CHAPTER 3
SPECIFIC STUDY OF THE COLLAPSE OF THE SOILS

3.1 Modeling of collapse

3.1.1 Geometry

We will present initially the various geometries of earth dam and cavities which we will study.

3.1.1.1 Circular cavity in a earth dam low height

The model of collapse of a circular conduit is taken in the longitudinal plan of an earth dam (Figure 15). The conduit thus has a direction upstream - downstream compared to the earth dam and one is placed in the longitudinal median plane of the earth dam. In this model low height, the height of the earth dam considered is of 10m.

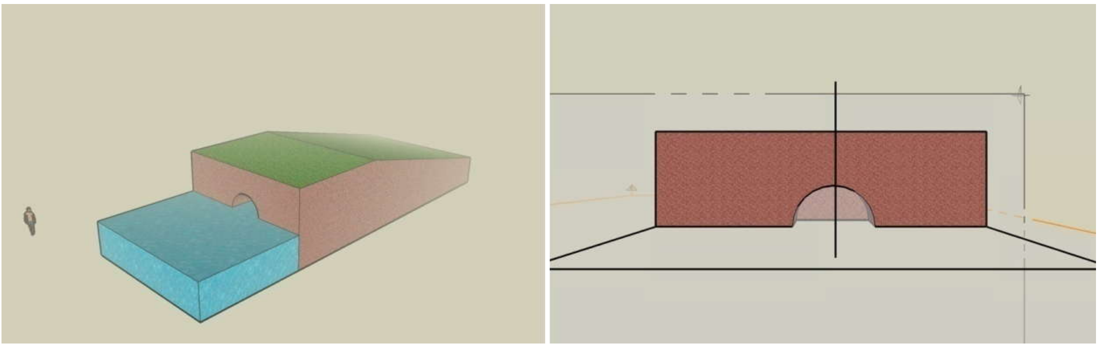


Figure 15 Transverse section of the conduit

In the first part, we consider the effect of cohesion with a circular cavity of form. To simplify calculation, we propose an axisymmetric geometry. The axisymmetric condition enables us to divide the transverse section of the cavity into two (Figure 16). In this basic model, we propose various rayon of cavity going of 1m with 9m, for various values of cohesion.

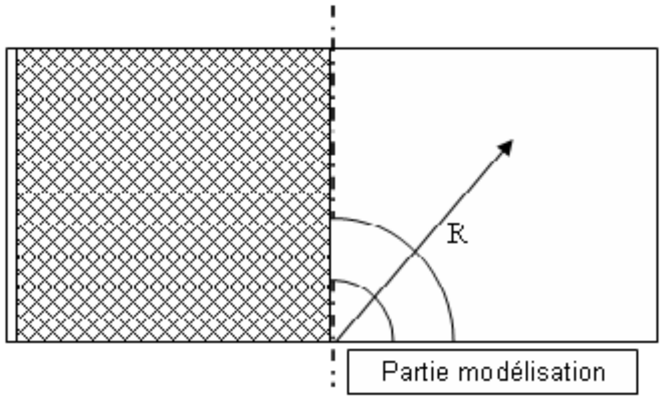


Figure 16 Axisymmetric model with a circular cavity

The limiting conditions of the system, are:

- fix in Y at the foundation of the earth dam
- fix in X at the edges (on the right and on the left)

3.1.1.2 Cavities of form of elliptic in a earth dam low height

The subsidence occurs normally according to a form which resembles an ellipse (Figure 17). In our second model, we take this geometry of cavity in the place of a circle. The various geometries of cavity enable us to distinguish the effects of the geometry on the process of collapse. The various shapes of cavities, which we model do not change only into height, but also in width.

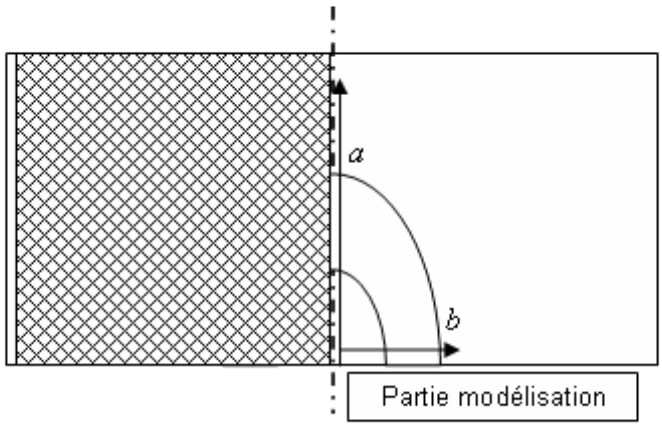


Figure 17 Axisymmetric model with an elliptic cavity

We introduce the coefficients *a* and *b*, which model the height and the width. The soil mechanics properties and the conditions of the edge are identical to the circular model of cavity.

3.1.1.3 Circular cavity in a earth dam great height

This model is interesting to see the effect of a earth dam great height. The conduit used has a circular form, but with variations of the rayon which are much more important. In this model of great dimension, we propose two cases:

1. Model with a height of 30m for a width of earth dam of 150m
2. Model with a height of 90m for a width of earth dam of 540m

These models have the same properties as the model of the circular cavity in a earth dam of 10m height.

3.1.2 Rheological models

The plastic model implies a certain degree of deformation (also rupture) which is a consequence of the non linearity of deformation - constraint. *FLAC* uses several types of plastic model characterized by their functions of output, of hardening/softening and law of flow.

The formulation of the plastic law of flow in *FLAC* rests on basic principles of the theory of plasticity which any increment of deformation can be broken up into two parts (elastic and plastic).

All the models of *FLAC* are based on the state of plane deformation, with the exception for the model of hardening/softening, which is also available with the option of plane constraint. Also note that all the plastic models are calculated in term of effective constraint, and not of total constraint.

The execution of numerical calculation is initialized by an elastic test (the elastic conjecture) where the increment of constraint is initially calculated in support that all the elastic deformation is natural. The corresponding constraints are then evaluated, if they exceed the criteria of rupture, where the state of stresses is with the top of the function of rupture, the plastic deformation occurs. A return of the state of stresses running on the criterion is then carried out just like an estimate of the associated plastic flow.

3.1.2.1 Presentation of the plastic model of Mohr Coulomb

Collapse that we explained in the preceding chapter will be modeled by using the model of Mohr Coulomb with a undrained cohesion. It is the most used conventional model to represent the rupture by shearing in the soils and the rocks. Moreover, Vermeer and deBorst (1984) showed that the model of Mohr Coulomb corresponds well to the test results in laboratory for sand and the concrete [25].

The model of Mohr Coulomb uses the principal constraints (*σ1*, *σ2*, *σ3*), and the three-dimensional constraint of the component (*σzz*) being recognized like one of these last. The principal constraints and the principal directions are evaluated starting from the components of the tensor of constraint.

$$\sigma_1 \leq \sigma_2 \leq \sigma_3 \tag{1.43}$$

The corresponding principal deformations *Δe1*, *Δe2*, *Δe3* are broken up into two, the elastic part and the plastic part, as follows:

$$\Delta e_i = \Delta e_i^e + \Delta e_i^p \qquad i = 1,3 \tag{1.44}$$

The criterion of rupture of Mohr Coulomb is presented to axis 1 and 3 as follows,

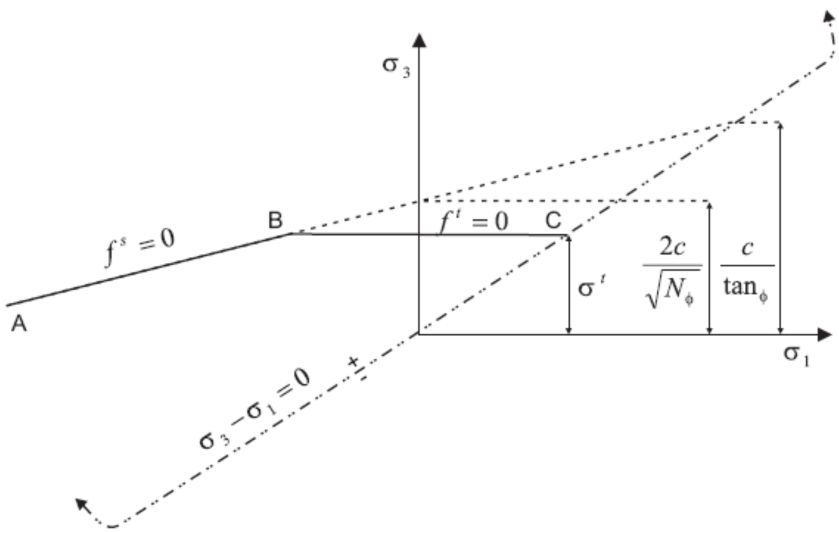


Figure 18 Criterion of rupture of Mohr Coulomb in *FLAC*

The envelope of rupture is defined point has at the point B by the function of rupture of Mohr Coulomb,

$$f^s = \sigma_1 - \sigma_3 N_\phi + 2c\sqrt{N_\phi} \tag{1.45}$$

$$N_\phi = \frac{1 + \sin\phi}{1 - \sin\phi} \tag{1.46}$$

And of the point B to the point C, the function takes the following form,

$$f^t = \sigma^t - \sigma_3 \tag{1.47}$$

With,

$\varnothing$: Natural angle of repose

c : Cohesion of the soil

σ^t : Tensile strength, which truncates the criterion in traction

The execution of the Mohr Coulomb Model in FLAC starts with a first estimation of the elastic constraint (elastic conjecture) σ_{ij}, while being added to the components of the former constraints calculated before by the application of the law of Hooke. Principal constraints (σ'_1, σ'_2, σ'_3) and corresponding principal directions are calculated. If these constraints violate the criterion of rupture, a correction must be applied to the elastic conjecture to give the new state of stress.

