CHAPTER 2
THE COLLAPSE OF CAVITIES: MODELLING

2.1 The stability of the cavities

The stability of the cavities is associated with the capacity of the soil to resist the collapse of the roof [11]. Tritsch (2005) distinguished a rupture from cavities according to two principal categories [12],

1. Mining method, which can influence the way in which one or one whole of cavities can crumble.
2. The initiating mechanisms, which can generate an increase of subsidence (localized rupture).

In the next part, we will develop the bibliography studies concerning collapse. Various approaches exist:

2.1.1 Observational approach

This approach makes it possible to formulate a judgment on the risk subsidence of an geographical area but does not allow to quantify it. It is based on:

1. The experiment
2. Measurement coming from the case of proven subsidences

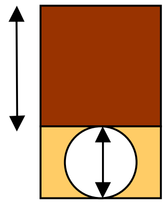
The installation of measuring instruments makes it possible to follow the evolution of the movements of the soils to the level of the cavity [13]. We introduce the procedure of risk management of convergence roof - wall of the galleries prone to such instability.

- For $V < 0,1$ mm/yr : satisfactory current stability
- For $0,1 < V < 0,3$ mm/yr : possibility of light short-term disorders (vigilance requires)
- For $V > 0,3$ mm/yr : nearest instability (requires to take security measures)

Beside its simplicity, this procedure does not adapt to any geographical condition (indeed, the studies are made in the north of France

There exist other approach:

1. The height of covering



It is possible to define a limit beyond which the risk to see the cavity going up until the surface becomes very weak. Two parameters are given by Eynon (1977) [14], the opening of the cavity - *O* and the thickness of covering - *H*. It specifies that the occurrence of a subsidence is strong for the ratio lower than 10.

2. The ratio the thickness (R)

This ratio is applied to a multi-layer soil whose deep layers are more rigid than the surface layers. This ratio is given by Hunt or Singh&Dahr where *R* is the ratio thickness of the movable rocks (soil) compared to the thickness of the coherent rocks of covering (the total thickness of soil in place to the top of the cavity) [15] [16]. This ratio must be in a given interval so that the risk of Figureance is weak.

- Hunt (1980): $0{,}1 \leq R \leq 1$
- Singh & Dhar (1996): $0{,}15 \leq R \leq 0{,}35$

3. The stratification of covering

It is the stratification of the soil to the correlation with the probability of rupture of the benches. A bench/layer slightly slim behaves better than a slimmer bench/layer [11].

4. The shape of the bell of subsidence

This proposal is based on a study which was made by Vachat (1982). This study takes the case of the coarse limestone quarries of the Paris basin. It highlights the following rule: “when ratio H/O is lower than 15 (reproducible on all the coarse limestone quarries of the Southern suburbs of Paris exclusively) the cavity is stable and will not evolve/move” (Figure 5).

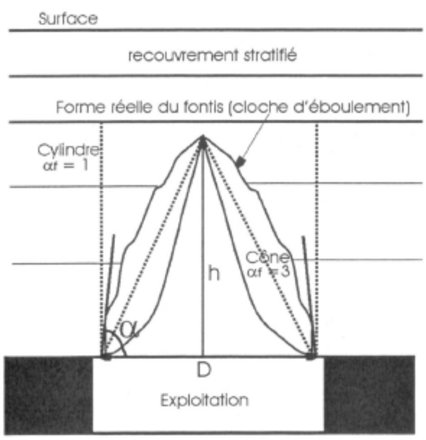


Figure 5 Diagrammatic cut of a cavity (Vachat’s Study)

These four various approaches require:

- Sufficient experiments in a geographical basin
- Homogeneity on the level of the nature of the cavities and covering.

