Hamburan Kaon-Nukleon
Dengan Potensial Separabel

TESIS

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SALEMBA
2008

UNIVERSITAS INDONESIA

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TESIS

Diajukan sebagai salah satu syarat untuk memperoleh gelar Magister Sains

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Telah berhasil dipertahankan di hadapan Dewan Penguji dan diterima sebagai bagian persyaratan yang diperlukan untuk memperoleh gelar Magister Sains pada Program Studi Magister Ilmu Fisika Fakultas Matematika dan Ilmu Pengetahuan Alam Universitas Indonesia.

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UCAPAN TERIMAKASIH

Puji syukur saya panjatkan kepada Alloh SWT, karena atas berkat dan rahmatNya, saya dapat menyelesaikan tesis ini. Penyusunan tesis ini dalam rangka memenuhi salah satu syarat untuk mencapai gelar magister Sains Jurusan Fisika pada Fakultas MIPA Universitas Indonesia. Saya menyadari bahwa tanpa bantuan, dukungan dan bimbingan dari berbagai pihak, mulai dari awal perkuliahan hingga pada penyusunan tesis ini. Untuk itu saya mengucapkan terima kasih kepada :

1. Bapak Dr. Agus Salam, selaku dosen pembimbing yang telah menyediakan waktu, tenaga dan pikiran didalam mengarahkan saya dalam penulisan tesis ini.
2. Dewan Penguji atas segala masukan-masukannya.
3. Orang tua dan adik-adik saya yang telah memberikan bantuan material maupun moril.
4. Tiga pria tampan: Alfian, Farhan dan Raihan yang selalu ada dalam hati saya, yang selalu menghibur di kala lelah dan resah melanda.
5. Para rekan dosen Universitas Pamulang dan Universitas Mercu Buana, sahabat serta para mahasiswa yang telah membantu menyemangati saya.
6. Pak Parman yang selalu siap membantu saya tanpa pamrih, tanpa lelah.

Akhir kata, saya berharap Alloh SWT membalas segala kebaikan kalian semua. Dan semoga tesis ini membawa manfaat bagi pengembangan ilmu.

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ABSTRAK

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Judul : Hamburan Kaon-Nukleon Dengan Potensial Separabel

Pada tulisan ini, interaksi kaon-nukleon dipelajari dengan pendekatan interaksi separabel dimana penurunan amplitudo dari hamburan kaon-nukleon pada tulisan ini akan menggunakan persamaan Lippman-Schwinger. Dari hasil perhitungan amplitudo transisi hamburan kaon-nukleon dengan potensial separabel rank-1 dan rank-2 untuk sembarang gelombang parsial, ditunjukkan bahwa penggunakan potensial separabel memudahkan perhitungan amplitudo transisi. Tetapi, tetap menyisakan beberapa parameter pada amplitudo transisi yang harus ditentukan melalui *fitting* dengan data eksperimen.

Kata kunci:
Pergeseran fase, potensial separabel, amplitude.

ABSTRACT

Name : Eva Siti Khuzaeva
Adviser : Dr. Agus Salam
Study Program: Magister of Physics
Title : Scattering of kaon-nukleon with separable potential

In this thesis, kaon-nucleon interaction has been studied by using a separable potential to the second order. The transition amplitudes of kaon-nucleon scattering has been calculated through solving the Lippmann-Schwinger equation for all partial waves. Application of separable potential makes the calculation of the transition amplitude more simple. The amplitude contains some parameters that should be determined through fitting by the experimental data.

Key words:

Phase shifts, separable potential, amplitude.

DAFTAR ISI

DAFTAR GAMBAR

DAFTAR LAMPIRAN

PENDAHULUAN

1.1. Latar Belakang dan Perumusan Masalah

Interaksi kaon-nukleon adalah subjek yang penting dalam studi interaksi kuat, sebab interaksi ini menjadi dasar dari banyak proses hadronik lainnya, terutama interaksi nukleon-hiperon, produksi elektromagnetik kaon, dan kondensasi kaon pada materi nuklir rapat. Interaksi kaon-nukleon dipelajari dengan beberapa pendekatan, seperti teori pertukaran meson [1], teori perturbasi *chiral* [2], model quark [3], dan pendekatan interaksi separabel [4].

Dalam penelitian ini, interaksi kaon-nukleon akan dipelajari melalui perhitungan pergeseran fase dengan pendekatan interaksi separabel. Keuntungan dari pendekatan ini adalah memberikan kemudahan perhitungan untuk masalah beberapa benda (misalnya. masalah tiga benda), seperti interaksi keadaan akhir pada reaksi $\gamma + d \rightarrow K + Y + N$, di mana ada tiga partikel pada keadaan akhir.

Interaksi kaon-nukleon dipelajari dengan percobaan hamburan kaon-nukleon yang reaksinya ditulis sebagai berikut :

$$K(\vec{p}_K) + N(\vec{p}_N) \rightarrow K(\vec{q}_K) + N(\vec{q}_N)$$

Pada energi lebih tinggi, muncul juga kanal inelastik yang menghasilkan partikel lain seperti eta(η), pion(π), lambda(Λ), dan sebagainya.

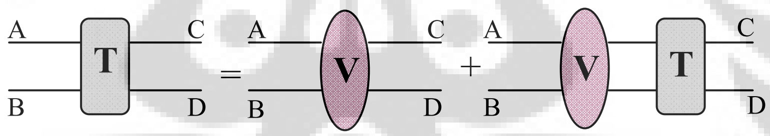
Penurunan amplitudo dari hamburan kaon-nukleon pada tulisan ini akan menggunakan persamaan Lippman-Schwinger [5], dengan hubungan antara amplitudo hamburan dan pergeseran fase (δ) juga akan dibahas [6]. Hasil perhitungan akan dibandingkan dengan data eksperimen yang ada [7].

Persamaan Lippmann-Schwinger dapat dituliskan sebagai berikut :

$$t(q,p;w) = v(q,p) + \int_0^\infty dkk^2 v(q,k) G_N(w;k) t(k,p;w) \qquad (1.1.1)$$

dimana t adalah *partial wave* untuk hamburan kaon-nukleon atau dapat pula dikatakan sebagai peluang transisi dari keadaan awal ke keadaan akhir, karena amplitudo hamburan harus memenuhi kondisi unitari yang menyatakan kekekalan probabilitas.

sedangkan :
$v(g,p)$ adalah potensial
$G_N(w;k)$ adalah fungsi Green
w adalah energi total
p adalah momentum relatif awal (*initial relative momenta*)
q adalah momentum relatif akhir (*final relative momenta*)



Gambar 1.1. Representasi dari Persamaan Lippmann-Schwinger untuk matriks-T dari reaksi dua partikel $A + B \rightarrow C + D$.

1.2. Metode dan Tujuan Penelitian

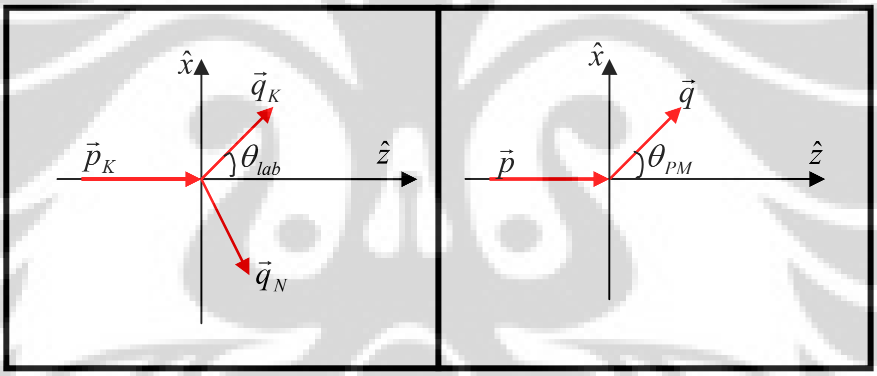
Metode yang dipakai dalam penelitian ini adalah perhitungan teoritis analitik yang dilanjutkan dengan kalkulasi numerik untuk mem-*fitting* perhitungan teoritis tersebut dengan data eksperimen.

Mempelajari hamburan *KN* dengan pendekatan potensial separabel rank-1 dan rank-2 dan melakukan *fitting* perhitungan teoritis dengan data pergeseran fase yang diekstrak dari eksperimen.

