin(\theta_0 + \Phi) - \dot{y}_{p0} \cos(\theta_0 + \Phi) - l\dot{\theta}_0 (\sin^2\theta_0 + \cos^2\theta_0)\cos\Phi = 0$$

$$\dot{x}_{p0} \sin(\theta_0 + \Phi) - \dot{y}_{p0} \cos(\theta_0 + \Phi) - l\dot{\theta}_0 \cos\Phi = 0 \quad \text{(B.6)}$$

Terlihat bahwa (B.6) ≡ (2.21)

B2. Pembuktian Persamaan 2.22

Menurut persamaan (2.20) diperoleh,

$$x_{p1} = x_{p0} - L\cos\theta_1 \qquad y_{p1} = y_{p0} - L\sin\theta_1$$
$$x_{p2} = x_{p1} - L\cos\theta_2 \qquad y_{p2} = y_{p1} - L\sin\theta_2$$
$$x_{p3} = x_{p2} - L\cos\theta_3 \qquad y_{p2} = y_{p2} - L\sin\theta_3$$

Differensiasi persaman diatas menghasilkan:

$$\dot{x}_{p1} = \dot{x}_{p0} + L\dot{\theta}_1 \sin\theta_1 \qquad \dot{y}_{p1} = \dot{y}_{p0} - L\dot{\theta}_1 \cos\theta_1$$
$$\dot{x}_{p2} = \dot{x}_{p1} + L\dot{\theta}_2 \sin\theta_2 \qquad \dot{y}_{p2} = \dot{y}_{p1} - L\dot{\theta}_2 \cos\theta_2$$
$$\dot{x}_{p3} = \dot{x}_{p2} + L\dot{\theta}_3 \sin\theta_3 \qquad \dot{y}_{p3} = \dot{y}_{p2} - L\dot{\theta}_3 \cos\theta_3$$

Berdasarkan (2.18), persamaan *constraint* untuk i = 1,2,3 menjadi

$$\dot{x}_{p1} \sin\theta_1 - \dot{y}_{p1} \cos\theta_1 = 0$$
$$\dot{x}_{p2} \sin\theta_2 - \dot{y}_{p2} \cos\theta_2 = 0$$
$$\dot{x}_{p3} \sin\theta_3 - \dot{y}_{p3} \cos\theta_3 = 0$$

Untuk i = 1, diperoleh persamaan *constraint*:

$$(\dot{x}_{p0} + L\dot{\theta}_1 \sin\theta_1)\sin\theta_1 - (\dot{y}_{p0} - L\dot{\theta}_1 \cos\theta_1)\cos\theta_1 = 0$$

$$\dot{x}_{p0} \sin\theta_1 - \dot{y}_{p0} \cos\theta_1 + L\dot{\theta}_1(\sin^2\theta_1 + \cos^2\theta_1) = 0$$

$$\dot{x}_{p0} \sin\theta_1 - \dot{y}_{p0} \cos\theta_1 + L\dot{\theta}_1 = 0 \qquad \text{(B.7)}$$

Untuk i = 2, diperoleh persamaan *constraint*:

$$(\dot{x}_{p1} + L\dot{\theta}_2 \sin\theta_2)\sin\theta_2 - (\dot{y}_{p1} - L\dot{\theta}_2 \cos\theta_2)\cos\theta_2 = 0$$

$$\dot{x}_{p1} \sin\theta_2 - \dot{y}_{p1} \cos\theta_2 + L\dot{\theta}_2(\sin^2\theta_2 + \cos^2\theta_2) = 0$$

$$(\dot{x}_{p0} + L\dot{\theta}_1 \sin\theta_1)\sin\theta_2 - (\dot{y}_{p0} - L\dot{\theta}_1 \cos\theta_1)\cos\theta_2 + L\dot{\theta}_2(\sin^2\theta_2 + \cos^2\theta_2) = 0$$

$$\dot{x}_{p0} \sin\theta_2 - \dot{y}_{p0} \cos\theta_2 + L\dot{\theta}_1(\sin\theta_1 \sin\theta_2 + \cos\theta_1 \cos\theta_2) + L\dot{\theta}_2 = 0$$

$$\dot{x}_{p0}\sin\theta_2 - \dot{y}_{p0}\cos\theta_2 + L\dot{\theta}_1\cos(\theta_1-\theta_2) + L\dot{\theta}_2 = 0 \qquad (B.8)$$