$$h = \sigma_3 - \sigma^t + \alpha^P(\sigma_1 - \sigma^P) \qquad (1.48)$$

With,

$$\alpha^P = \sqrt{1 + N_\phi^2} + N_\phi \qquad (1.49)$$

$$\sigma^P = \sigma^t N_\phi - 2c\sqrt{N_\phi} \qquad (1.50)$$

The function $h(\sigma_1, \sigma_3) = 0$ is presented by

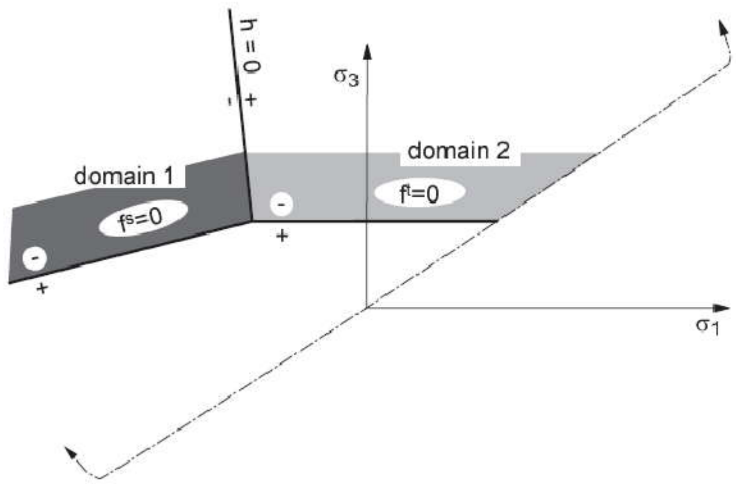


Figure 19 Field used for the rule of flow

We then have two criteria according to Figure 19 where $h(\sigma_1^I,\sigma_3^I)\leq 0$ or $h(\sigma_1^I,\sigma_3^I)\geq 0$. In the first case, we have the rupture due to shearing (*f^s=0*), and in the second case, the rupture due to traction (*f^t=0*).

3.1.2.2 Model with softening

It is about a model in conformity with reality based on a model of Mohr Coulomb modified with a coefficient of softening. The radoucissant model makes it possible to change cohesion, the angle of friction, dilatancy, and the ultimate stress with traction. Contrary to the model of Mohr Coulomb, where its mechanical properties remain constant, the radoucissant model can have softening (or hardening) mechanical properties, which takes place after the beginning of plasticity.

In *FLAC*, this function is integrated in language *FISH*. This softening suggested is a linear function of a parameter measuring the plastic deformation of shearing. How to make if softening is not linear?

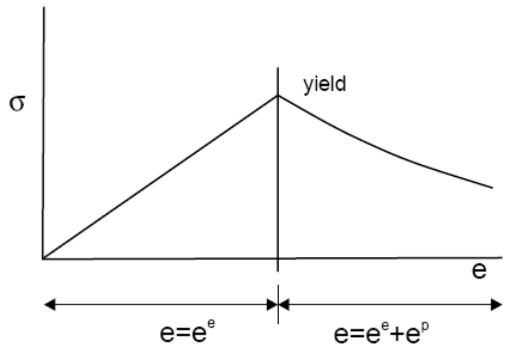


Figure 20 Radoucissant model

Figure 20 shows the relation of stress-strain which radoucit just after the limit elastic and reached a residual value. The curve is linear until the plastic limit. In this field, the part of the deformation is elastic only (*E* = *ee*). When we extinguish the plastic limit, the deformation breaks up into two elastic parts and plastic (*E* = *ee* + *ep*). The linear approach of small piece (small segment) is thus used to model a coefficient of softening (Figure 21).

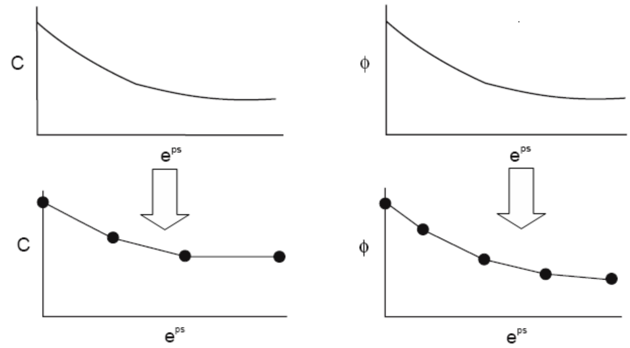


Figure 21 Approaches the model radoucissant in *FLAC2D*

The plastic shearing strain is measured by the parameter hardening of shearing (*shear hardening parameter*) e^{ps}, whose form is defined like,

$$\Delta e^{ps} = \left\{ \frac{1}{2}\left(\Delta e_1^{ps} - \Delta e_m^{ps}\right) + \frac{1}{2}\left(\Delta e_m^{ps}\right)^2 + \frac{1}{2}\left(\Delta e_3^{ps} - \Delta e_m^{ps}\right)^2 \right\}^{\frac{1}{2}} \qquad (1.51)$$

With,

$$\Delta e_m^{ps} = \frac{1}{3}\left(\Delta e_1^{ps} + \Delta e_3^{ps}\right) \qquad (1.52)$$

And $\Delta e_j^{ps}\ ; j = 1,3$ are the main axes of the second invariant of plastic deformation (γ^p).

For the hardening parameter of the force to traction, it measures the total of the plastic deformation of the force to traction. Its increment is defined as,

$$\Delta e^{pt} = \Delta e_3^{pt} \qquad (1.53)$$

Where Δe_3^{pt} is the increment of the plastic deformation of the force to traction along its major axis.

Numerical calculation starts with the calculation of the new constraints as described in the description of model of Mohr Coulomb. The values of the hardening parameters are calculated as the average of surface of values obtained starting from the equations (1.51) and (1.53) for each element in the zone. The hardening parameters are evaluated with each step and the new properties of model are evaluated by linear interpolation.

For a material having the angle of friction, the maximum value of the tensile strength is evaluated starting from the equation (1.54) using the angle of friction (ø) and cohesion (*c*). This value is maintained by the code if it is smaller than the tensile strength updated of the table.

$$\sigma^{t}_{\max} = \frac{c}{\tan\phi} \tag{1.54}$$

3.1.3 Numerical models

3.1.3.1 Earth dam - low height with circular cavity and Mohr-Coulomb

The mechanical behaviors suggested to use the model of Mohr Coulomb with following properties:

- The height of the modelled earth dam (of the impermeable rigid soil to surface) is of 10m.
- Undrained cohesion (*Cu*) is fixed for each model, but it will take successively values going of 300 kPa (maximum) with 30 kPa (minimum)
- The density (*ρ*) is given by

$$\rho = \frac{\gamma}{g} \tag{1.55}$$

With, γ = 20 kN/m3

- The Poisson's ratio (*ν*) is fixed 0,45
- The Young modulus proposed is given by

$$E = 200 \times C_u \tag{1.56}$$

- The module of compressibility (*K*) is given by

$$K = \frac{E}{3(1-2\nu)} \tag{1.57}$$

- The modulus of rigidity (*G*) is given by

$$G = \frac{E}{2(1+\nu)} \tag{1.58}$$

We generate a simple geometry of a rectangular form with sufficiently fine mesh. The size of the grid must have a compromise between precision and computing time. Indeed, a finer grid gives more precision, but it takes more time computing. The shape of mesh that we propose is a square of 0,25m.

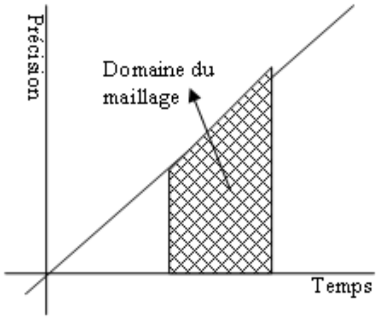


Figure 22 Cut grid: compromise meanwhile and precision

We consider that the width is 2,5 times larger than the height to minimize the effects edge, which gives us a model of 100 X 40 squares.