2. TEORI UMUM

2.1. Kinematika Hamburan Kaon-Nukleon

Dalam kerangka laboratorium, kita melihat sistem dua partikel dengan momentum awal partikel kaon $\vec{p}_K$ dan partikel nukleon $\vec{p}_N$. Selanjutnya persoalan dalam perhitungan bisa dipermudah apabila kita menggunakan kerangka pusat massa. Dengan demikian kita bisa melihat sistem *KN* sebagai sistem satu partikel dengan momentum relatif awal $\vec{p}$. Untuk keadaan akhir, kita gunakan $\vec{q}_K$ dan $\vec{q}_N$ sebagai momentum akhir partikel kaon dan nukleon dalam kerangka laboratorium, serta $\vec{q}$ sebagai momentum relatif akhir dalam kerangka pusat massa.



Gambar 2.1. Kinematika Hamburan Kaon-Nukleon

Secara umum, hubungan antara momentum dalam kerangka laboratorium dan momentum dalam kerangka pusat massa adalah sebagai berikut :

$$\vec{p} = \frac{m_N \vec{p}_K - m_K \vec{p}_N}{m_K + m_N}, \tag{2.1.1}$$

Pada keadaan awal kita berurusan dengan sistem dimana Kaon bergerak menumbuk Nukleon yang berada dalam keadaan diam relatif terhadap kerangka laboratorium. Karenanya $\vec{p}_N = 0$. Maka

$$\vec{p} = \frac{\mu}{m_K}\vec{p}_K \tag{2.1.2}$$

Dengan μ adalah massa tereduksi :

$$\mu = \frac{m_K m_N}{m_K + m_N} \tag{2.1.3}$$

Rumus untuk energi total dalam kerangka pusat massa w_{PM} dan energi total dalam kerangka laboratorium w_{lab} adalah :

$$w_{lab} = \sqrt{\vec{p}_K^{\,2} + m_K^2} + \frac{\vec{p}_N^{\,2}}{2m_N} + m_N \;;\; \vec{p}_N = 0.$$

$$w_{lab} = \sqrt{\vec{q}_K^{\,2} + m_K^2} + \frac{\vec{q}_N^{\,2}}{2m_N} + m_N \tag{2.1.4}$$

$$w_{PM} = \sqrt{\vec{p}^{\,2} + m_K^2} + \frac{\vec{p}^{\,2}}{2m_N} + m_N$$

$$w_{PM} = \sqrt{\vec{q}^{\,2} + m_K^2} + \frac{\vec{q}^{\,2}}{2m_N} + m_N \tag{2.1.5}$$

2.2. Persamaan Lippmann-Schwinger

Jika hamburan dari *KN* pada pusat massa dimana memiliki energi total *w* dan momentum relatif awal dan akhir masing-masing $\vec{p}$ dan $\vec{q}$, maka amplitudo *t* untuk hamburan *KN* ini diberikan oleh persamaan Lippmann-Schwinger sebagai berikut :

$$t(q,p;w) = v(q,p) + \int_0^\infty dkk^2 v(q,k) G_N(w;k) t(k,p;w) \tag{2.2.1}$$

dimana $v(q,p)$ adalah potensial interaksi antar partikel, *k* adalah momentum relatif, $G_N(w;k)$ fungsi Green, dan N kontribusi partikel pada keadaan antara.

Untuk hamburan tunggal, fungsi Green dapat dituliskan sebagai berikut :

$$G_N(w;k) = [w + i\varepsilon - w_N(k)]^{-1} \quad (2.2.2)$$

dimana

$$w_N(k) = \sqrt{k^2 + m_K{}^2} + \frac{k^2}{2m_N} + m_N. \quad (2.2.3)$$

Untuk rank-1, potensial separabel (*separable potential*) dapat dituliskan menjadi :

$$v(q,p) = \lambda g(q) g(p); \lambda = \pm 1 \quad (2.2.4)$$

dengan :

λ : parameter fase

g : faktor bentuk

p : momentum relatif awal

q : momentum relatif akhir

Dimana solusi untuk persamaan Lippmann-Schwinger diatas diberikan oleh persamaan :

$$t(q,p;w) = g(q)\tau(w)g(p), \quad (2.2.5)$$

dengan

$$\tau(w) = \left[\lambda^{-1} - \int_0^\infty dkk^2 g(k) G_N(w;k) g(k)\right]^{-1} \quad (2.2.6)$$

Untuk rank-2, potensial separabel (*separable potential*) dapat dituliskan menjadi:

$$v(q,p)=\lambda_1 g_1(q)g_1(p)+\lambda_2 g_2(q)g_2(p) \qquad (2.2.7)$$

sehingga diperoleh solusi yang berbentuk :

$$t(q,p;w)=\sum_{i,j=1,2} g_i(q)\frac{N_{ij}(w)}{D(w)}g_j(p), \qquad (2.2.8)$$

dimana

$$N_{3-i,3-i}(w)=\lambda_i^{-1}-X_{ii}(w), \quad (i=1,2), \quad N_{12}(w)=N_{21}(w)=X_{12}(w) \qquad (2.2.9)$$

dengan

$$X_{ij}(w)=\int_0^\infty dkk^2 g_i(k)G_N(w;k)g_j(k) \qquad (2.2.10)$$

dan

$$D(w)=N_{11}(w)N_{22}(w)-[N_{12}(w)]^2. \qquad (2.2.11)$$

Untuk hamburan *multi-channel,* Fungsi Green akan berbentuk :

$$G_i(k;w)=[w+i\varepsilon-w_i(k)]^{-1}, \qquad (i=1,2,...,n) \qquad (2.2.12)$$

dimana

$$w_i(k)=\sqrt{k^2+m_i^2}+\frac{k^2}{2M_i}+M_i. \quad (i=1,2,...,n) \qquad (2.2.13)$$

Pada kasus ini diasumsikan potensial rank-1 sebagai berikut :

$$v(q,p;w)=\lambda^{-1}\begin{bmatrix} g_N(q) \\ g_1(q) \\ \vdots \\ g_N(q) \end{bmatrix}\cdot[g_N(p) \quad g_1(p) \quad \cdots \quad g_n(p)]. \qquad (2.2.14)$$

Sehingga didapatkan solusi

$$t(q,p;w)=\begin{bmatrix} g_N(q) \\ g_1(q) \\ \vdots \\ g_N(q) \end{bmatrix}\tau(w)\left[g_N(p) \quad g_1(p) \quad \cdots \quad g_n(p)\right]. \qquad (2.2.15)$$

dengan $\tau(w)$ pada persamaan ini didefinisikan sebagai

$$\tau(w)=\left[\lambda^{-1}-\sum_{i=N,1,2,\ldots,n}\int_0^{\infty} dkk^2 g_i(k)G_i(w;k)g_i(k)\right]^{-1}. \qquad (2.2.16)$$

Hamburan *multi-channel* rank-2 diasumsikan sebagai berikut

$$v(q,p;w)=\begin{bmatrix} \lambda_1 g_1(q)g_1(p)+g_2(q)g_2(p) & \lambda_2 g_2(q)\gamma_1(p) & \cdots & \lambda_2 g_2(q)\gamma_n(p) \\ \lambda_2\gamma_1(q)g_2(p) & \lambda_2\gamma_1(q)\gamma_1(p) & \cdots & \lambda_2\gamma_1(q)\gamma_n(p) \\ \vdots & \vdots & \vdots & \vdots \\ \lambda_2\gamma_n(q)g_2(p) & \lambda_2\gamma_n(q)\gamma_1(p) & \cdots & \lambda_2\gamma_n(q)\gamma_n(p) \end{bmatrix}.$$

(2.2.17)

Solusi untuk amplitudo hamburan KN diberikan oleh persamaan yang mirip dengan persamaan-persamaan di atas, tetapi dengan

$$X_{11}(w)=\int_0^{\infty} dkk^2 g_1(k)G_N(w,k)g_1(k),$$

$$X_{22}(w)=\int_0^{\infty} dkk^2 g_2(k)G_N(w,k)g_2(k)+\sum_{i=1,\ldots,n}\int_0^{\infty} dkk^2\gamma_i(k)G_i(w)\gamma_i(k), \qquad (2.2.18)$$

$$X_{12}(w)=\int_0^{\infty} dkk^2 g_1(k)G_N(w,k)g_2(k).$$

2.3. Pergeseran Fase (Phase Shifts)

Phase Shifts dapat menunjukkan sifat atraktif atau repulsif dari gaya nuklir. Sesuai dengan relasi antara momentum relatif *p*, parameter tumbukan *b* dan momentum angular *l* :

$$pb = \left(L + \tfrac{1}{2}\right) \qquad (2.3.1)$$

kita dapat menemukan nilai radius $(\approx b)$ daerah gaya nuklir yang memiliki sifat atraktif atau repulsif. Misalnya berdasarkan persamaan (2.3.1), kita tahu bahwa pada nilai energi proyektil yang sama (*p* sama), untuk nilai *l* yang kecil kita memiliki nilai *b* yang kecil. Sehingga, untuk melihat bagaimana sifat interaksi nuklir pada radius yang pendek kita bisa melihat pada nilai *phase shifts* $S(l=0)$ di energi yang tidak terlalu tinggi.Lebih lanjut dengan persamaan (2.3.1) kita dapat mengetahui nilai maksimum dari *L*, yang dapat kita ikut sertakan untuk meneliti sifat gaya nuklir pada radius tertentu *b* dan pada besar energi tertentu.