2.1.1.1 Model of Piggott & Eynon (1977)

Piggott and Eynon (1977) simplify the relation between the height of increase and the height of void, while explaining in a simple ratio of proportionality describes like [17]

$$h = \frac{3w}{(k-1)} \tag{1.9}$$

With,

h : Height of increase

w : Height of void

k : Coincidence factor

Piggott and Eynon explain why the Figureance of a subsidence occurs when a cavity reached certain a diameter, but also that "the rupture of a cavity can stop automatically if there is a layer (in particular rigid) having a thickness of 1,75 times larger than the size of void" [18] (Figure 6). They also show that, "the height of the rupture can rise up to 10 times that height of void, but frequently it goes up up to 3 - 5 times the height of void".

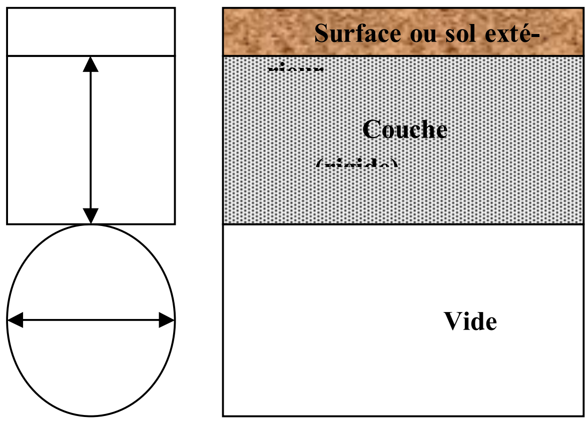


Figure 6 Model of Piggott and Eynon

We also find the equation of Piggott and Eynon which involves the volume of rupture. Into this equation, we introduce a factor b_f. Indeed, this factor considers the maximum height of collapse before a void is blocked. There exist 3 various models of rupture,

1. Model in cone (Figure 7)

Volume of intact

$$V_0 = \frac{\pi B^2}{4} \times \frac{D_c}{3} \tag{1.10}$$

With,

B : Diameter of the subsidence (here is a circle)

D_c : Height of the rupture

Total volume of the zone of rupture

$$V_c = V_0 + \frac{\pi B^2}{4} \times h \tag{1.11}$$

With,

h : Height of void of the cavity

Like,

$$b_f = \frac{V_c - V_0}{V_0} \tag{1.12}$$

b_f : Coefficient of *Piggott and Eynon*

Thus, by using the equations

$$b_f = \frac{3h}{4D_c} \qquad D_c = \frac{3h}{4b_f} \tag{1.13}$$

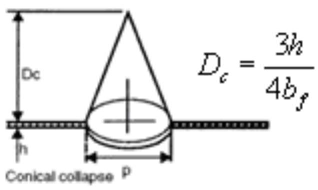


Figure 7 Model in cone (Piggott and Eynon)

2. Model in triangle (Figure 8)

$$V_0 = \frac{p \times D_c \times l}{2} \tag{1.14}$$

With,

p : width of triangle

D_c : height of triangle

l : depth of the triangle

<u>Total volume of the zone of rupture</u>

$$V_c = V_0 + p \times l \times h \tag{1.15}$$

We have then,

$$\begin{aligned} b_f &= \frac{2h}{D_c} \\ D_c &= \frac{2h}{b_f} \end{aligned} \tag{1.16}$$

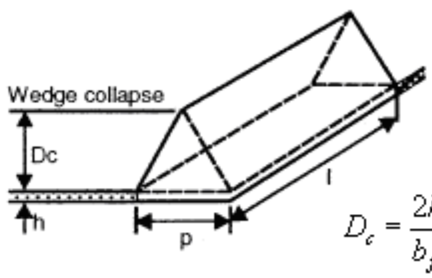


Figure 8 Model in triangle (Piggott and Eynon)

3. Model in rectangle (Figure 9)

$$V_0 = p \times l \times h \tag{1.17}$$

$$V_c = V_0 + p \times l \times h \tag{1.18}$$

Thus,

$$b_f = \frac{h}{D_c}$$
$$D_c = \frac{h}{b_f} \tag{1.19}$$

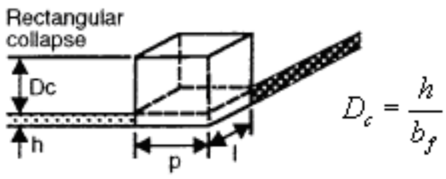


Figure 9 Model in rectangle (Piggott and Eynon)