Table 4 Mohr Coulomb Model with a circular cavity in low height

Numerical data	
Height of the earth dam	10m
Width of the earth dam	25m
Type of grid	Quadrilateral of 0,25 m
Elements	*100 X 40 squares*
Form cavity	Circular, with rayons proposed (1m, 2m, 3m, 4m, 5m, and 9m)
Undrained cohesion (*Cu*)	30kPa - 300 kPa
Note	• The limiting conditions of earth dam are fixed in Y at the base of the earth dam and are fixed in X at the flat rims and left. • The model used is of Mohr Coulomb.

The initial balance of the system without cavity is obtained when the ratio of the imbalance force is lower than 10^{-5} which is 100 times weaker than the ratio recommended. Then, we generate two zones of soil, one with a fixed cohesion and the other with a cohesion which decreases on the level of the wall of the cavity. This reduction is automated with a very small step (1kPa) so that the state of the soil does not change brutally. When cohesion reaches a zero value in a zone, the associated elements disFigure from the model.

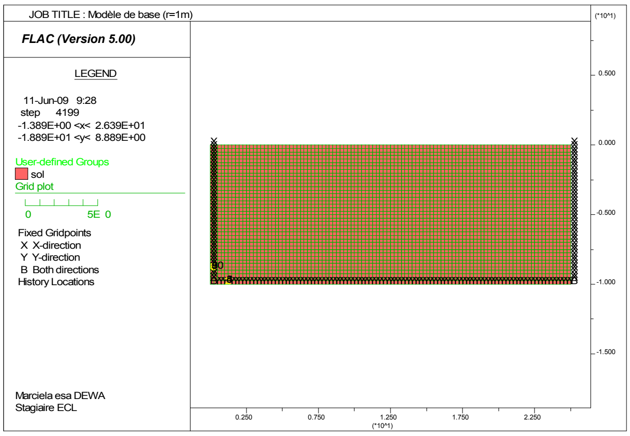


Figure 23 Mohr Coulomb Model at the balance state

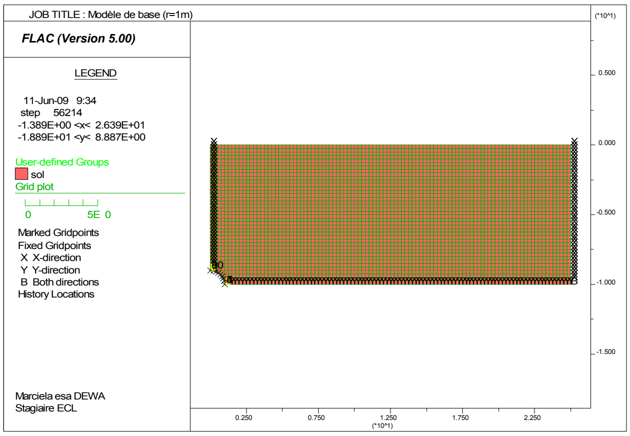


Figure 24 Mohr Coulomb Model with a circular cavity (here *r=1m*)

3.1.3.2 Earth dam - low height with elliptic cavity and Mohr-Coulomb

For the earth dam having an elliptic cavity, we list the numerical model on Table 5,

Table 5 Mohr Coulomb Model with elliptic cavity

Numerical data	
Height of the earth dam	10m
Width of the earth dam	25m
Type of grid	Quadrilateral of 0,25 m
Elements	*100 X 40 squares*

Form cavity	Elliptic, with following values: *= 1m has; B = 3m, 5m* *= 3m has; B = 1m, 5m* *= 5m has; B = 1m, 3m* *= 9m has; B = 1m, 3m, 5m, 7m*
Undrained cohesion (*Cu*)	• 30kPa - 300 kPa • 1100 kPa maximum for the cavity of *a=9m; b=1m* • 1300 kPa maximum for the cavity of *a=1m; b=5m*
Note	• The limiting conditions of earth dam are fixed in Y at the base of the earth dam and are fixed in X at the flat rims and left. • The model used is of Mohr Coulomb.

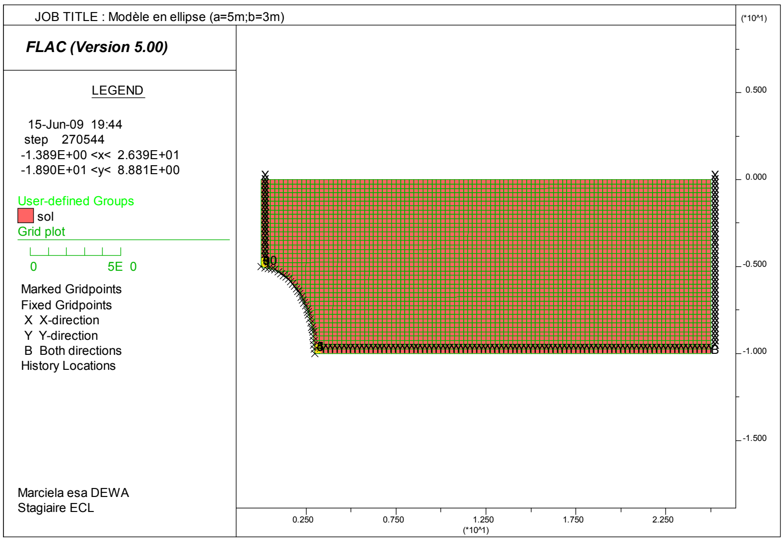


Figure 25 Mohr Coulomb Model with an elliptic cavity (here, *a=5m; b=3m*)

3.1.3.3 Earth dams - great height with a circular cavity and Mohr-Coulomb

The earth dam is either 30m or90 m The mechanical properties used are the same ones as the model of earth dam low height, except that the value of maximum undrained cohesion used is larger to compensate for the change of the geometry (because the actual weight of the soil above the cavity is much heavier than the model of the earth dam having a height of 10m).

Table 6 Mohr Coulomb Model great height

Numerical data		
Height of the earth dam	30m	90m
Width of the earth dam	75m	270m
Type of grid	Quadrilateral of 0,5m	Quadrilateral of 1m
Elements	*150 X 60 squares*	*100 X 40 squares*
Form cavity	Circular, with rayons proposed (*1m, 3m, 9m, 18m, 27m*)	Circular, with rayons proposed (1m, 5m, 9m, 18m, 27m, 54m, 81m)
Undrained cohesion (*Cu*)	• 80kPa - 2700kPa	• 150kPa - 5000kPa • 10000kPa for the cavity of 81m
Note	• The limiting conditions of earth dam are fixed in Y at the base of the earth dam and are fixed in X at the flat rims and left. • The model used is of Mohr Coulomb.	

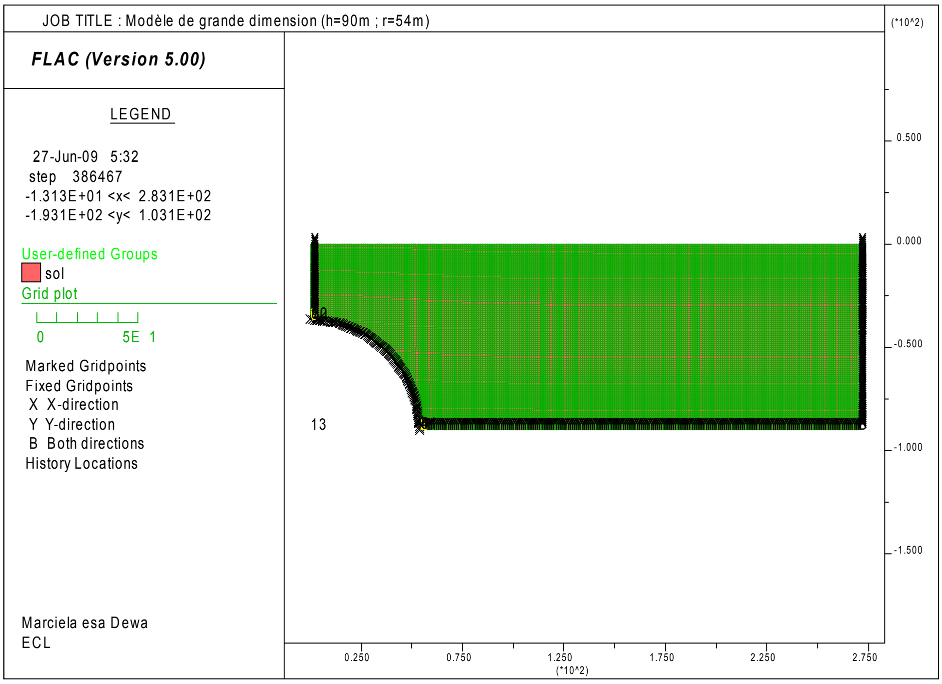


Figure 26 Model of great dimension (*h=90m; r=54m*)

In Figure 26, we notice that the model has much more elements than the basic model.