Dari *phase shifts* kita dapat memperoleh pengertian yang lebih dalam untuk membuat model. Namun sebelumnya, kita perlu untuk mencari hubungan antara matriks-T dan *phase shifts*. Hubungan antara matriks-S dan matriks-T dapat dituliskan sebagai:

$$S^{j}_{l'l} = \delta_{l'l} - 2\pi\, i\, \rho T^{j}_{l'l} \qquad (2.3.2)$$

dimana :

j = momentum angular total ; $J = L + S$

$l'l$ = bilangan kuantum orbital akhir dan awal

μ = massa tereduksi

p = nilai momentum *on-shell* ; $p = \sqrt{2\,\mu\, w}$ (2.3.3)

= nilai momentum relatif ; $p = \dfrac{m_2 k_1 - m_1 k_2}{m_1 + m_2}$

$S^{j}_{l'l}$ = matriks-S dalam basis PW

$T^{j}_{l'l}$ = matriks-T dalam basis PW

Karena spin total $S = \frac{1}{2}$, maka :

$$j = l \pm \tfrac{1}{2} \quad (2.3.4)$$

atau

$$l = j \pm \tfrac{1}{2} \quad (2.3.5)$$

sehingga nilai minimum untuk *j* adalah $j_{\min} = \frac{1}{2}$ yaitu untuk $l = 0$. Maka untuk nilai $l = 0,1,2,3,\ldots$ kita akan memperoleh nilai $j = \frac{1}{2},\frac{3}{2},\frac{5}{2},\frac{7}{2},\ldots$

Nilai untuk *l* dan *l'* adalah $j \pm \frac{1}{2}$. Karena dalam proses tumbukan *KN*, momentum angular total *J*, nilai spin total *S* dan paritas $II = (-1)^l$ adalah tetap, maka untuk nilai *j* tertentu berlaku :

$$l = l' \quad (2.3.6)$$

Sehingga yang ada hanya keadaan *uncoupled*. Dalam notasi:

$$^{2S+1}L_j = {}^2L_j \,,\; L = S, P, D, F, \ldots \quad (2.3.7)$$

Keadaan yang kita miliki adalah :

$$^2S_{\frac{1}{2}}, {}^2P_{\frac{1}{2}}, {}^2P_{\frac{3}{2}}, {}^2D_{\frac{3}{2}}, \ldots \quad (2.3.8)$$

Notasikan amplitudo elastik hamburan *KN* dengan t_{NN}, sehingga didapat matriks-*S* untuk amplitudo elastik dengan momentum angular total *J* dan paritas *P* sehingga

$$S_{ll}^{j} = 1 - i2\pi\rho(p)t_{NN}, \quad (2.3.9)$$

dimana

$$\frac{1}{\mu(p)} = \frac{1}{\sqrt{p^2 + m_K{}^2}} + \frac{1}{m_N}. \tag{2.3.10}$$

Sebagai sebuah bilangan kompleks, S_{ll}^{j} dapat dinyatakan :

$$\begin{aligned} S_{ll}^{j} &= \mathrm{Re}\left(S_{ll}^{j}\right) + i\,\mathrm{Im}\left(S_{ll}^{j}\right) \\ &= \cos\left(2\delta_l^{j}\right) + i\sin\left(2\delta_l^{j}\right) \end{aligned} \tag{2.3.11}$$

Sehingga dapat dituliskan :

$$\begin{aligned} \tan\left(2\delta_l^{j}\right) &= \frac{\mathrm{Im}\left(S_{ll}^{j}\right)}{\mathrm{Re}\left(S_{ll}^{j}\right)} \\ &= \frac{-2\pi\,\rho\,\mathrm{Re}\left(T_{ll}^{j}\right)}{1 + 2\,\pi\rho\,\mathrm{Im}\left(T_{ll}^{j}\right)} \end{aligned} \tag{2.3.12}$$

Maka, kita peroleh hubungan antara phase shifts dengan matriks-T sebagai berikut :

$$\delta_l^{j} = \tfrac{1}{2}\arctan\left(X_l^{j}\right) \tag{2.3.13}$$

dengan

$$X_l^{j} \equiv \frac{-2\pi\,\rho\,\mathrm{Re}\left(T_{ll}^{j}\right)}{1 + 2\pi\,\rho\,\mathrm{Im}\left(T_{ll}^{j}\right)} \tag{2.3.14}$$

3. PERHITUNGAN ANALITIK

3.1. Pencarian Solusi Lippmann-Schwinger Pada Potensial Separabel Rank-1

Akan dihitung solusi untuk persamaan Lippmann-Schwinger pada potensial separabel (*separable potential*) rank-1.

Dari persamaan umum Lippmann-Schwinger, pada (2.2.1)

$$t(q,p;w) = v(q,p) + \int_0^\infty dkk^2 v(q,k) G_N(w;k) t(k,p;w)$$

Karena potensial separabel dan rank-1, maka pada (2.2.4)

$$v(q,p) = \lambda g(q) g(p); \lambda = \pm 1$$

sehingga dari (2.2.1) dan (2.2.4)

$$t(q,p;w) = \lambda g(q) g(p) + \int_0^\infty dkk^2 \lambda g(q) g(k) G_N(w;k) t(k,p;w)$$

$$= g(q)\left[\lambda g(p) + \lambda \int_0^\infty dkk^2 g(k) G_N(w;k) t(k,p;w)\right]$$

$$t(q,p;w) = g(q) a(p;w) \qquad (3.1.1)$$

dimana

$$a(p;w) = \lambda g(p) + \lambda \int_0^\infty dkk^2 g(k) G_N(w;k) t(k,p;w)$$

$$= \lambda g(p) + \lambda \int_0^\infty dkk^2 g(k) G_N(w;k) g(k) a(p;w)$$

$$= \lambda g(p) + \lambda a(p;w) \int_0^\infty dkk^2 g(k) G_N(w;k) g(k)$$

$$\left[1 - \lambda \int_0^\infty dkk^2 g(k) G_N(w;k) g(k)\right] a(p,w) = \lambda g(p)$$

$$a(p;w) = \frac{\lambda g(p)}{1 - \lambda \int_0^\infty dkk^2 g(k) G_N(w;k) g(k)}$$

$$a(p;w) = \frac{g(p)}{\lambda^{-1} - \int_0^\infty dkk^2 g(k) G_N(w;k) g(k)} \qquad (3.1.2)$$

Dari (3.1.1) dan (3.1.2) diperoleh

$$t(q,p;w) = g(q) \frac{1}{\lambda^{-1} - \int_0^\infty dkk^2 g(k) G_N(w;k) g(k)} g(p) \qquad (3.1.3)$$

yang merupakan solusi dari persamaan Lippmann-Schwinger, pada (2.2.5)

$$t(q,p;w) = g(q) \tau(w) g(p)$$

dimana pada (2.2.6)

$$\tau(w) = \left[\lambda^{-1} - \int_0^\infty dkk^2 g(k) G_N(w;k) g(k)\right]^{-1}.$$

Akan dicari

$$A = \int_0^\infty dkk^2 g(k) G_N(w;k) g(k) \qquad (3.1.4)$$

dimana

$$G_N(w;k) = [w + i\varepsilon - w_N(k)]^{-1}$$

$$w_N(k) = \sqrt{k^2 + m_K{}^2} + \frac{k^2}{2m_N} + m_N .$$

$$g(k) = k^l \sum_{i=1}^{N} \frac{c_i}{k^2 + \beta_i^2}$$; l = momentum angular = 0 atau 1

$$N = 1 \Rightarrow g(k) = \frac{k^l c}{k^2 + \beta^2}; c_1 = c, \beta_1 = \beta$$

sehingga persamaan (3.1.4) dapat dibuat menjadi

$$A = \int_0^\infty dk k^2 g^2(k) G_N(w;k)$$

$$= \int_0^\infty dk k^2 \left(\frac{k^l c}{k^2 + \beta^2}\right)^2 \frac{1}{w + i\varepsilon - w_N(k)}$$

$$= \int_0^\infty dk \frac{k^{2+2l} c^2}{\left(k^2 + \beta^2\right)^2 \left(w + i\varepsilon - \sqrt{k^2 + m_K^2} - \frac{k^2}{2m_N} - m_N\right)}$$

$$= c^2 \int_0^\infty dk \frac{k^{2+2l}}{\left(k^2 + \beta^2\right)^2 \left(w + i\varepsilon - \sqrt{k^2 + m_K^2} - \frac{k^2}{2m_N} - m_N\right)} \qquad (3.1.5)$$