Dengan cara yang sama untuk $i = 2$, diperoleh persamaan *constraint*:

$$\dot{x}_{p0}\sin\theta_3 - \dot{y}_{p0}\cos\theta_3 + L\dot{\theta}_1\cos(\theta_1-\theta_3) + L\dot{\theta}_2\cos(\theta_2-\theta_3) + L\dot{\theta}_2 = 0 \qquad (B.9)$$

Persamaan (B.7) – (B.9) adalah penjabaran dari persamaan (2.22) untuk $i = 1,2,3$.

B3. Pembuktian Persamaan 2.23

Persamaan (B.6) dapat diuraikan menjadi:

$$\dot{x}_{p0}(\sin\theta_0\cos\Phi + \cos\theta_0\sin\Phi) - \dot{y}_{p0}(\cos\theta_0\cos\Phi - \sin\theta_0\sin\Phi) = l\dot{\theta}_0\cos\Phi$$

$$(\dot{x}_{p0}\sin\theta_0 - \dot{y}_{p0}\cos\theta_0)\cos\Phi + (\dot{x}_{p0}\cos\theta_0 + \dot{y}_{p0}\sin\theta_0)\sin\Phi = l\dot{\theta}_0\cos\Phi$$

$$(0)\cos\Phi + (v\cos^2\theta_0 + v\sin^2\theta_0)\sin\Phi = l\dot{\theta}_0\cos\Phi$$

$$v\tan\Phi = l\dot{\theta}_0$$

$$\therefore\ \dot{\theta}_0 = \frac{v}{l}\tan\Phi \qquad (B.10)$$

Persamaan (B.7) dapat diuraikan menjadi:

$$\dot{x}_{p0}\sin\theta_1 - \dot{y}_{p0}\cos\theta_1 + L\dot{\theta}_1 = 0$$

$$-v\cos\theta_0\sin\theta_1 + v\sin\theta_0\cos\theta_1 = L\dot{\theta}_1$$

$$v\sin(\theta_0-\theta_1) = L\dot{\theta}_1$$

$$\therefore\ \dot{\theta}_1 = \frac{v}{L}\sin(\theta_0-\theta_1) \qquad (B.11)$$

Persamaan (B.8) dapat diuraikan menjadi:

$$\dot{x}_{p0}\sin\theta_2 - \dot{y}_{p0}\cos\theta_2 + L\dot{\theta}_1\cos(\theta_1-\theta_2) + L\dot{\theta}_2 = 0$$

$$-v\cos\theta_0\sin\theta_2 + v\sin\theta_0\cos\theta_2 - v\sin(\theta_0-\theta_1)\cos(\theta_1-\theta_2) = L\dot{\theta}_2$$

$$v\sin(\theta_0-\theta_2) - v\sin(\theta_0-\theta_1)\cos(\theta_1-\theta_2) = L\dot{\theta}_2$$

$$v\sin((\theta_0-\theta_1)+(\theta_1-\theta_2)) - v\sin(\theta_0-\theta_1)\cos(\theta_1-\theta_2) = L\dot{\theta}_2$$

$$v\sin(\theta_0-\theta_1)\cos(\theta_1-\theta_2)+v\cos(\theta_0-\theta_1)\sin(\theta_1-\theta_2)$$
$$-v\sin(\theta_0-\theta_1)\cos(\theta_1-\theta_2)=L\dot{\theta}_2$$

$$v\cos(\theta_0-\theta_1)\sin(\theta_1-\theta_2)=L\dot{\theta}_2$$

$$\therefore\ \dot{\theta}_2=\frac{v}{L}\cos(\theta_0-\theta_1)\sin(\theta_1-\theta_2) \qquad \text{(B.12)}$$

$$v\sin((\theta_0-\theta_1)+(\theta_1-\theta_2))-v\sin(\theta_0-\theta_1)\cos(\theta_1-\theta_2)=L\dot{\theta}_2$$