3.1.3.4 Earth dam of 10m, 30m and 90m with circular hole and radoucissant model

The radoucissant model proposed is based on the model of Mohr Coulomb and follows the geometrical properties exactly (the condition limits, the size of the model, the elements used) that was seen previously. However, there is variation of the mechanical properties when the criterion of plasticity is reached. We use a radoucissant model applied for all the heights (10m, 30m, 90m). The mechanical behavior is described as righter because it gives an account of real phenomena that to return account a model of simple Mohr Coulomb is not able. The model radoucissant 1 uses the same mechanical properties like Mohr Coulomb Model except its value of cohesion. Cohesion falls and reaches a zero value when the criterion of plasticity (γ^p) of 10% is reached (Figure 27). The model radoucissant 2 indicates a fall of cohesion of material but with creation of a natural angle of repose. The material behaves then like a rubbing material becoming deformed in drained condition. It models the effect of the cracking of clay when great deformations déviatoires are created. The parameterized mechanical ones are shown in Figure 28.

Table 7 Model radoucissant 1 with a circular cavity

Numerical data

Height of the earth dam	10m	90m
Width of the earth dam	25m	270m
Type of grid	Quadrilateral of 0,25m	Quadrilateral of 1m
Elements	*100 X 40 squares*	*100 X 40 squares*
Form cavity	Circular, with rayons proposed (*1m, 3m, 5m, 9m*)	Circular, with rayons proposed (1m, 9m, 18m, 54m, 81m)
Undrained cohesion (*Cu*)	70kPa - 500kPa	200kPa - 5000kPa (exception for the cavity of 81m, Cu max is 10.000kPa)
Note	• The limiting conditions of earth dam are fixed in Y at the base of the earth dam and are fixed in X at the flat rims and left. • Use model radoucissant 1, the mechanical properties as from the moment one reaches the criterion of plasticity is shown on Figure 27	

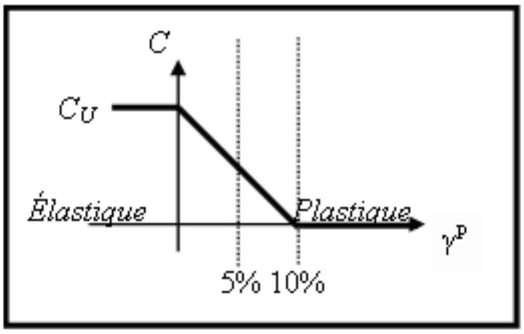


Figure 27 Mechanical properties of the model radoucissant 1

Table 8 Model radoucissant 2 with a circular cavity

Numerical data			
Height of the earth dam	10m	30m	90m
Width of the earth dam	25m	75m	270m
Type of grid	Quadrilateral of 0,25m	Quadrilateral of 0,5m	Quadrilateral of 1m

Elements	*100 X 40 squares*	*150 X 60 squares*	*100 X 40 squares*
Form cavity	Circular, with rayons proposed (*1m, 3m, 5m, 9m*)	Circular, with rayons proposed (*1m, 3m, 9m, 18m, 27m*)	Circular, with rayons proposed (1m, 5m, 9m, 18m, 27m, 54m, 81m)
Undrained cohesion (*Cu*)	50kPa - 500kPa	125kPa - 2300kPa	200kPa - 5000kPa (exception for the cavity of 81m, Cu max is 10.000kPa)
Note	• The limiting conditions of earth dam are fixed in Y at the base of the earth dam and are fixed in X at the flat rims and left. • Use model radoucissant 2, the mechanical properties as from the moment one reaches the criterion of plasticity is shown on Figure 28		

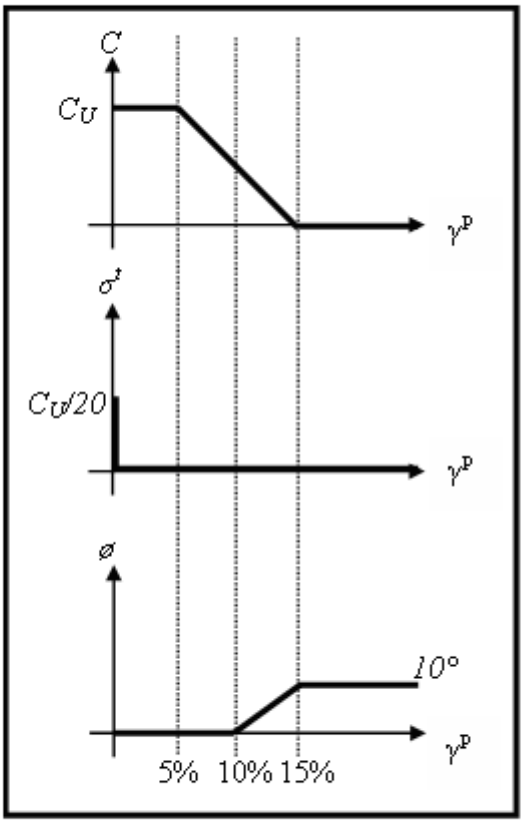


Figure 28 Mechanical properties of the model radoucissant 2

3.2 Criteria of collapse

3.2.1 Analysis of Adimensional Parameters.

In this analysis, we use several assumptions to simplify, explain and compare the results of numerical calculations. First of all, we introduce the coefficient of reduction *R* which is shown on the equation (1.59) This adimensional coefficient is useful to compare the various values of the undrained cohesion which we use.

$$R = \frac{C_{ref}}{C_u} \tag{1.59}$$

With C_{ref} is the maximum value reference of the cohesion undrained for a model, and *Cu* is the undrained cohesion which one varies in calculations to see which *Cu* value collapse occurs for a height of earth dam and a given diameter of conduit. A model in our case means an earth dam with a height and depth of cavity given. This coefficient of reduction thus has a minimal value which is equal to 1 (*Cref* = *Cu*), and becomes larger because undrained cohesions taken thereafter are smaller than the cohesion of reference.

For better comparing the results with various heights, rayons of cavity, and cohesions references, we introduce the number of stability (*NR*). It is an adimensional value like the coefficient of reduction.