Persamaan (3.1.5) akan diselesaikan dengan Teorema Residu sebagai berikut :

$$\left(k^2 + \beta^2\right)^2 \left(w + i\varepsilon - \sqrt{k^2 + m_K^2} - \frac{k^2}{2m_N} - m_N\right) = 0$$

(i) $\left(k^2 + \beta^2\right)^2 = 0$

$$\Rightarrow k^2 + \beta^2 = 0$$

$$k^2 = -\beta^2$$

$$k = \pm\sqrt{-\beta^2}$$

$$k_{1,2} = \pm i\beta$$

(ii) $\left(w + i\varepsilon - \sqrt{k^2 + m_K^2} - \frac{k^2}{2m_N} - m_N \right) = 0$

Dengan program Mathematica didapat :

$$k_3 = -\sqrt{2}\sqrt{m_N w + m_N i\varepsilon - \sqrt{m_K^2 m_N^2 - m_N^4 + 2m_N^3 w + 2m_N^3 i\varepsilon}}$$
$$k_4 = \sqrt{2}\sqrt{m_N w + m_N i\varepsilon - \sqrt{m_K^2 m_N^2 - m_N^4 + 2m_N^3 w + 2m_N^3 i\varepsilon}}$$
$$k_5 = -\sqrt{2}\sqrt{m_N w + m_N i\varepsilon + \sqrt{m_K^2 m_N^2 - m_N^4 + 2m_N^3 w + 2m_N^3 i\varepsilon}}$$
$$k_6 = \sqrt{2}\sqrt{m_N w + m_N i\varepsilon + \sqrt{m_K^2 m_N^2 - m_N^4 + 2m_N^3 w + 2m_N^3 i\varepsilon}}$$

Dengan mengasumsikan $\varepsilon = 0$, maka akan diperoleh

$$k_3 = -\sqrt{2}\sqrt{m_N w - \sqrt{m_K^2 m_N^2 - m_N^4 + 2m_N^3 w}}$$
$$k_4 = \sqrt{2}\sqrt{m_N w - \sqrt{m_K^2 m_N^2 - m_N^4 + 2m_N^3 w}}$$
$$k_5 = -\sqrt{2}\sqrt{m_N w + \sqrt{m_K^2 m_N^2 - m_N^4 + 2m_N^3 w}}$$
$$k_6 = \sqrt{2}\sqrt{m_N w + \sqrt{m_K^2 m_N^2 - m_N^4 + 2m_N^3 w}}$$

Sehingga didapat:

$$k_{1,2} = \pm i\beta$$

$$k_{3,4,5,6} = \mp\sqrt{2}\sqrt{m_N w \mp \sqrt{m_K^2 m_N^2 - m_N^4 + 2m_N^3 w}}$$

dengan :

$k_{1,2}$ adalah *pole second order*

$k_{3,4,5,6}$ adalah *simple pole*

Nilai *k* akan digambarkan dengan menggunakan program Matlab pada Bab.IV.

3.1.1. PERHITUNGAN NILAI RESIDU UNTUK TIAP NILAI *k*

Dengan memisalkan $m_K = m$ dan $m_N = n$, maka :

1. Pencarian residu dari $f(k_1)$ pada *double pole* $k_1 = i\beta$ pada lampiran 1.

   Sehingga diperoleh residu dari $f(k_1)$ pada *double pole* $k_1 = i\beta$ adalah

$$= -\frac{(2+2l)(i\beta)^{1+2l}}{4\beta^2\left(w-\sqrt{m^2-\beta^2}+\frac{\beta^2}{2n}-n\right)} + \frac{2(i\beta)^{2+2l}}{18\beta^3\left(w-\sqrt{m^2-\beta^2}+\frac{\beta^2}{2n}-n\right)} - \frac{\left(\frac{i\beta}{\sqrt{m^2-\beta^2}}+\frac{i\beta}{n}\right)(i\beta)^{2+2l}}{4\beta^2\left(w-\sqrt{m^2-\beta^2}+\frac{\beta^2}{2n}-n\right)^2}. \quad (3.1.1.3)$$

2. Pecarian residu dari $f(k_2)$ pada *double pole* $k_2 = -i\beta$ pada lampiran 2

   Sehingga diperoleh residu dari $f(k_2)$ pada *double pole* $k = -i\beta$ adalah

$$= -\frac{(2+2l)(-i\beta)^{1+2l}}{4\beta^2\left(w-\sqrt{m^2-\beta^2}+\frac{\beta^2}{2n}-n\right)} + \frac{2(-i\beta)^{2+2l}}{18\beta^3\left(w-\sqrt{m^2-\beta^2}+\frac{\beta^2}{2n}-n\right)} - \frac{\left(\frac{i\beta}{\sqrt{m^2-\beta^2}}+\frac{i\beta}{n}\right)(-i\beta)^{2+2l}}{4\beta^2\left(w-\sqrt{m^2-\beta^2}+\frac{\beta^2}{2n}-n\right)^2}. \quad (3.1.1.5)$$

3. Pecarian residu dari $f(k_3)$ pada *simple pole* $k_3 = -\sqrt{2}\sqrt{nw-\sqrt{m^2n^2-n^4+2n^3w}}$ pada lampiran 3.

   Sehingga diperoleh residu dari $f(k_3)$ pada *simple pole* $k_3 = -\sqrt{2}\sqrt{nw-\sqrt{m^2n^2-n^4+2n^3w}}$ adalah

$$=\frac{n\left(\sqrt{2}\sqrt{nw-\sqrt{m^2n^2-n^4+2n^3w}}\right)^{1+2l}\sqrt{\left(\sqrt{2}\sqrt{nw-\sqrt{m^2n^2-n^4+2n^3w}}\right)^2+m^2}}{\left(\left(\sqrt{2}\sqrt{nw-\sqrt{m^2n^2-n^4+2n^3w}}\right)^2+\beta^2\right)^2\left(n+\sqrt{\left(\sqrt{2}\sqrt{nw-\sqrt{m^2n^2-n^4+2n^3w}}\right)^2+m^2}\right)}$$

(3.1.1.6)

4. Pecarian residu dari $f(k_4)$ pada *simple pole*

   $k_4=\sqrt{2}\sqrt{nw-\sqrt{m^2n^2-n^4+2n^3w}}$ pada lampiran 4.

   Sehingga diperoleh residu dari $f(k_4)$ pada *simple pole* $k_4=\sqrt{2}\sqrt{nw-\sqrt{m^2n^2-n^4+2n^3w}}$ adalah

$$-\frac{n\left(\sqrt{2}\sqrt{nw-\sqrt{m^2n^2-n^4+2n^3w}}\right)^{1+2l}\sqrt{\left(\sqrt{2}\sqrt{nw-\sqrt{m^2n^2-n^4+2n^3w}}\right)^2+m^2}}{\left(\left(\sqrt{2}\sqrt{nw-\sqrt{m^2n^2-n^4+2n^3w}}\right)^2+\beta^2\right)^2\left(n+\sqrt{\left(\sqrt{2}\sqrt{nw-\sqrt{m^2n^2-n^4+2n^3w}}\right)^2+m^2}\right)}$$

(3.1.1.7)

5. Pecarian residu dari $f(k_5)$ pada *simple pole*

   $k_5=-\sqrt{2}\sqrt{nw+\sqrt{m^2n^2-n^4+2n^3w}}$ pada lampiran 5.