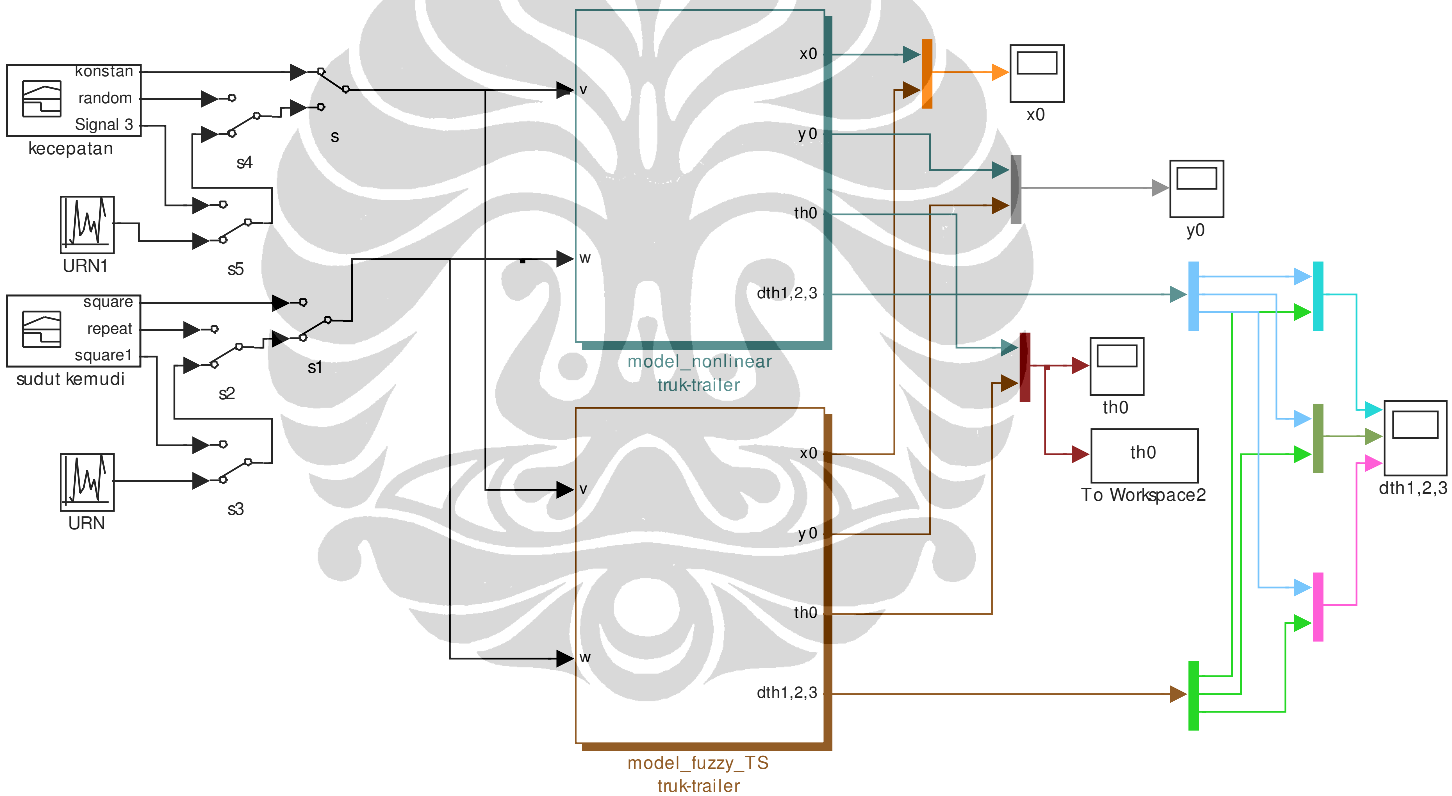
Dengan cara yang sama untuk (B.9) diperoleh:

$$\dot{\theta}_3=\frac{v}{L}\cos(\theta_0-\theta_1)\cos(\theta_1-\theta_2)\sin(\theta_2-\theta_3) \qquad \text{(B.13)}$$