$$N = \frac{\gamma h}{C} \tag{1.60}$$

With,

γ : Unit weight of the soil

h : Height of a earth dam

C : Cohesion

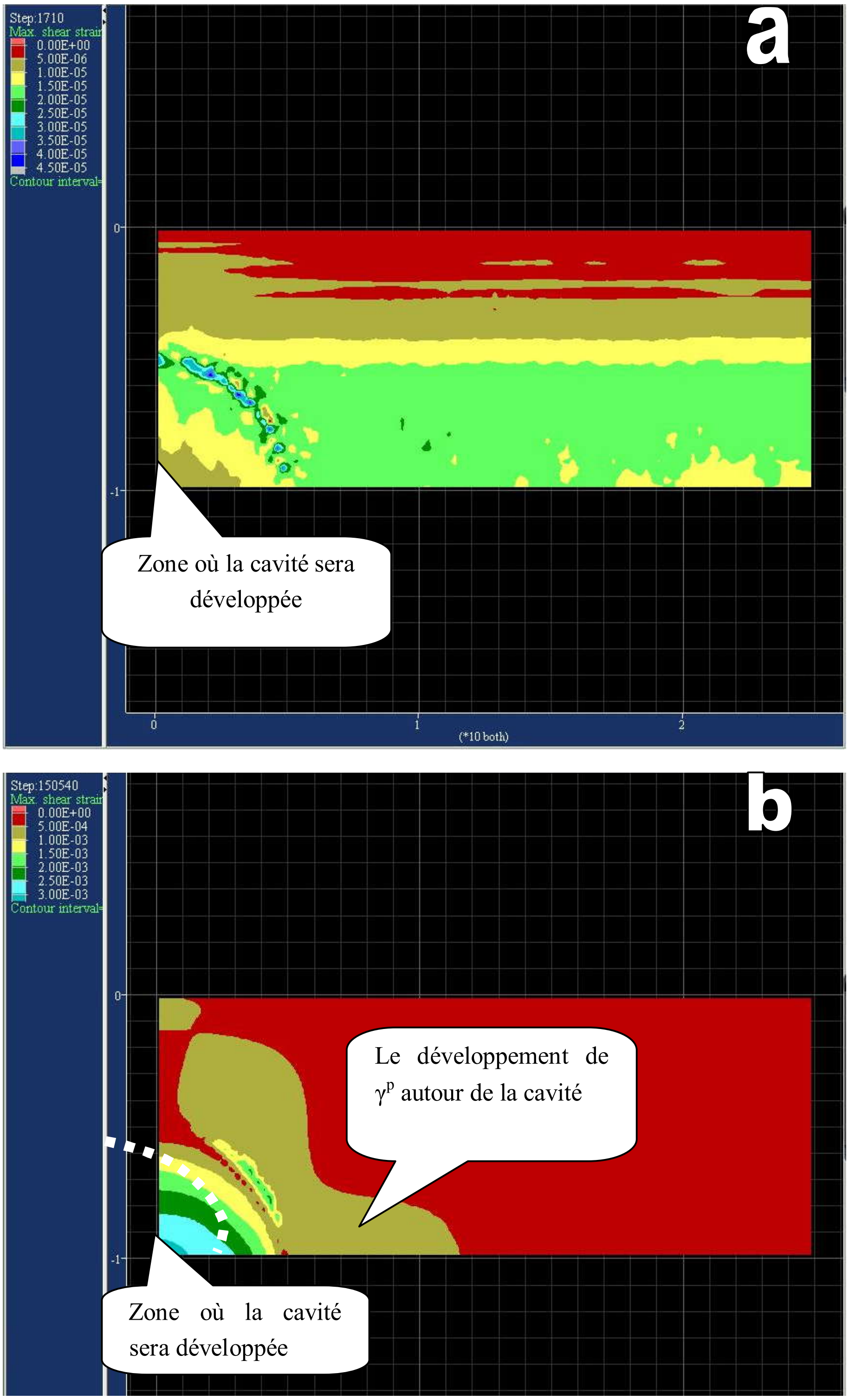
3.2.2 Criteria of rupture

We define two criteria: first criterion is associated with a starter of rupture (qualified of limit), the other corresponds to a total rupture crossing the system (qualified the ultimate one).

3.2.2.1 Limiting criteria

In practice of the geotechnician, the deformation becomes destructive for a system, when the value of the second invariant of the plastic deformations (γ^p) becomes larger than 2,5%. In our case, we use γ^p of 5% (an approach based on the observation of real case) (i.e when the deformations are sufficiently large to consider that the collapse of the roof of the cavity is inescapable).

For better including/understanding on this fracture topography, let us try to see on Figure 29 (a), (b), and (c) the evolution of the second invariant when the undrained cohesion of the soil decreases.



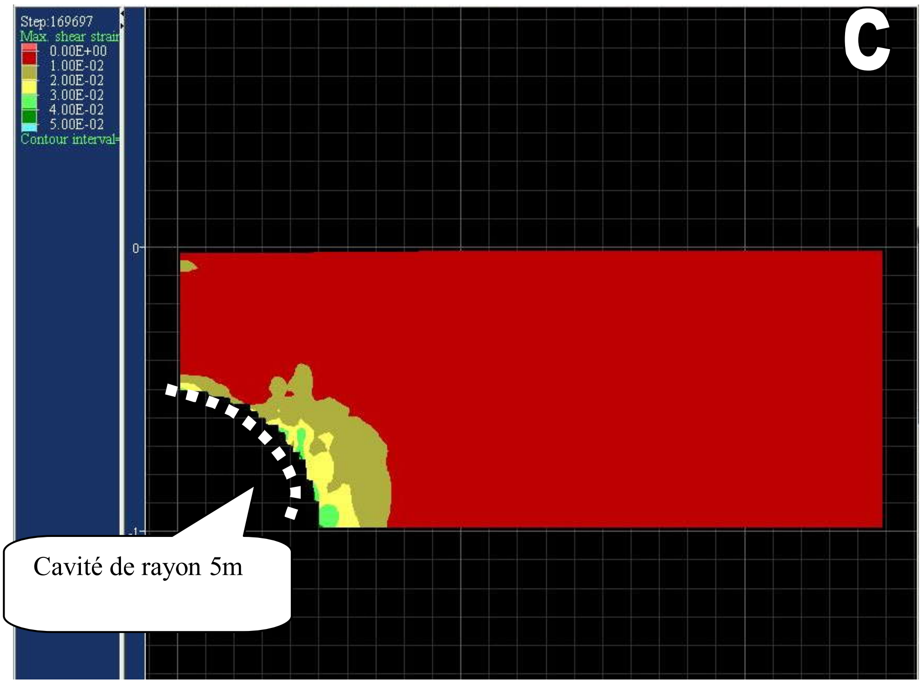
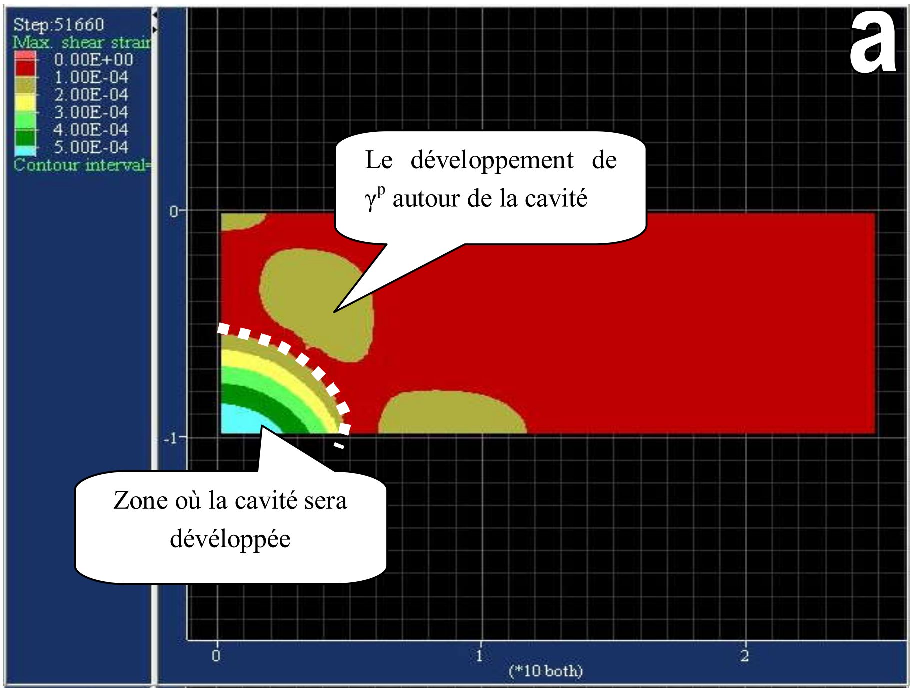


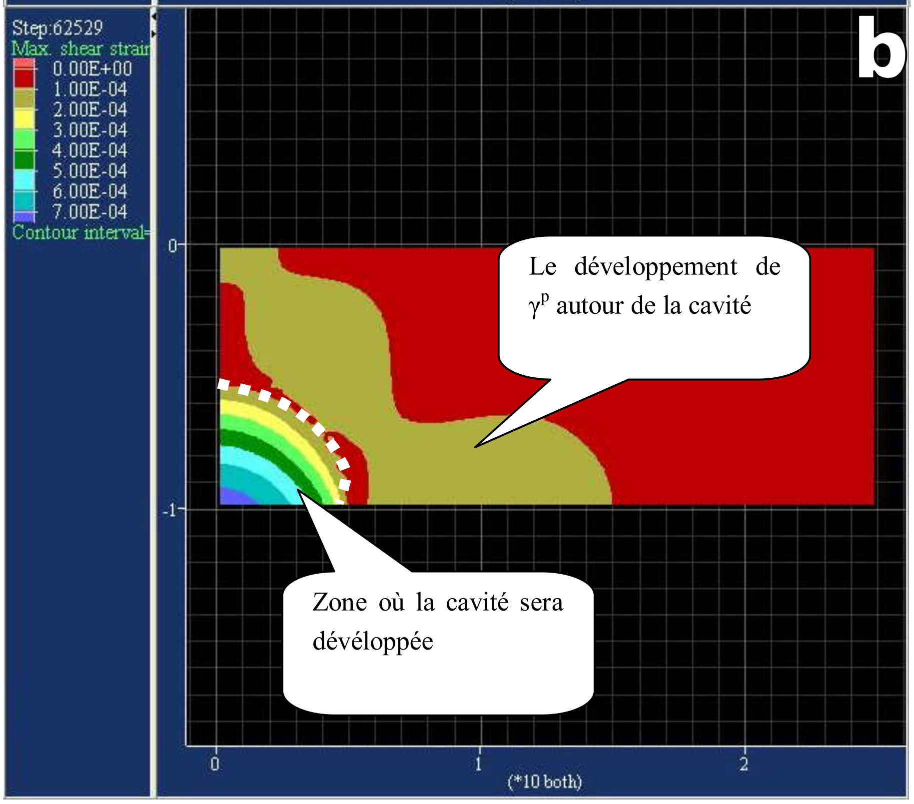
Figure 29 Development of γ^p during the creation of a cavity (value obtained with *r=5m; C=100kPa*)

These figures make it possible to understand that the second invariant on the soil does not develop vertically (with axis Z). There exists an arching above the cavity which tends to deviate from the fracture topography of the mode considered. It should also be noted that we will not take account of possible important values of γ^p on the surrounding area of the cavity. These local ruptures will not be taken into account of total ruptures.