   Sehingga diperoleh residu dari $f(k_4)$ pada *simple pole* $k_4=\sqrt{2}\sqrt{nw-\sqrt{m^2n^2-n^4+2n^3w}}$ adalah

$$\frac{n\left(\sqrt{2}\sqrt{nw+\sqrt{m^2n^2-n^4+2n^3w}}\right)^{1+2l}\sqrt{\left(\sqrt{2}\sqrt{nw+\sqrt{m^2n^2-n^4+2n^3w}}\right)^2+m^2}}{\left(\left(\sqrt{2}\sqrt{nw+\sqrt{m^2n^2-n^4+2n^3w}}\right)^2+\beta^2\right)^2\left(n+\sqrt{\left(\sqrt{2}\sqrt{nw+\sqrt{m^2n^2-n^4+2n^3w}}\right)^2+m^2}\right)}$$

(3.1.1.8)

6. Pecarian residu dari $f(k_6)$ pada *simple pole*

$k_6 = \sqrt{2}\sqrt{nw + \sqrt{m^2 n^2 - n^4 + 2n^3 w}}$ pada lampiran 6.

Sehingga diperoleh residu dari $f(k_6)$ pada *simple pole* $k_6 = \sqrt{2}\sqrt{nw + \sqrt{m^2 n^2 - n^4 + 2n^3 w}}$ adalah

$$-\frac{n\left(\sqrt{2}\sqrt{nw+\sqrt{m^2n^2-n^4+2n^3w}}\right)^{1+2l}\sqrt{\left(\sqrt{2}\sqrt{nw+\sqrt{m^2n^2-n^4+2n^3w}}\right)^2+m^2}}{\left(\left(\sqrt{2}\sqrt{nw+\sqrt{m^2n^2-n^4+2n^3w}}\right)^2+\beta^2\right)^2\left(n+\sqrt{\left(\sqrt{2}\sqrt{nw+\sqrt{m^2n^2-n^4+2n^3w}}\right)^2+m^2}\right)} \quad (3.1.1.9)$$

3.2. Pencarian Solusi Lippmann-Schwinger Pada Potensial Separabel Rank-2

Akan dihitung solusi untuk persamaan Lippmann-Schwinger pada potensial separabel (*separable potential*) rank-2.

Dari persamaan umum Lippmann-Schwinger pada (2.2.1)

$$t(q,p;w) = v(q,p) + \int_0^\infty dkk^2 v(q,k) G_N(w;k) t(k,p;w)$$

Karena potensial separabel dan rank-2, maka pada (2.2.7)

$$v(q,p) = \lambda_1 g_1(q) g_1(p) + \lambda_2 g_2(q) g_2(p)$$

Sehingga dengan mensubstitusi (2.2.7) ke (2.2.1) didapatkan

$$t(q,p;w)=\lambda_1 g_1(q)g_1(p)+\lambda_2 g_2(q)g_2(p)$$
$$+\int_0^\infty dkk^2\left(\lambda_1 g_1(q)g_1(k)+\lambda_2 g_2(q)g_2(k)\right)G_N(w;k)t(k,p;w)$$
$$=\lambda_1 g_1(q)g_1(p)+\lambda_2 g_2(q)g_2(p)$$
$$+\int_0^\infty dkk^2\lambda_1 g_1(q)g_1(k)G_N(w;k)t(k,p;w)+\lambda_2 g_2(q)g_2(k)G_N(w;k)t(k,p;w)$$

$$=g_1(q)\left(\lambda_1 g_1(p)+\int_0^\infty dkk^2\lambda_1 g_1(k)G_N(w;k)t(k,p;w)\right)$$
$$+g_2(q)\left(\lambda_2 g_2(p)+\int_0^\infty dkk^2\lambda_2 g_2(k)G_N(w;k)t(k,p;w)\right)$$

$$t(q,p;w)=g_1(q)a_1(p;w)+g_2(q)a_2(p;w) \qquad (3.2.1)$$

dimana

$$a_1(p,w)=\lambda_1 g_1(p)+\lambda_1\int_0^\infty dkk^2 g_1(k)G_N(w;k)t(k,p;w)$$

$$=\lambda_1 g_1(p)+\lambda_1\int_0^\infty dkk^2 g_1(k)G_N(w;k)\left(g_1(k)a_1(p,w)+g_2(k)a_2(p,w)\right)$$
$$=\lambda_1 g_1(p)+\lambda_1\int_0^\infty dkk^2 g_1(k)G_N(w;k)g_1(k)a_1(p,w)+g_1(k)G_N(w;k)g_2(k)a_2(p,w)$$
$$=\lambda_1 g_1(p)+\lambda_1\int_0^\infty dkk^2 g_1(k)G_N(w;k)g_1(k)a_1(p,w)+\lambda_1\int_0^\infty dkk^2 g_1(k)G_N(w;k)g_2(k)a_2(p,w)$$
$$=\lambda_1 g_1(p)+\lambda_1 a_1(p;w)\int_0^\infty dkk^2 g_1(k)G_N(w;k)g_1(k)+\lambda_1 a_2(p;w)\int_0^\infty dkk^2 g_1(k)G_N(w;k)g_2(k)$$

$$=\frac{\lambda_1 g_1(p)+\lambda_1 a_2(p;w)\int_0^\infty dkk^2 g_1(k)G_N(w;k)g_2(k)}{1-\lambda_1\int_0^\infty dkk^2 g_1(k)G_N(w;k)g_1(k)}$$

$$a_1(p;w) = \frac{g_1(p) + a_2(p;w)\int_0^\infty dkk^2 g_1(k) G_N(w;k) g_2(k)}{\lambda_1^{-1} - \int_0^\infty dkk^2 g_1(k) G_N(w;k) g_1(k)} \tag{3.2.2}$$

dan

$$a_2(p,w) = \lambda_2 g_2(p) + \lambda_2 \int_0^\infty dkk^2 g_2(k) G_N(w;k) t(k,p;w)$$

$$= \lambda_2 g_2(p) + \lambda_2 \int_0^\infty dkk^2 g_2(k) G_N(w;k)(g_2(k) a_2(p,w) + g_1(k) a_1(p,w))$$

$$= \lambda_2 g_2(p) + \lambda_2 \int_0^\infty dkk^2 g_2(k) G_N(w;k) g_2(k) a_2(p,w) + g_2(k) G_N(w;k) g_1(k) a_1(p,w)$$

$$= \lambda_2 g_2(p) + \lambda_2 \int_0^\infty dkk^2 g_2(k) G_N(w;k) g_2(k) a_2(p,w) + \lambda_2 \int_0^\infty dkk^2 g_2(k) G_N(w;k) g_1(k) a_1(p,w)$$

$$= \lambda_2 g_2(p) + \lambda_2 a_2(p;w) \int_0^\infty dkk^2 g_2(k) G_N(w;k) g_{21}(k) + \lambda_2 a_1(p;w) \int_0^\infty dkk^2 g_2(k) G_N(w;k) g_1(k)$$

$$= \frac{\lambda_2 g_2(p) + \lambda_2 a_1(p;w)\int_0^\infty dkk^2 g_2(k) G_N(w;k) g_1(k)}{1 - \lambda_2 \int_0^\infty dkk^2 g_2(k) G_N(w;k) g_2(k)}$$

$$a_2(p;w) = \frac{g_2(p) + a_1(p;w)\int_0^\infty dkk^2 g_2(k) G_N(w;k) g_1(k)}{\lambda_2^{-1} - \int_0^\infty dkk^2 g_2(k) G_N(w;k) g_2(k)} \tag{3.2.3}$$

Dengan memisalkan

$$X_{11}(w)=\int_0^\infty dkk^2 g_1(k)G_N(w,k)g_1(k),$$

$$X_{22}(w)=\int_0^\infty dkk^2 g_2(k)G_N(w,k)g_2(k)$$

$$X_{12}(w)=\int_0^\infty dkk^2 g_1(k)G_N(w,k)g_2(k)$$

$$X_{21}(w)=\int_0^\infty dkk^2 g_2(k)G_N(w,k)g_1(k)$$

$$X_{12}(w)=X_{21}(w)$$

Maka persamaan (3.2.2) dan (3.2.3) dapat ditulis menjadi

$$a_1(p;w)=\frac{g_1(p)+a_2(p;w)X_{12}(w)}{N_{22}(w)} \qquad (3.2.4)$$

$$a_2(p;w)=\frac{g_2(p)+a_1(p;w)X_{21}(w)}{N_{11}(w)}$$

$$=\frac{g_2(p)+a_1(p;w)X_{12}(w)}{N_{11}(w)} \qquad (3.2.5)$$

dengan memisalkan :

$$N_{11}(w)=\lambda_2^{-1}-X_{22}(w)$$
$$N_{22}(w)=\lambda_1^{-1}-X_{11}(w)$$
$$X_{12}=X_{21}=N_{12}$$