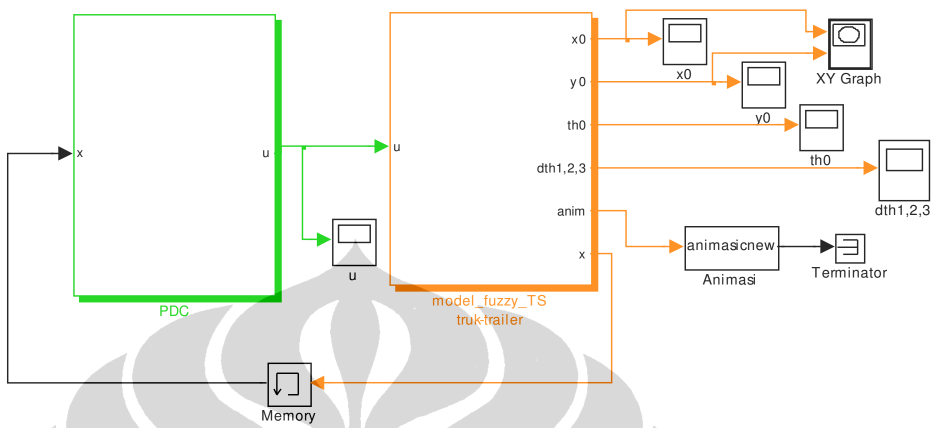
Untuk $d_1 = d_2 = d_3 = L$, maka (B.10) – (B.13) identik sama dengan 4 baris terakhir dari persamaan (2.23).

LAMPIRAN C

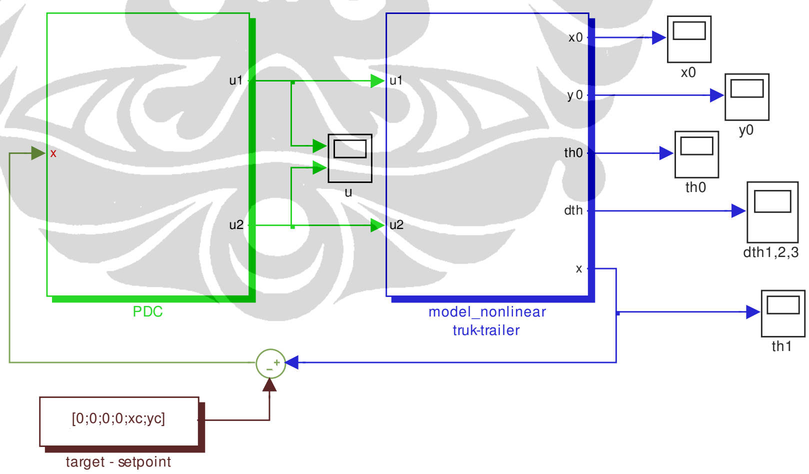
C.1 Simulink Diagram - Validasi



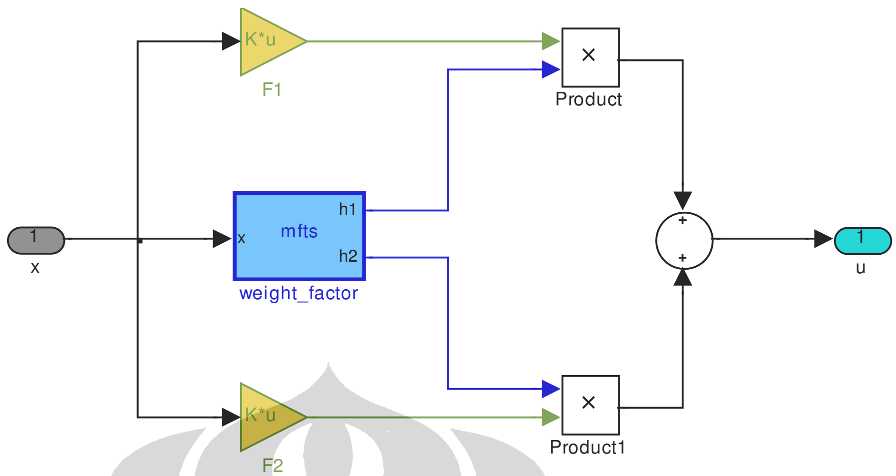
C2. Simulink Diagram: Simulasi PDC – Model Fuzzy