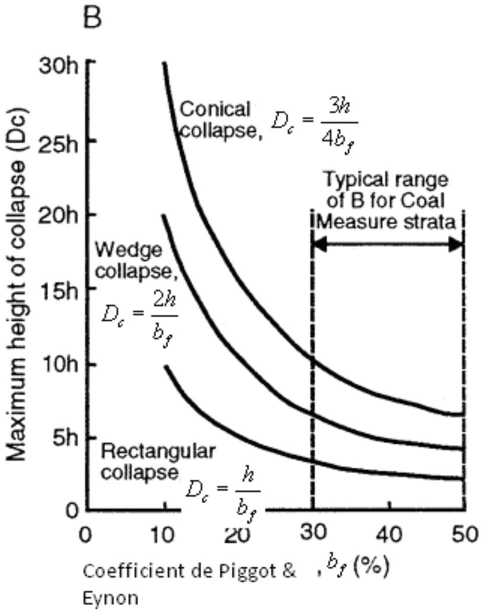


Figure 10 Results of the model of Piggott and Eynon

2.1.1.2 Model of Whittaker & Reddish (1989)

This model compares the "bell" of subsidence to a cylinder of constant rayon (half diameter) which is equal to the rayon of collapse on the level of the roof of the cavity (surface). This model is based to the measure in laboratory of the physical models of long size [19]. The model uses various layers of sand colored and of the special products which separate the various layers of sand. Therefore, they found the following equations, (related with Figure 11)

$$V(x) = \frac{Am}{2}\left(1 - \tanh\left(\frac{2x}{h \times \tan(\gamma)}\right)\right) \tag{1.20}$$

With,

A_m : Maximum vertical subsidence

x : Distance to the maximum point of depression

h : Depth of exploitation

γ : Angle of influence

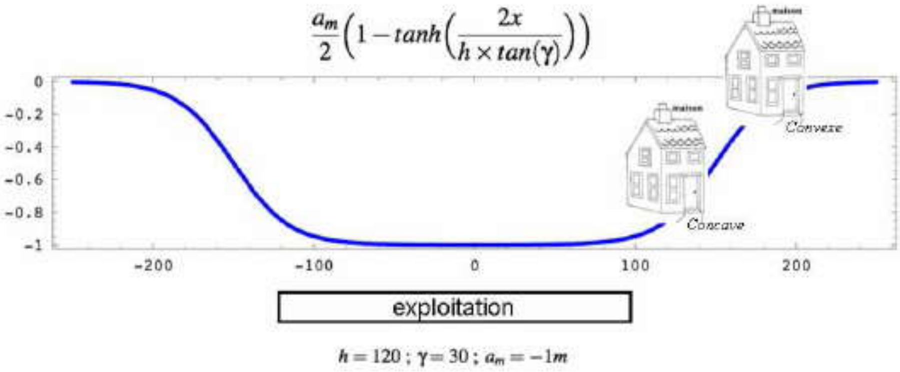


Figure 11 Vertical displacement according to Whittaker and Reddish

In the case of subsidence, by systematically supposing rayon of cylinder for the subsidence equal to the half diagonal of the crossroads of a bord and pillar system, we don't take the variation of height of the subsidence. The larger the rayon of the subsidence is, the less the cylinder must develop to fill the volume of void available

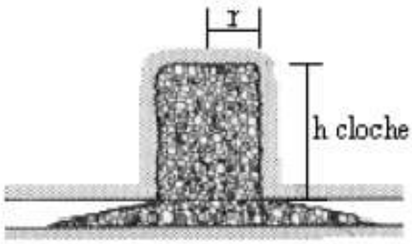


Figure 12 Model of subsidence by Whittaker and Reddish

$$h = \frac{4}{\pi r^2 (k-1)}\left(2aw^2 \cot\theta + wa^2\right) \tag{1.21}$$

With,

h : Height of bell (m)

k : Coincidence factor

r : Initiating rayon at the roof (m)

w : Height of void (m)

a : Width between two pillars (m)

θ : Angle of slope of material at rest (°)

2.1.1.3 Model of Vachat (1982)

This model calculates the height of increase of a bell of subsidence starting from the estimated volume of the bell and volume available for the ploughed up material slope. Two coefficients are introduced to estimate the shape of bell, and the condition of site [20].