Maka diperoleh dari persamaan (3.2.4) dan (3.2.5) bahwa

$$a_1(p;w)N_{22}(w)=g_1(p)+a_2(p;w)N_{12}(w) \qquad (3.2.6)$$

$$\frac{a_2(p;w)N_{11}(w)-g_2(p)}{N_{12}(w)}=a_1(p;w) \qquad (3.2.7)$$

Dan dari persamaan (3.2.6) dan (3.2.7) akan didapatkan

$$
\begin{aligned}
&[a_2(p;w)N_{11}(w)-g_2(p)]N_{22}(w)=g_1(p)N_{12}(w)+a_2(p;w)N_{12}^2(w)\\
&a_2(p;w)N_{11}(w)N_{22}(w)-g_2(p)N_{22}(w)=g_1(p)N_{12}(w)+a_2(p;w)N_{12}^2(w)\\
&a_2(p;w)\left(N_{11}(w)N_{22}(w)-N_{12}^2(w)\right)=g_1(p)N_{12}(w)+g_2(p)N_{22}(w)\\
&a_2(p;w)=\frac{g_1(p)N_{12}(w)+g_2(p)N_{22}(w)}{N_{11}(w)N_{22}(w)-N_{12}^2(w)}
\end{aligned}
$$

Misalkan $D(w)=N_{11}(w)N_{22}(w)-N_{12}^2(w)$, sehingga diperoleh

$$
a_2(p;w)=\frac{g_1(p)N_{12}(w)+g_2(p)N_{22}(w)}{D(w)} \tag{3.2.8}
$$

Dengan memasukkan persamaan (3.2.8) ke (3.2.7), akan didapatkan

$$
\begin{aligned}
a_1(p;w)&=\frac{\left(\frac{g_1(p)N_{12}(w)}{D(w)}+\frac{g_2(p)N_{22}(w)}{D(w)}\right)N_{11}(w)-g_2(p)}{N_{12}(w)}\\
&=\frac{g_1(p)N_{12}(w)N_{11}(w)}{D(w)N_{12}(w)}+\frac{g_2(p)N_{22}(w)N_{11}(w)}{D(w)N_{12}(w)}-\frac{g_2(p)}{N_{12}(w)}\\
&=\frac{g_1(p)N_{11}(w)}{D(w)}+g_2(p)\left(\frac{N_{22}(w)N_{11}(w)}{D(w)N_{12}(w)}-\frac{1}{N_{12}(w)}\right)\\
&=\frac{g_1(p)N_{11}(w)}{D(w)}+g_2(p)\left(\frac{N_{22}(w)N_{11}(w)-D(w)}{D(w)N_{12}(w)}\right)\\
&=\frac{g_1(p)N_{11}(w)}{D(w)}+g_2(p)\left(\frac{N_{22}(w)N_{11}(w)-\left(N_{11}(w)N_{22}(w)-N_{12}^2(w)\right)}{D(w)N_{12}(w)}\right)\\
&=\frac{g_1(p)N_{11}(w)}{D(w)}+g_2(p)\left(\frac{N_{12}(w)}{D(w)}\right)
\end{aligned}
$$

sehingga

$$a_1(p;w) = \frac{g_1(p)N_{11}(w)}{D(w)} + \frac{g_2(p)N_{12}(w)}{D(w)} \tag{3.2.9}$$

Dengan memasukkan persamaan (3.2.8) dan (3.2.9) ke (3.2.1) maka diperoleh

$$t(q,p;w) = g_1(q)a_1(p;w) + g_2(q)a_2(p;w)$$

$$= g_1(q)\left(\frac{g_1(p)N_{11}(w)}{D(w)} + \frac{g_2(p)N_{12}(w)}{D(w)}\right) + g_2(q)\left(\frac{g_1(p)N_{12}(w)}{D(w)} + \frac{g_2(p)N_{22}(w)}{D(w)}\right)$$

$$= g_1(q)\frac{N_{11}(w)}{D(w)}g_1(p) + g_1(q)\frac{N_{12}(w)}{D(w)}g_2(p) + g_2(q)\frac{N_{12}(w)}{D(w)}g_1(p) + g_2(q)\frac{N_{22}(w)}{D(w)}g_2(p)$$

Sehingga pada (2.2.8)

$$t(q,p;w) = \sum_{i,j=1,2} g_i(q)\frac{N_{ij}(w)}{D(w)}g_j(p)$$

3.2.1. Pencarian nilai $X_{12}(w) = \int_0^\infty dkk^2 g_1(k)G_N(w,k)g_2(k)$

dimana

$$G_N(w;k) = \left[w + i\varepsilon - w_N(k)\right]^{-1}$$

$$w_N(k) = \sqrt{k^2 + m_K^{\ 2}} + \frac{k^2}{2m_N} + m_N.$$

$$g_n(k) = k^l \sum_{i=1}^{N} \frac{c_{n,i}}{k^2 + \beta_{n,i}^2}\text{; } l = \text{momentum angular} = 0 \text{ atau } 1$$

$$N = 1 \Rightarrow g_n(k) = \frac{k^l c_n}{k^2 + \beta_n^2}; c_{n,1} = c_n, \beta_{n,1} = \beta_n$$

Sehingga

$$X_{12}(w)=\int_0^\infty dkk^2\left(\frac{k^l c_1}{k^2+\beta_1^2}\right)\left(\frac{1}{w+i\varepsilon-\left(\sqrt{k^2+m_K^2}+\frac{k^2}{2m_N}+m_N\right)}\right)\left(\frac{k^l c_2}{k^2+\beta_2^2}\right)$$

$$=c_1c_2\int_0^\infty dk\frac{k^2k^lk^l}{\left(k^2+\beta_1^2\right)\left(k^2+\beta_2^2\right)\left(w+i\varepsilon-\sqrt{k^2+m_K^2}-\frac{k^2}{2m_N}-m_N\right)}$$

$$=c_1c_2\int_0^\infty dk\frac{k^{2+2l}}{\left(k^2+\beta_1^2\right)\left(k^2+\beta_2^2\right)\left(w+i\varepsilon-\sqrt{k^2+m_K^2}-\frac{k^2}{2m_N}-m_N\right)} \quad (3.2.1.1)$$

Akan diselesaikan dengan Teorema Residu :

$$\left(k^2+\beta_1^2\right)\left(k^2+\beta_2^2\right)\left(w+i\varepsilon-\sqrt{k^2+m_K^2}-\frac{k^2}{2m_N}-m_N\right)=0$$

(i) $k^2+\beta_1^2=0$

$\Rightarrow k^2=-\beta_1^2$

$k=\pm\sqrt{-\beta_1^2}$

$k_{1,2}=\pm i\beta_1^2$

(ii) $\left(w+i\varepsilon-\sqrt{k^2+m_K^2}-\frac{k^2}{2m_N}-m_N\right)=0$

Dengan program Mathematica didapat :

$$k_3=-\sqrt{2}\sqrt{m_Nw+m_Ni\varepsilon-\sqrt{m_K^2m_N^2-m_N^4+2m_N^3w+2m_N^3i\varepsilon}}$$

$$k_4=\sqrt{2}\sqrt{m_Nw+m_Ni\varepsilon-\sqrt{m_K^2m_N^2-m_N^4+2m_N^3w+2m_N^3i\varepsilon}}$$

$$k_5=-\sqrt{2}\sqrt{m_Nw+m_Ni\varepsilon+\sqrt{m_K^2m_N^2-m_N^4+2m_N^3w+2m_N^3i\varepsilon}}$$

$$k_6=\sqrt{2}\sqrt{m_Nw+m_Ni\varepsilon+\sqrt{m_K^2m_N^2-m_N^4+2m_N^3w+2m_N^3i\varepsilon}}$$

Dengan mengasumsikan $\varepsilon = 0$, maka akan diperoleh

$$k_3 = -\sqrt{2}\sqrt{m_N w - \sqrt{m_K^2 m_N^2 - m_N^4 + 2m_N^3 w}}$$
$$k_4 = \sqrt{2}\sqrt{m_N w - \sqrt{m_K^2 m_N^2 - m_N^4 + 2m_N^3 w}}$$
$$k_5 = -\sqrt{2}\sqrt{m_N w + \sqrt{m_K^2 m_N^2 - m_N^4 + 2m_N^3 w}}$$
$$k_6 = \sqrt{2}\sqrt{m_N w + \sqrt{m_K^2 m_N^2 - m_N^4 + 2m_N^3 w}}$$

(iii) $k^2 + \beta_2^2 = 0$

$$\Rightarrow k^2 = -\beta_2^2$$
$$k = \pm\sqrt{-\beta_2^2}$$
$$k_{7,8} = \pm i\beta_2^2$$

Sehingga didapat:

$$k_{1,2} = \pm i\beta_1$$
$$k_{7,8} = \pm i\beta_2$$
$$k_{3,4,5,6} = \mp\sqrt{2}\sqrt{m_N w \mp \sqrt{m_K^2 m_N^2 - m_N^4 + 2m_N^3 w}}$$

dimana :

$k_{1,2,3,4,5,6,7,8}$ adalah *simple pole*

Nilai untuk setiap *k* dicari dengan cara serupa pada potensial separabel rank-1.