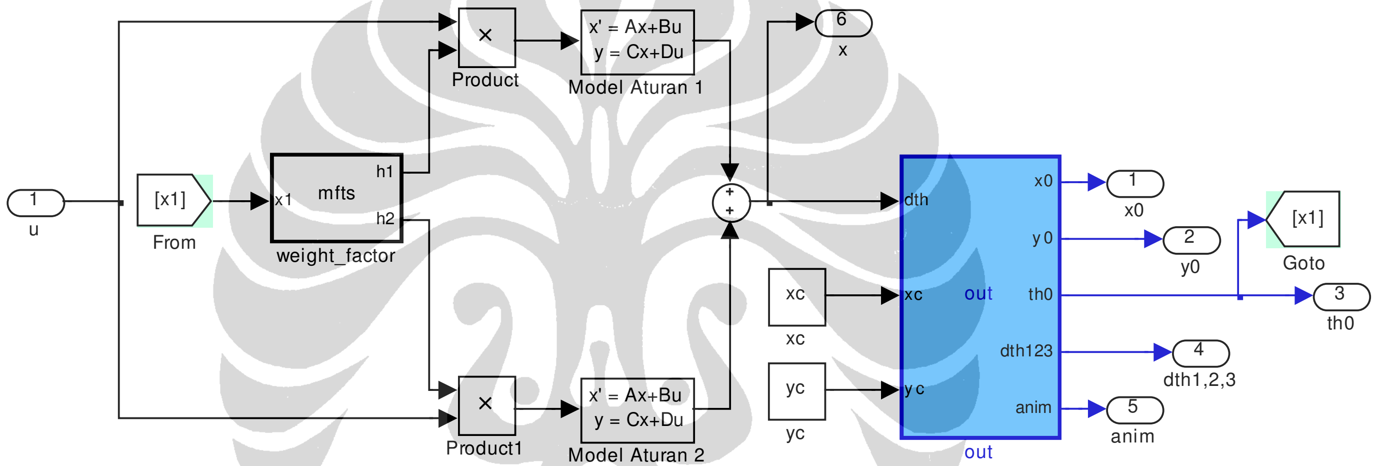
C3. Simulink Diagram: Simulasi PDC – Model Kinematik Truk-Trailer