3.2.2.2 Ultimate criterion

Figure 30 (a), (b), and (c) show the evolution of γ^p when the limiting criterion is exceeded. In this case, the deformation of the roof of the cavity is large, an oblique line of rupture is formed and crossed completely the system. All the points of this crossing line are in states of γ^p close to 5% (Figure 30 (c)).


a
Step:51660
Max. shear strain
0.00E+00
1.00E-04
2.00E-04
3.00E-04
4.00E-04
5.00E-04
Contour interval=
Le développement de γ^p autour de la cavité
Zone où la cavité sera dévéloppée
(*10 both)


b
Step:62529
Max. shear strain
0.00E+00
1.00E-04
2.00E-04
3.00E-04
4.00E-04
5.00E-04
6.00E-04
7.00E-04
Contour interval=
Le développement de γ^p autour de la cavité
Zone où la cavité sera dévéloppée
(*10 both)

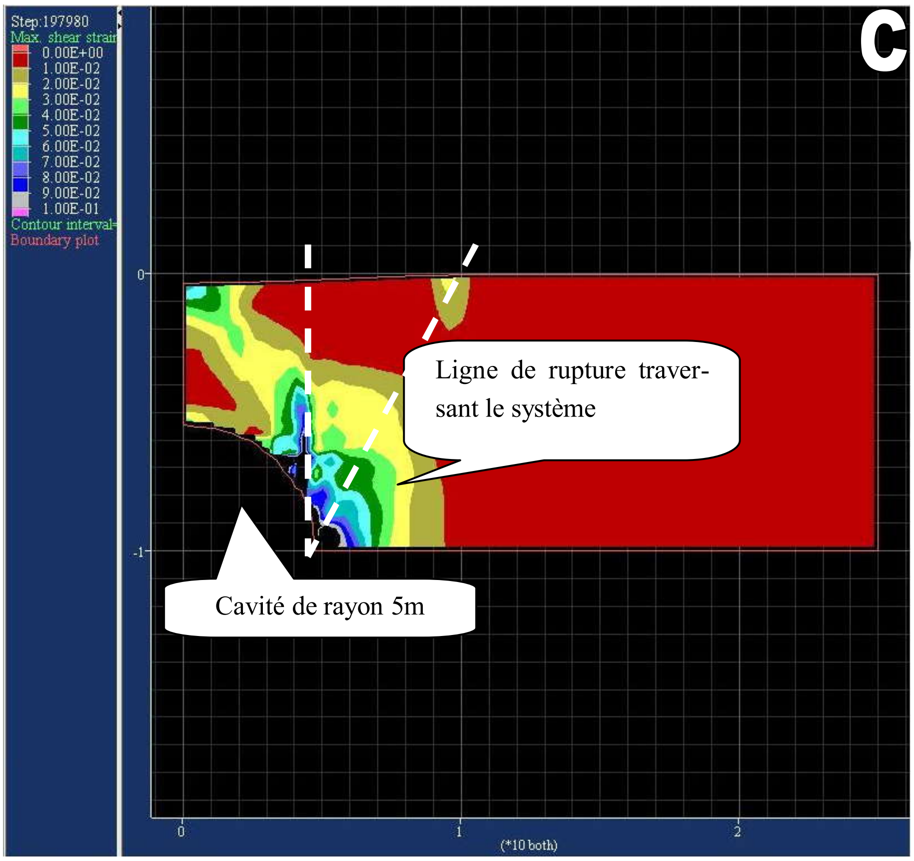


Figure 30 Development of γp during the creation of a cavity with total rupture (value obtained with *r=5m; C=80kPa*)

We associate the parameters (*R* and *N*) with these two criteria of rupture. The first criterion is taken starting from the coefficient of reduction (*R*), which is coupled with the second invariant of the plastic deformations (γ^p). We introduce the maximum value of the coefficient of reduction (R_{max}). This maximum value of *R* corresponds to the ultimate collapse (total) of an earth dam.

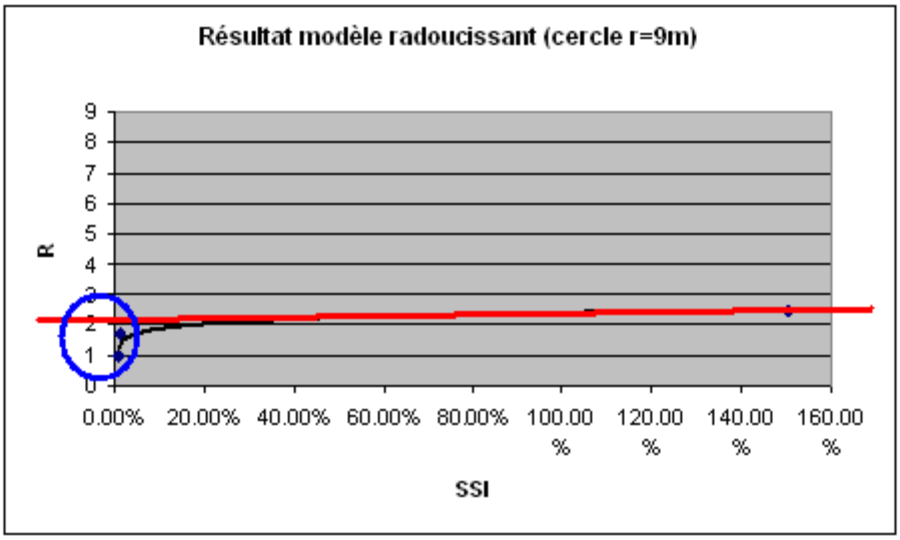


Figure 31 Maximum coefficient of Reduction, taken of a curve γp - R

Figure 31 shows a tangent with the curve which one associates with the value of R involving a total rupture (ultimate criterion). The value of R associated with this tangent is R_{max}.

From these two criteria of rupture, we distinguish two types of cohesion,

1. Cohesion limits (C_{lim}), is the value of the cohesion to which one has a value of γ^pmax equal to 5% in the system. This cohesion corresponds to a phase of starter of a collapse of the conduit.

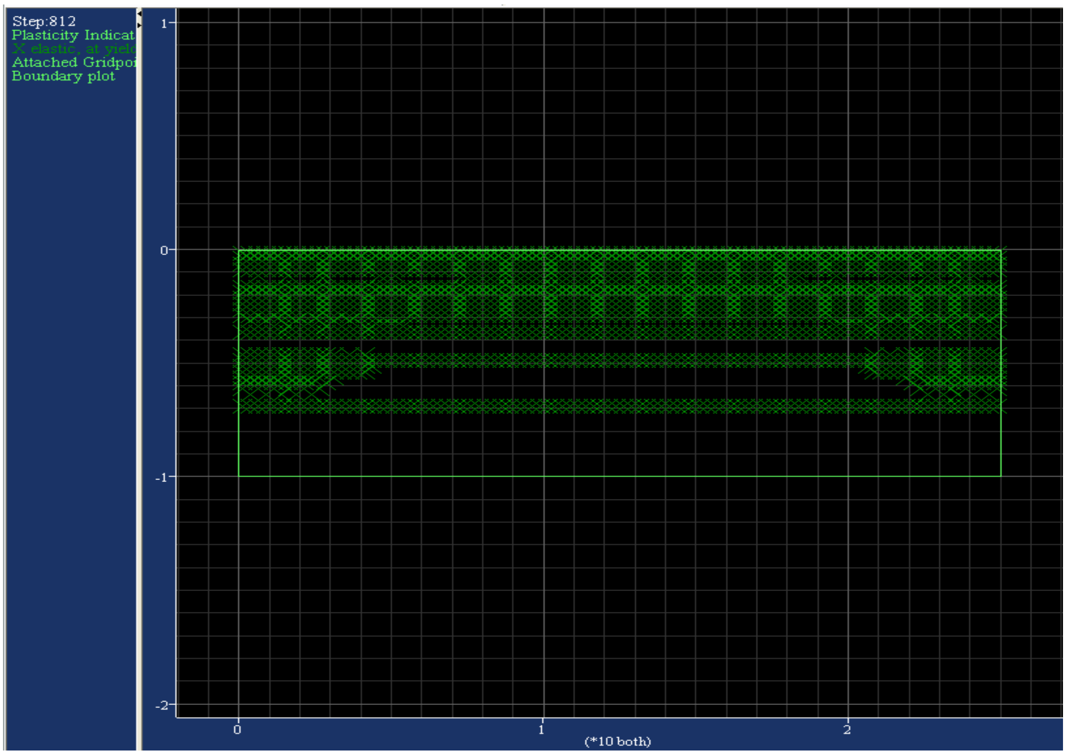
   This value of 5% is still ambiguous, but in reality we start to have an excessive deformation when γ^p reaches this value.