3.2.1.1. PERHITUNGAN NILAI RESIDU UNTUK TIAP NILAI *k*

i. Pencarian dari $f(k_1)$ pada *simple pole* $k_1 = i\beta_1$ pada lampiran 7.

   Sehingga diperoleh residu dari $f(k_1)$ pada *simple pole* $k_1 = i\beta_1$ adalah

$$= \frac{(i\beta_1)^{2+2l}}{(2i\beta_1)(-\beta_1{}^2+\beta_2^2)\left(w-\sqrt{-\beta_1{}^2+m_K^2}-\frac{-\beta_1{}^2}{2m_N}-m_N\right)} \qquad (3.2.1.1.1.1)$$

ii. Pencarian dari $f(k_2)$ pada *simple pole* $k_2 = i\beta_2$ pada lampiran 8

Sehingga diperoleh residu dari $f(k_2)$ pada *simple pole* $k_2 = i\beta_2$ adalah

$$= \frac{(-i\beta_1)^{2+2l}}{(-2i\beta_1)(-\beta_1{}^2+\beta_2^2)\left(w-\sqrt{-\beta_1{}^2+m_K^2}+\frac{\beta_1{}^2}{2m_N}-m_N\right)} \qquad (3.2.1.1.1.2)$$

iii. Pencarian dari $f(k_3)$ pada *simple pole* $k_3 = -\sqrt{2}\sqrt{nw-\sqrt{m^2n^2-n^4+2n^3w}}$ pada lampiran 9

Sehingga diperoleh residu dari $f(k_3)$ pada *simple pole* $k_3 = -\sqrt{2}\sqrt{nw-\sqrt{m^2n^2-n^4+2n^3w}}$ adalah

$$\frac{n\left(\sqrt{2}\sqrt{nw-\sqrt{m^2n^2-n^4+2n^3w}}\right)^{1+2l}\sqrt{\left(\sqrt{2}\sqrt{nw-\sqrt{m^2n^2-n^4+2n^3w}}\right)^2+m^2}}{\left(\begin{array}{l}\left(\left(\sqrt{2}\sqrt{nw-\sqrt{m^2n^2-n^4+2n^3w}}\right)^2+\beta_1{}^2\right)\left(\left(\sqrt{2}\sqrt{nw-\sqrt{m^2n^2-n^4+2n^3w}}\right)^2+\beta_2{}^2\right)\\ \left(n+\sqrt{\left(\sqrt{2}\sqrt{nw-\sqrt{m^2n^2-n^4+2n^3w}}\right)^2+m^2}\right)\end{array}\right)}$$

(3.2.1.1.1.3)

iv. Pencarian dari $f(k_4)$ pada *simple pole* $k_4 = \sqrt{2}\sqrt{nw-\sqrt{m^2n^2-n^4+2n^3w}}$ pada lampiran 10

Sehingga diperoleh residu dari $f(k_4)$ pada *simple pole* $k_4 = \sqrt{2}\sqrt{nw-\sqrt{m^2n^2-n^4+2n^3w}}$ adalah

$$-\frac{n\left(\sqrt{2}\sqrt{nw-\sqrt{m^2n^2-n^4+2n^3w}}\right)^{1+2l}\sqrt{\left(\sqrt{2}\sqrt{nw-\sqrt{m^2n^2-n^4+2n^3w}}\right)^2+m^2}}{\left(\begin{array}{l}\left(\left(\sqrt{2}\sqrt{nw-\sqrt{m^2n^2-n^4+2n^3w}}\right)^2+\beta_1{}^2\right)\left(\left(\sqrt{2}\sqrt{nw-\sqrt{m^2n^2-n^4+2n^3w}}\right)^2+\beta_2{}^2\right)\\ \left(n+\sqrt{\left(\sqrt{2}\sqrt{nw-\sqrt{m^2n^2-n^4+2n^3w}}\right)^2+m^2}\right)\end{array}\right)}$$

(3.2.1.1.1.4)

v. Pencarian dari $f(k_5)$ pada *simple pole* $k_5 = -\sqrt{2}\sqrt{nw+\sqrt{m^2n^2-n^4+2n^3w}}$ pada lampiran 11.

Sehingga diperoleh residu dari $f(k_5)$ pada *simple pole* $k_5 = -\sqrt{2}\sqrt{nw+\sqrt{m^2n^2-n^4+2n^3w}}$ adalah

$$\frac{n\left(\sqrt{2}\sqrt{nw+\sqrt{m^2n^2-n^4+2n^3w}}\right)^{1+2l}\sqrt{\left(\sqrt{2}\sqrt{nw+\sqrt{m^2n^2-n^4+2n^3w}}\right)^2+m^2}}{\left(\begin{array}{l}\left(\left(\sqrt{2}\sqrt{nw+\sqrt{m^2n^2-n^4+2n^3w}}\right)^2+\beta_1{}^2\right)\left(\left(\sqrt{2}\sqrt{nw+\sqrt{m^2n^2-n^4+2n^3w}}\right)^2+\beta_2{}^2\right)\\ \left(n+\sqrt{\left(\sqrt{2}\sqrt{nw+\sqrt{m^2n^2-n^4+2n^3w}}\right)^2+m^2}\right)\end{array}\right)}$$

(3.2.1.1.1.5)

vi. Pencarian dari $f(k_6)$ pada *simple pole* $k_6 = \sqrt{2}\sqrt{nw+\sqrt{m^2n^2-n^4+2n^3w}}$ pada lampiran 12

Sehingga diperoleh residu dari $f(k_6)$ pada *simple pole* $k_6 = \sqrt{2}\sqrt{nw+\sqrt{m^2n^2-n^4+2n^3w}}$ adalah

$$-\frac{n\left(\sqrt{2}\sqrt{nw+\sqrt{m^2n^2-n^4+2n^3w}}\right)^{1+2l}\sqrt{\left(\sqrt{2}\sqrt{nw+\sqrt{m^2n^2-n^4+2n^3w}}\right)^2+m^2}}{\left(\left(\left(\sqrt{2}\sqrt{nw+\sqrt{m^2n^2-n^4+2n^3w}}\right)^2+\beta_1^2\right)\left(\left(\sqrt{2}\sqrt{nw+\sqrt{m^2n^2-n^4+2n^3w}}\right)^2+\beta_2^2\right)\left(n+\sqrt{\left(\sqrt{2}\sqrt{nw+\sqrt{m^2n^2-n^4+2n^3w}}\right)^2+m^2}\right)\right)}$$

(3.2.1.1.1.6)

vii. Pencarian dari $f(k_7)$ pada *simple pole* $k_7 = i\beta_2$ pada lampiran 13

Sehingga diperoleh residu dari $f(k_7)$ pada *simple pole* $k_7 = i\beta_2$ adalah

$$=\frac{(i\beta_2)^{2+2l}}{(2i\beta_2)(\beta_1{}^2-\beta_2^2)\left(w-\sqrt{-\beta_2{}^2+m_K^2}+\frac{\beta_2{}^2}{2m_N}-m_N\right)} \quad (3.2.1.1.1.7)$$

viii. Pencarian dari $f(k_8)$ pada *simple pole* $k_8 = -i\beta_2$ pada lampiran 14

Sehingga diperoleh residu dari $f(k_8)$ pada *simple pole* $k_8 = -i\beta_2$ adalah

$$=\frac{(-i\beta_1)^{2+2l}}{(-2i\beta_2)(\beta_1{}^2-\beta_2^2)\left(w-\sqrt{-\beta_2{}^2+m_K^2}+\frac{\beta_2{}^2}{2m_N}-m_N\right)} \quad (3.2.1.1.1.8)$$