1. Coefficient of form (α)
   - Cylinder → 1
   - Parabolic → 2
   - Cone → 3
2. Coefficient of site (β)

The value varies from 0 <β<1 according to the case. This coefficient takes account of the density of the pillars in the vicinity of the bell of subsidence. The value increases if the influence of the pillars decreases.

The condition of the auto filling is also considered where the volume of the bell which must fall is equal to the volume of the bell to fill.

The height of increase of the subsidence, is written like following,

$$h = \frac{\alpha w}{k-1}\left(\frac{\beta w^2}{3r^2 \tan^2\theta} + \frac{\beta w}{r \tan\theta} + 1\right) \tag{1.22}$$

With,

h : Height of bell (m)

k : Coincidence factor

r : Initiating rayon at the roof (m)

w : Height of void (m)

θ : Angle of slope of material at rest (°)

α : Coefficient of form

β : Coefficient of site

2.1.1.4 Model of Meier (2001)

Meier compared the general path of a cylindrical type with a parabolic type during the rupture of subsidence. In this model we can also distinguish the exploitations in rooms with those in pillars from the isolated galleries [21].

- Isolated Gallery and Cylindrical form

$$h = \frac{w}{k-1}\left(1 + \frac{w}{D\tan\theta}\right) \tag{1.23}$$

With,

w : Height of void

k : Coincidence factor

D : Initiating diameter with the roof

Θ : Angle of slope of material at rest

- Isolated gallery and forms parabolic

$$h = 1{,}274 \times \text{equation (1.23)} \tag{1.24}$$

- Rooms & pillars and cylindrical form

$$h = \frac{w}{k-1}\left(1 + \frac{w}{r\tan\theta} + \frac{w}{3r^2\tan^2\theta}\right) \tag{1.25}$$

- Rooms & pillars and parabolic form

$$h = 1{,}5 \times \text{equation (1.25)} \tag{1.26}$$

2.1.2 Analytical approach

This approach is based on the resolution of the equilibrium of mechanical equations connecting the states of stress-strains in the medium concerned. It is in particular used for the analysis of the stability of the roof of the cavity [11]. We present here some approaches to give ideas on the analytical approach,

- Approach of the thin slab type

Tritsch (1987 & 2002) [12] proposed 2 models, studying the stability of the roof of the cavity according to the thickness of the first bench.

1. 1st model - the plate of Timoschenko (1961 & 1968), which supposes an entirely heavy monolithic flagstone.
2. 2nd model - a flagstone embedded in the compressible edges.

- Analytical approach of a software implementation

Proposed by Abbas - Fayad (2004) [22], this approach uses various benches constituting the solid mass of soil by superimposed beams. The deformation of beam is ensured by the actual weight of beam and the weight of overlying materials. This approach can determine the deformation and the state of stress in each layer of material. The concept of induced fracturing can be simplified on Figure 13

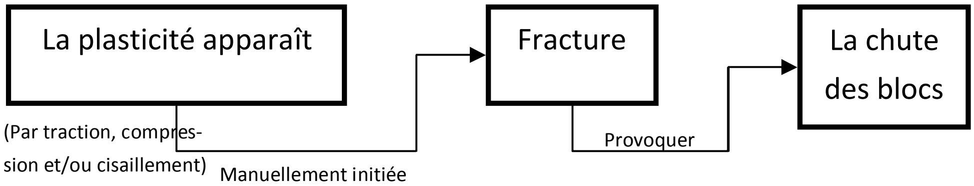


Figure 13 Concept of induced fracturing

The whole of this step was implemented in an analytical computer code able to take account of the auto filling by expansion of voids. The iteration suggested is as follows