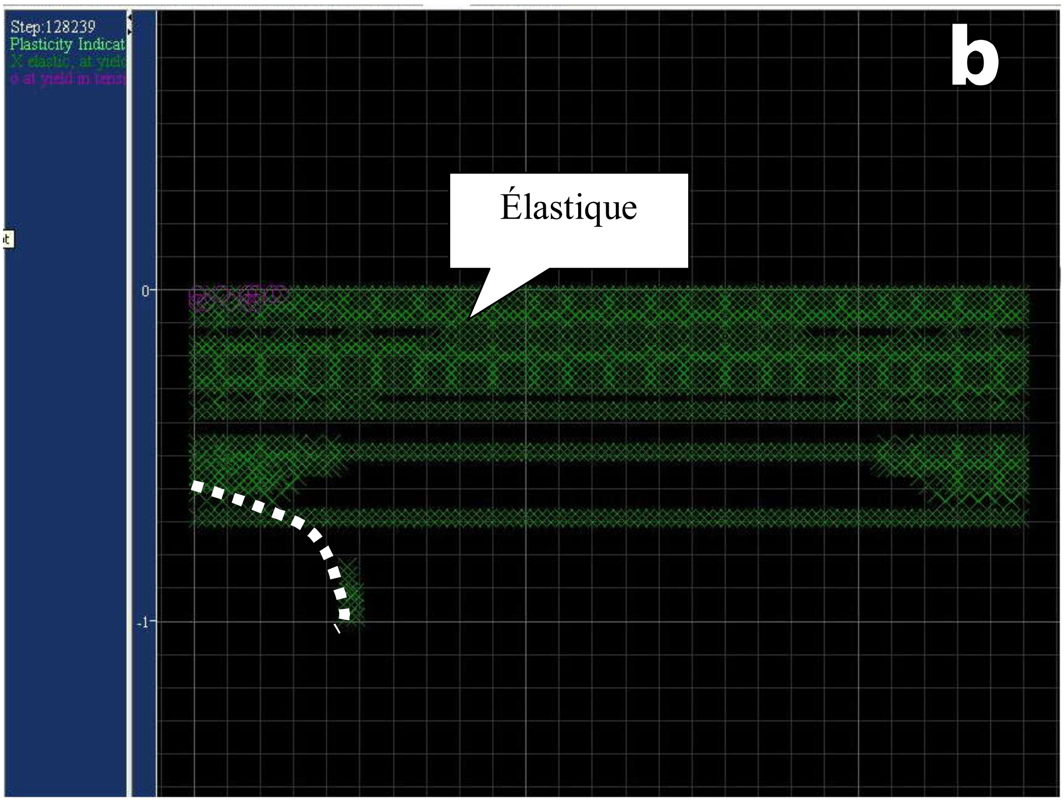
2. Ultimate cohesion (C_{ult}), is the minimal cohesion involving the total collapse of the conduit and the formation of a breach. Ultimate cohesion can be calculated with the equation(1.61). This value of ultimate cohesion can be associated with a value of γ^p_{max} larger than 5% in the system.

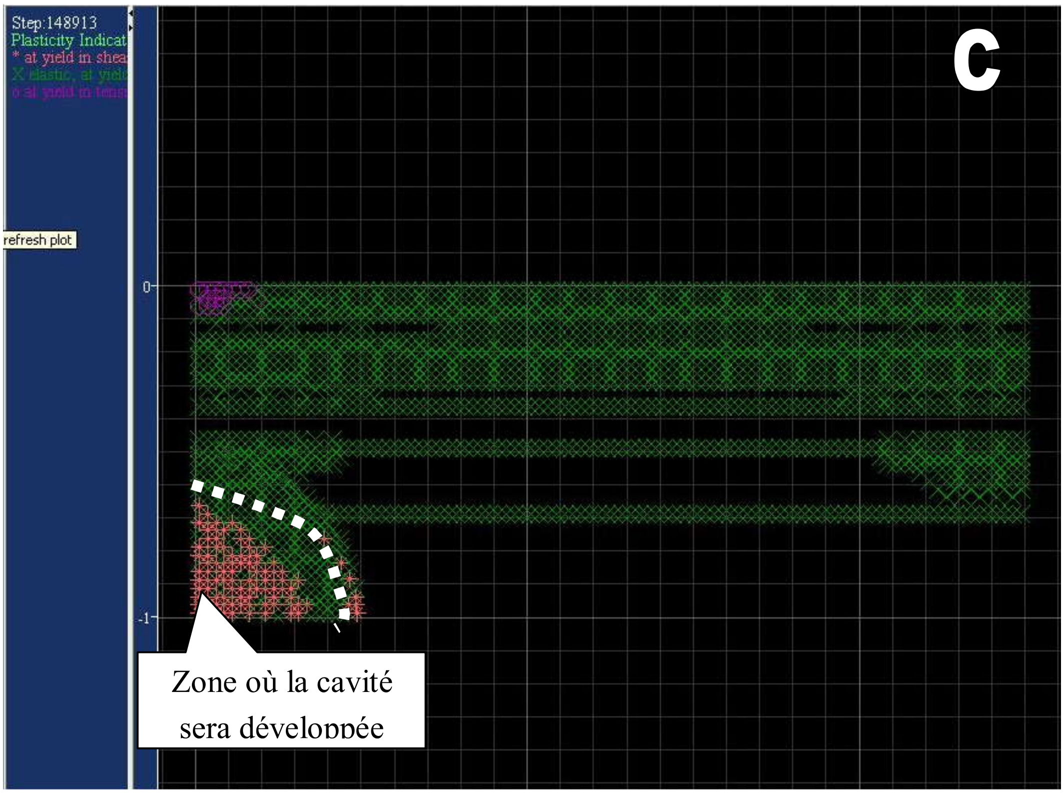
$$C_{ult} = \frac{C_{ref}}{R_{\max}} \tag{1.61}$$

3.2.3 Cartography of the plastic zones

We can distinguish collapse from a cavity by using the index of plasticity. By looking at this index, we can estimate the zone where the rupture is likely to occur. The evolution of the index of plasticity is shown on Figure 32 (of (A) with (*E*)). The index of plasticity when the system is stable is shown by the Figure 32a, where all the elements have only elastic zones. Figure 32e watch the two zones of plasticity: the plastic zone due to the attack of the plastic criterion by shearing which is around the cavity and the plastic zone due to the attack of the plastic criterion by traction which is close to the surface of the earth dam (Figure 32 (E)).




Step:128239
Plasticity Indicat
b
Élastique
0
-1


Step:148913
Plasticity Indicat
refresh plot
c
0
-1
Zone où la cavité
sera développée