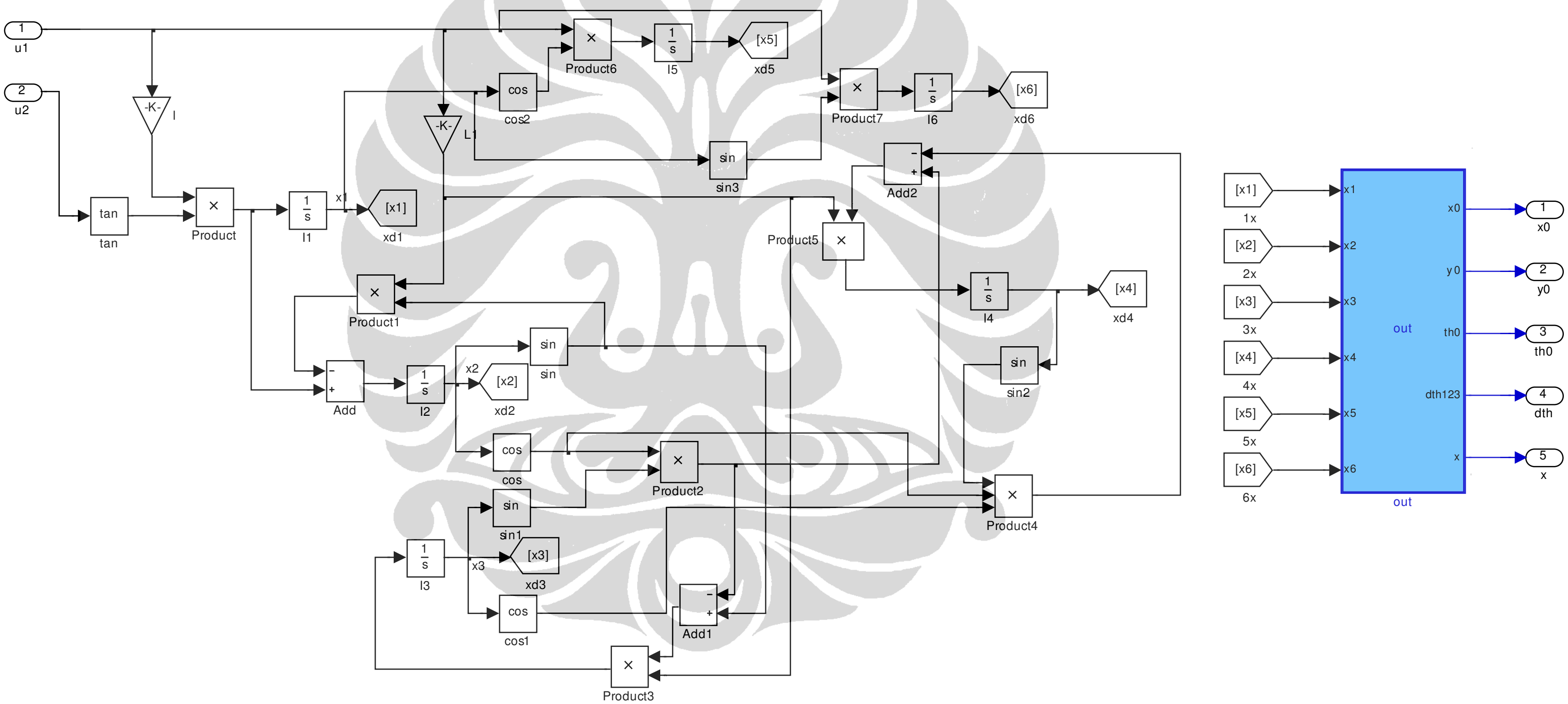
C4. Simulink Diagram: PDC Subsistem


K*u
F1
Product
1
x
mfts
h1
h2
weight_factor
1
u
K*u
F2
Product1

C5. Simulink Diagram: Model Fuzzy Subsistem



C6. Simulink Diagram: Model Kinematik Subsistem



LAMPIRAN D

D1. Matlab Source Code: Weight Factor

```
function [h1,h2] = mfts(x1)
% ------------------------------------------------------
% Program untuk mencari fungsi keangotaan dari
% model fuzzy Takagi Segeno dari sistem Truk dengan
% tiga Trailer.
% h1 adalah defuzz mf untuk model aturan 1
% h2 adalah defuzz mf untuk model aturan 2
% ------------------------------------------------------

q1 = 1;                        %faktor pengali untuk nilai
q2 = cos (88*pi/180);          %max dan min dari cos x1
b1 = 1;                        %faktor pengali untuk sector
b2 = 2/pi;                     %local dari grafik sn x1

m1 = (cos (x1) - q2)/(q1 - q2);
m2 = (q1 - cos (x1))/(q1 - q2);
if x1~=0
    n1 = (sin (x1) - b2*x1)/((b1 - b2)*x1);
    n2 = (b1*x1 - sin (x1))/((b1 - b2)*x1);
else
    n1 = 1;
    n2 = 0;
end
w1 = m1*n1;
w2 = m2*n2;
h1 = w1/(w1+w2);
h2 = w2/(w1+w2);
```