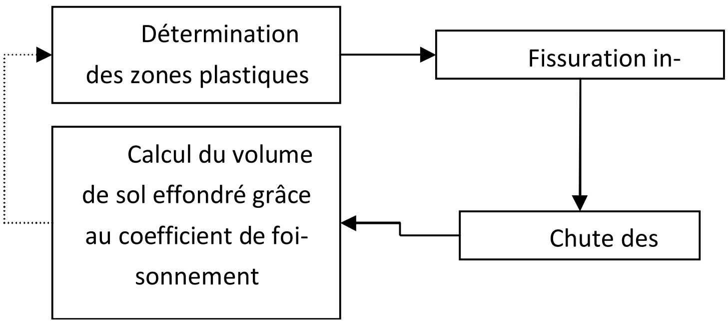


Figure 14 Iteration suggested by Abbas-Fayad on the analytical application of calculation

Calculation is carried out until the attack of one of the three conditions of stop,

1. Absence of new plastic zone→ there is a sufficiently resistant bench
2. Rupture of the last bench of covering→ subsidence emerges on the surface
3. Auto filling

$$V_{matériau\ effondré} \geq V_{cavité} + V_{cloche\ de\ fontis} \quad (1.27)$$

Some remarks on this analytical approach:

1. The covering division in a certain number of beams conditions in an important way the Figureance of instabilities.

2. Require the knowledge of the constitution and mechanical characteristics of the various layers.
3. The induced fracturing is created manually according to a bibliographical study as well of the shape of the bell of subsidence and increase of this one in covering.

2.2 The expansion of cavity

We introduce the work of Yu and Houlsby (1991), named the theory of the expansion of cavity, which predicts the displacement of the soil during the digging. This theory uses the analysis in large-displacements [23].

The expansion of cavities is not a new theory in spite of lack of information. Gibson & Anderson (1961) worked on the application of the expansion of the cavities in geotechnics. The work of Gibson & Anderson is then used in the more practical field in particular to autory out a test of pressure gauge. For the spherical problem of cavity, some studies were undertaken in various types of materials:

- Hill (1950) gave a general solution of the finished expansion of spherical cavity by using a material of Tresca.
- Chadwick (1959) presented a solution of a more complete spherical cavity in an elastic-plastic material conforming to the rule associated with flow with Mohr - Coulomb.
- Vesic (1972) detailed the analysis of compressible soils by taking account of the possibility that the voluminal constraint is not null, and also presented an approximate solution for the limiting pressure for the spherical expansion of cavity.

Yu & Houlsby (1991), which presents this method, employ an analytical solution unified with cylindrical and/or spherical expansion of the cavities in the soils dilating elastic-plastic with the criterion of Mohr - Coulomb and the rule of flow not - associated.

2.2.1 The application of the expansion of cavities

Compared to the displacement of soil, we distinguish the elastic zone and the plastic zone where displacement is permanent. For the plastic zone, its external dimension depends on the existing state of soil, of the initial rayon of the cavity, and the ratio of expansion (the relationship between the width of digging and the width of cavity) [24]. Compared to these risks, some theorems were proposed:

- O.Rourke (1985) proposed a solution to envisage the rayon of the zone plastic, depend on the quantity of expansion on cavity and rigidity of the soil.

- Atalah and Al (1997) use the theory of expansion of cavity developed by Vesic (1972) to envisage the rayon of plasticity of soil. (the solution of Vesic does not allow calculations of plastic deformation in the plastic zone).
- Chapman and Rogers (1992) use a model of flow to envisage displacements of soil and compared it with their experimental results (not adapted to any type of soil).

The use of the theory of expansion of a cavity simplifies things by neglecting the nonhomogeneous properties of soil, the initial gradient of force geostatics, the local stress concentrations, and the longitudinal effects during the digging of soil. The answer of soil will be evaluated on the plane axisymmetric constraints.