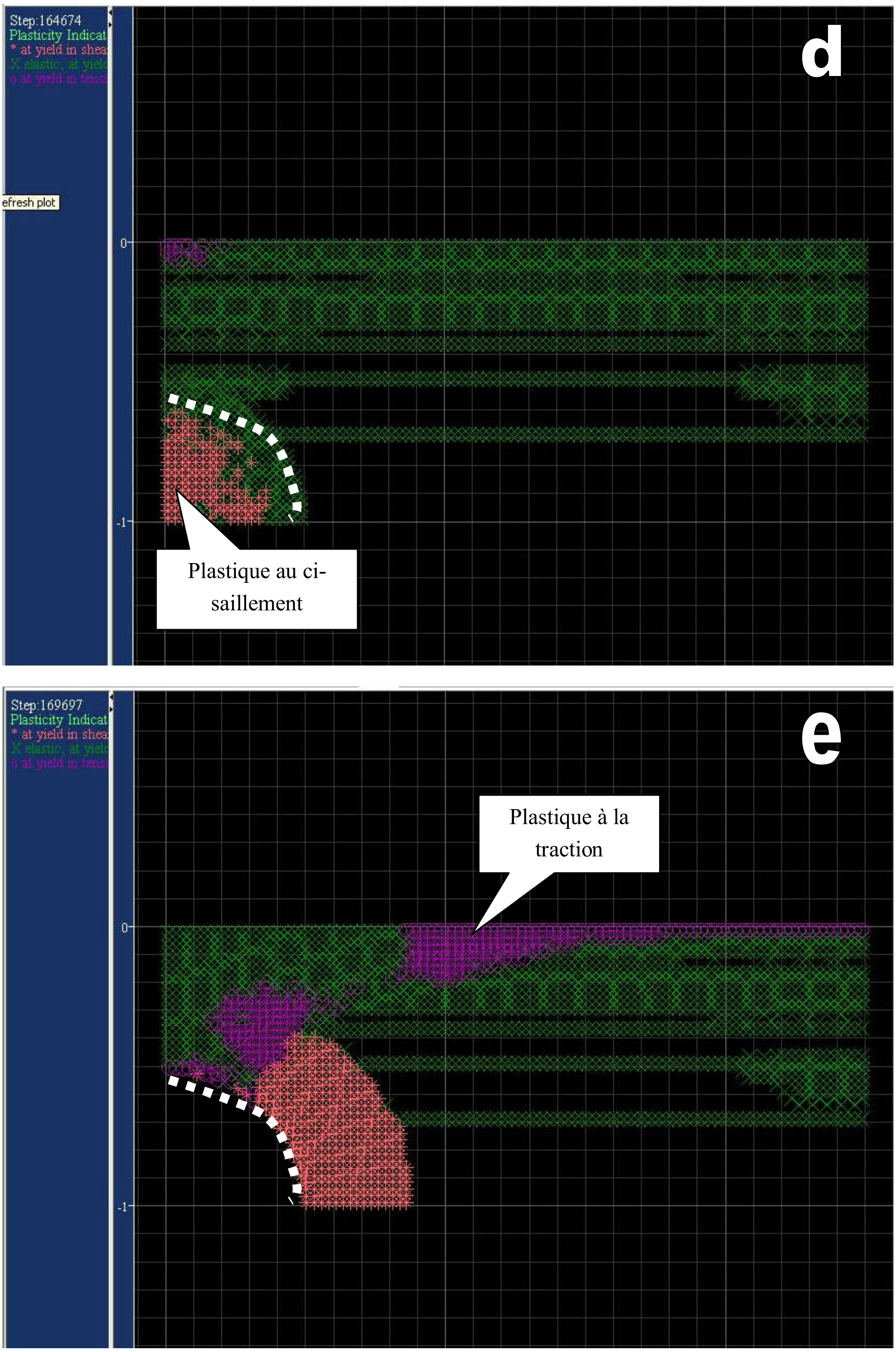
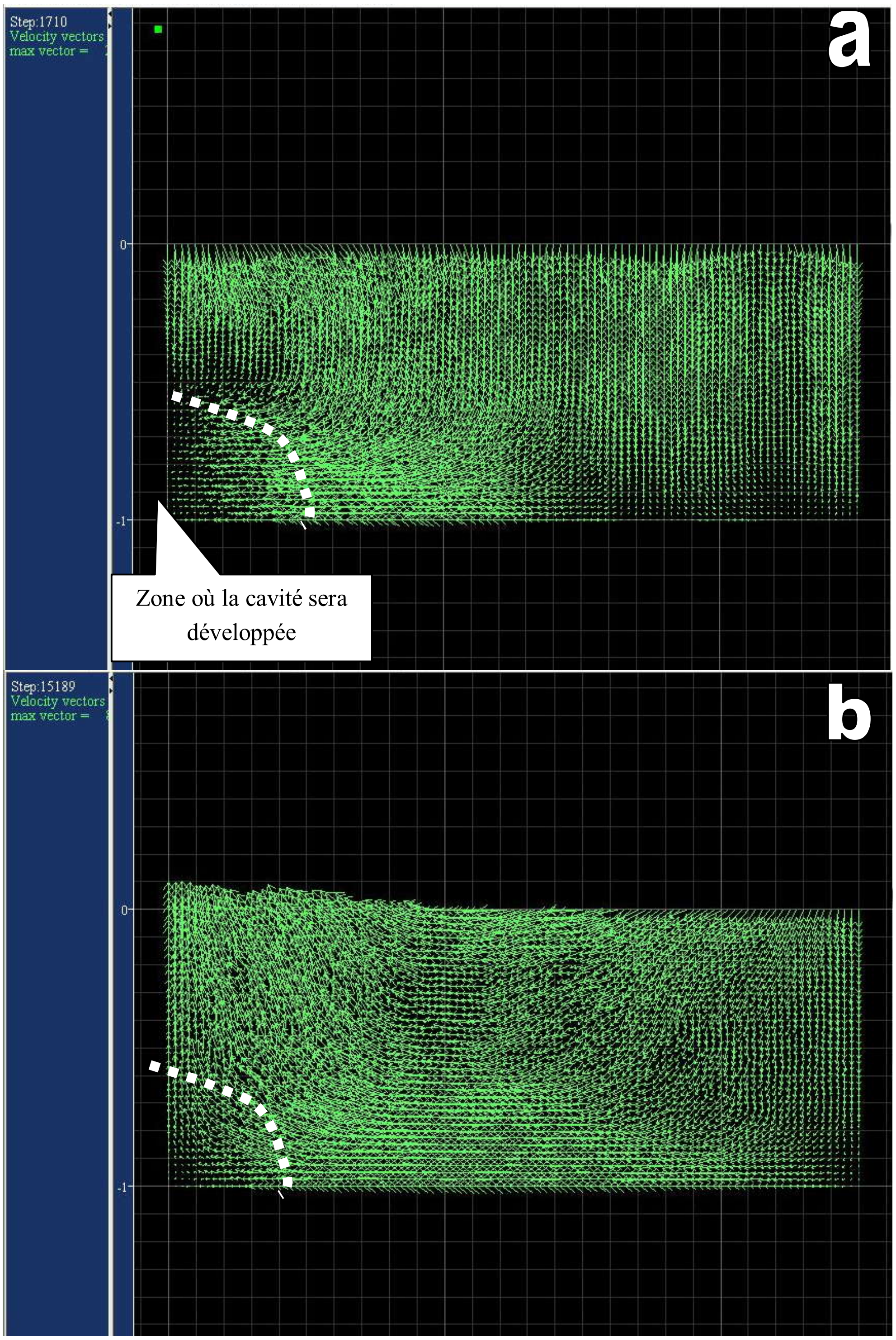


Figure 32 Evolution of the index of plasticity

3.2.4 Cartography of vector of deformation

The zone of rupture can be also presented by cartography of vector of deformation. The evolution of the vectors speed is shown on Figure 33 (from a to *c)*.



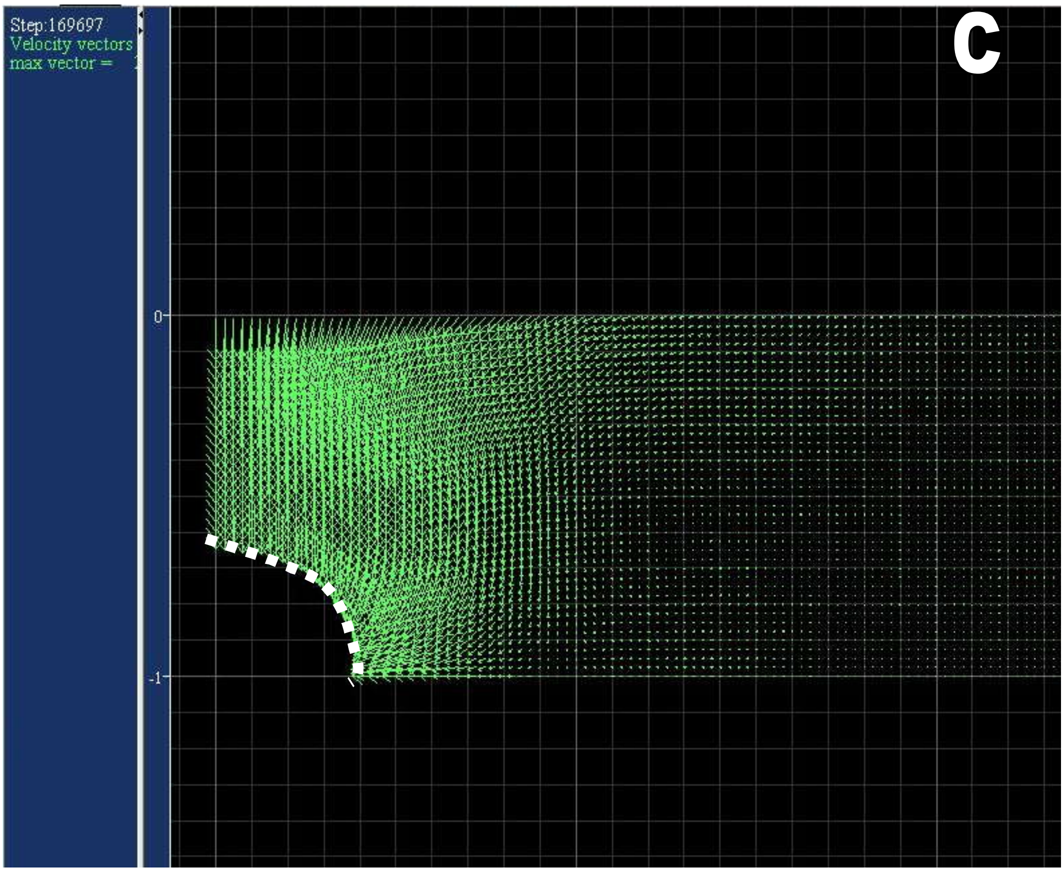


Figure 33 Evolution of the vectors speed