D2. Matlab Source Code: pdc.m

```
% ------------------------------------------------------------
% Program pengendali fuzzy PDC
% ------------------------------------------------------------

disp(' pilih ');
disp('(1) basic ');
disp('(2) input constraint ');
disp('(3) initial state ');
disp('(4) i/o constraint ');

pilih = input('pilihan : --> ');

switch pilih
    case 1
        lmi_basic;
    case 2
        lmi_consti;
    case 3
        lmi_init_independen;
    case 4
        lmi_constio;
    otherwise
        disp('salah input coy..');
end;

xc = input('x =  ');
yc = input('y =  ');

th0 = 0; th01 = 0; th12 = 0; th23 = 0; %sudut (derajat)

x5 = -xc;                %compensasion of accumulation
x6 = -yc;                %of 2 integrator
x1 = th0*pi/180;
x2 = th01*pi/180;
x3 = th12*pi/180;
x4 = th23*pi/180;

xawal = [x1;x2;x3;x4;x5;x6];
```

D3. Matlab Source Code: lmi_basic.m

```
% ------------------------------------------------------------
% Program mencari matrix positif definit P dan feedback gain
% Fi yang memenuhi kondisi kestabilan LMI
% ------------------------------------------------------------

clear all;
clc;

v = 0.126;                                   %kecepatan max mobil
l = .3;                                      %panjang truk
d = .15;                                     %panjang trailer

%Matriks untuk model fuzzy aturan 1

A1 = [0 0 0 0 0 0;            %th0 :sdt truk trhdp sb-x
      0 -v/d 0 0 0 0;         %th0 - th1 : sdt antara truk-tr1
      0 v/d -v/d 0 0 0;       %th1 - th2 : sdt antara tr1 - tr2
      0 0 v/d -v/d 0 0;       %th2 - th3 : sdt antara tr2 - tr3
      0 0 0 0 0 0;            %xcar : posisi truk
      v 0 0 0 0 0];           %ycar : posisi truk
B1 = [0 v/l;                  %matrix input
      0 v/l;
      0 0;
      0 0;
      1 0;
      0 0];

%Matriks untuk model fuzzy aturan 2

a = pi/2;  b = cosd(88);

A2 = [0 0 0 0 0 0;            %th0 :sdt truk trhdp sb-x
      0 -v/d 0 0 0 0;         %th0 - th1 : sdt antara truk-tr1
      0 v/d -v/d 0 0 0;       %th1 - th2 : sdt antara tr1 - tr2
      0 0 v/d -v/d 0 0;       %th2 - th3 : sdt antara tr2 - tr3
      0 0 0 0 0 0;            %xcar : posisi truk
      a*v 0 0 0 0 0];         %ycar : posisi truk

B2 = [0 v/l;                  %matrix input
      0 v/l;
      0 0;
      0 0;
      b 0;
      0 0];

C1 = eye(6);
C2 = C1;
CI = C1(2:4,:);
```

```
D1 = zeros(6,2);
D2 = D1;

%LMI editor
%LMI matrix variables
  setlmis([]);
  X=lmivar(1,[6 1]);
  M1=lmivar(2,[2 6]);
  M2=lmivar(2,[2 6]);

%LMI expressions
  lmiterm([-1 1 1 X],1,1);

  lmiterm([-2 1 1 X],1,-A1','s');
  lmiterm([-2 1 1 -M1],.5*1,B1','s');
  lmiterm([-2 1 1 M1],.5*B1,1,'s');

  lmiterm([-3 1 1 X],1,-A2','s');
  lmiterm([-3 1 1 -M2],.5*1,B2','s');
  lmiterm([-3 1 1 M2],.5*B2,1,'s');

  lmiterm([-4 1 1 X],1,-A1','s');
  lmiterm([-4 1 1 X],1,-A2','s');
  lmiterm([-4 1 1 -M2],.5*1,B1','s');
  lmiterm([-4 1 1 M2],.5*B1,1,'s');
  lmiterm([-4 1 1 -M1],.5*1,B2','s');
  lmiterm([-4 1 1 M1],.5*B2,1,'s');

  sys=getlmis;

%LMI Solver
  [tmin,xfeas]=feasp(sys);
  Xf=dec2mat(sys,xfeas,X);
  M1f=dec2mat(sys,xfeas,M1);
  M2f=dec2mat(sys,xfeas,M2);

%Feedback Gain & Common P
  P = inv(Xf);
  F1 = M1f*P;
  F2 = M2f*P;
```