2.2.2 Simplified calculation

First of all the rayon of the cavity is equal to a_0 and a hydrostatic pressure p_0 which acts in all surface of the soil that we suppose homogeneous. The pressure inside the cavity is then increased gently to p (thus the dynamic effect is negligible). We are interested in the distribution of the effort and of displacement in the soil when the pressure increases p_0 up to a certain value p. limits.

When the pressure applied p increases p_0, the first deformation of the soil is purely elastic. We have,

$$\sigma_r = -p_0 - (p - p_0)\left(\frac{a}{r}\right)^{m+1} \tag{1.28}$$

$$\sigma_\theta = -p_0 + \frac{(p - p_0)}{m}\left(\frac{a}{r}\right)^{m+1} \tag{1.29}$$

With,

m : Form cavity (1 for cylindrical, 2 for spherical)

a : Radius of the cavity

r : Rayon of the point of reference

And for displacement,

$$u = \frac{p - p_0}{2mG}\left(\frac{a}{r}\right)^{m+1} r \tag{1.30}$$

With,

G : Modulus of rigidity

2.3 Crolet’s Model

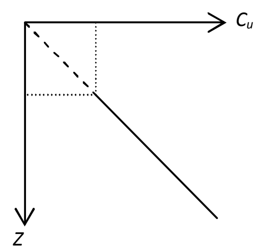
Here the existing calculation suggested by Crolet (2006) [1], estimating the time of erosion as well as the output hydrogramme. The data input of calculation used are the time of erosion and the flow of exit. This calculation also requires laws of erosion of the model which are still to date difficult to check. By writing the implying balance of the forces of the forces of cohesion and weight, the author finds an approach on the equation (1.31)

To estimate the force of cohesion we consider:

- The case of undrained cohesion varying linearly with the depth.
- A surcohésion on the surface related to the compaction.

$$\left(2HR - \frac{\pi R^2}{2}\right)\gamma = F_{cohésion} \tag{1.31}$$

Cohesion being constant with the top height of compaction and a function depth in lower part.



The equation (1.31) becomes

<u>For Z ≥ Zc</u>

$$\left(2HR - \frac{\pi R^2}{2}\right)\gamma = 2\int_0^H \sin(\phi')\gamma z dz + 2\frac{Z_c C_{uc}}{2} \tag{1.32}$$

<u>For Z < Zc</u>

$$\left(2HR - \frac{\pi R^2}{2}\right)\gamma = 2ZC_{uc} \tag{1.33}$$

Without the quadratic term

$$2HR = \sin(\phi')H^2 + C_{uc}Z_c \tag{1.34}$$

Undrained cohesion

$$C_{uc} = \sin(\phi')\gamma Z_c \tag{1.35}$$

We substitute (1.35) → (1.34)

$$R = \frac{\sin(\phi')H^2}{2H}\left(H^2 + \gamma Z_c^{\ 2}\right) \tag{1.36}$$

With the quadratic term

For Z ≥ Zc

$$\pi R^2 - 4HR + 2\sin(\phi')\left(H^2 + Z_c^{\ 2}\right) = 0 \tag{1.37}$$

$$R = \frac{1}{\pi}\left(2H - \sqrt{4H^2 - 2\pi\sin(\phi')\left(H^2 + Z_c^{\ 2}\right)}\right) \tag{1.38}$$

For Z < Zc

$$\left(2HR - \frac{\pi R^2}{2}\right)\gamma = 2ZC_{uc} \tag{1.39}$$

$$R = \frac{1}{\pi}\left(2H - \sqrt{4H^2 - 2\sin(\phi')\pi(Z)(Z_c)}\right) \tag{1.40}$$

To consider the height critical (*Ingold*), It is necessary to take into account:

- The coefficient of earth pressure
- Pressure generated by the compactor

$$Z_c = \frac{1}{K_a}\sqrt{\frac{2p}{\pi\gamma}} \tag{1.41}$$

With,

$$K_a = 1 - \sin(\phi') \tag{1.42}$$

Where the values of pressure (p) depend on the date of construction