3.2.2. Pencarian nilai $X_{21}(w)=\int_0^\infty dkk^2 g_2(k)G_N(w,k)g_1(k)$

$$X_{21}^l(w)=\int_0^\infty dkk^2 g_2(k)G_N(w,k)g_1(k)$$

$$=\int_0^\infty dkk^2 g_1(k)G_N(w,k)g_2(k)$$

$$=X_{12}^l(w)$$

3.2.3. Pencarian nilai $X_{11}(w)=\int_0^\infty dkk^2 g_1(k)G_N(w,k)g_1(k)$

Dengan asumsi yang sama dengan $X_{12}(w)$, yaitu

$$G_N(w;k)=\left[w+i\varepsilon-w_N(k)\right]^{-1}$$

$$w_N(k)=\sqrt{k^2+m_K{}^2}+\frac{k^2}{2m_N}+m_N.$$

$$g_n(k)=k^l\sum_{i=1}^{N}\frac{c_{n,i}}{k^2+\beta_{n,i}^2};\ l = \text{momentum angular} = 0 \text{ atau } 1$$

$$N=1\Rightarrow g_n(k)=\frac{k^l c_n}{k^2+\beta_n^2};c_{n,1}=c_n,\beta_{n,1}=\beta_n$$

maka didapatkan :

$$X_{11}^l(w)=\int_0^\infty dkk^2\left(\frac{k^l c_1}{k^2+\beta_1^2}\right)\left(\frac{1}{w+i\varepsilon-\left(\sqrt{k^2+m_K^2}+\frac{k^2}{2m_N}+m_N\right)}\right)\left(\frac{k^l c_1}{k^2+\beta_1^2}\right)$$

$$=\int_0^\infty dk\frac{k^{2+2l}c_1^2}{\left(k^2+\beta_1^2\right)^2\left(w+i\varepsilon-\sqrt{k^2+m_K^2}-\frac{k^2}{2m_N}-m_N\right)}$$

$$=c_1^2\int_0^\infty dk\frac{k^{2+2l}}{\left(k^2+\beta_1^2\right)^2\left(w+i\varepsilon-\sqrt{k^2+m_K^2}-\frac{k^2}{2m_N}-m_N\right)}$$

Akan diselesaikan dengan Teorema Residu :

$$\left(k^2+\beta_1^2\right)^2\left(w+i\varepsilon-\sqrt{k^2+m_K^2}-\frac{k^2}{2m_N}-m_N\right)=0$$

(i) $\left(k^2+\beta_1^2\right)^2=0$

$$\Rightarrow k^2+\beta_1^2=0$$
$$k^2=-\beta_1^2$$
$$k=\pm\sqrt{-\beta_1^2}$$
$$k_{1,2}=\pm i\beta_1^2$$

(ii) $\left(w+i\varepsilon-\sqrt{k^2+m_K^2}-\dfrac{k^2}{2m_N}-m_N\right)=0$

Dengan program Mathematica didapat :

$$k_3=-\sqrt{2}\sqrt{m_N w+m_N i\varepsilon-\sqrt{m_K^2 m_N^2-m_N^4+2m_N^3 w+2m_N^3 i\varepsilon}}$$
$$k_4=\sqrt{2}\sqrt{m_N w+m_N i\varepsilon-\sqrt{m_K^2 m_N^2-m_N^4+2m_N^3 w+2m_N^3 i\varepsilon}}$$
$$k_5=-\sqrt{2}\sqrt{m_N w+m_N i\varepsilon+\sqrt{m_K^2 m_N^2-m_N^4+2m_N^3 w+2m_N^3 i\varepsilon}}$$
$$k_6=\sqrt{2}\sqrt{m_N w+m_N i\varepsilon+\sqrt{m_K^2 m_N^2-m_N^4+2m_N^3 w+2m_N^3 i\varepsilon}}$$

Dengan mengasumsikan $\varepsilon=0$, maka akan diperoleh

$$k_3=-\sqrt{2}\sqrt{m_N w-\sqrt{m_K^2 m_N^2-m_N^4+2m_N^3 w}}$$
$$k_4=\sqrt{2}\sqrt{m_N w-\sqrt{m_K^2 m_N^2-m_N^4+2m_N^3 w}}$$
$$k_5=-\sqrt{2}\sqrt{m_N w+\sqrt{m_K^2 m_N^2-m_N^4+2m_N^3 w}}$$
$$k_6=\sqrt{2}\sqrt{m_N w+\sqrt{m_K^2 m_N^2-m_N^4+2m_N^3 w}}$$

Sehingga didapat:

$$k_{1,2}=\pm i\beta_1$$
$$k_{3,4,5,6}=\mp\sqrt{2}\sqrt{m_N w\mp\sqrt{m_K^2 m_N^2-m_N^4+2m_N^3 w}}$$

dimana :

$k_{1,2}$ adalah *pole second order*

$k_{3,4,5,6}$ adalah *simple pole*

Nilai untuk setiap *k* dicari dengan cara serupa pada potensial separabel rank-1.

3.2.3.1. PERHITUNGAN NILAI RESIDU UNTUK TIAP NILAI *k*

Dengan memisalkan $m_K = m$ dan $m_N = n$, maka :

i. Akan dicari residu dari $f(k_1)$ pada *double pole* $k_1 = i\beta_1$

Dengan mengganti $\beta \to \beta_1$ pada lampiran 1 diperoleh residu dari $f(k_1)$ pada *double pole* $k_1 = i\beta_1$ adalah

$$= -\frac{(2+2l)(i\beta_1)^{1+2l}}{4\beta_1^{\,2}\left(w-\sqrt{m^2-\beta_1^{\,2}}+\frac{\beta_1^2}{2n}-n\right)} + \frac{2(i\beta_1)^{2+2l}}{18\beta_1^{\,3}\left(w-\sqrt{m^2-\beta_1^{\,2}}+\frac{\beta_1^2}{2n}-n\right)} - \frac{\left(\frac{i\beta_1}{\sqrt{m^2-\beta_1^{\,2}}}+\frac{i\beta_1}{n}\right)(i\beta_1)^{2+2l}}{4\beta_1^{\,2}\left(w-\sqrt{m^2-\beta_1^{\,2}}+\frac{\beta_1^2}{2n}-n\right)^2} \qquad (3.2.3.1.2)$$

ii. Akan dicari residu dari $f(k_2)$ pada *double pole* $k_2 = -i\beta_1$

Dengan mengganti $\beta \to \beta_1$ pada lampiran 2 diperoleh residu dari $f(k_2)$ pada *double pole* $k_2 = -i\beta_1$ adalah

$$= -\frac{(2+2l)(-i\beta_1)^{1+2l}}{4\beta_1^{\,2}\left(w-\sqrt{m^2-\beta_1^{\,2}}-\frac{\beta_1^2}{2n}-n\right)}+\frac{2(-i\beta_1)^{2+2l}}{18\beta_1^{\,3}\left(w-\sqrt{m^2-\beta_1^{\,2}}-\frac{\beta_1^2}{2n}-n\right)}$$

$$-\frac{\left(\frac{i\beta_1}{\sqrt{m^2-\beta_1^{\,2}}}+\frac{i\beta_1}{n}\right)(-i\beta_1)^{2+2l}}{4\beta_1^{\,2}\left(w-\sqrt{m^2-\beta_1^{\,2}}-\frac{\beta_1^2}{2n}-n\right)^2}. \qquad (3.2.3.1.4)$$

iii. Akan dicari residu dari $f(k_3)$ pada *simple pole* $k_3=-\sqrt{2}\sqrt{nw-\sqrt{m^2n^2-n^4+2n^3w}}$

Dengan mengganti $\beta\to\beta_1$ pada lampiran 3 diperoleh residu dari $f(k_3)$ pada *simple pole* $k_3=-\sqrt{2}\sqrt{nw-\sqrt{m^2n^2-n^4+2n^3w}}$ adalah

$$\frac{n\left(\sqrt{2}\sqrt{nw-\sqrt{m^2n^2-n^4+2n^3w}}\right)^{1+2l}\sqrt{\left(\sqrt{2}\sqrt{nw-\sqrt{m^2n^2-n^4+2n^3w}}\right)^2+m^2}}{\left(\left(\sqrt{2}\sqrt{nw-\sqrt{m^2n^2-n^4+2n^3w}}\right)^2+\beta_1^{\,2}\right)^2\left(n+\sqrt{\left(\sqrt{2}\sqrt{nw-\sqrt{m^2n^2-n^4+2n^3w}}\right)^2+m^2}\right)}$$

(3.2.3.1.5)

iv. Akan dicari residu dari $f(k_4)$ pada *simple pole* $k_4=\sqrt{2}\sqrt{nw-\sqrt{m^2n^2-n^4+2n^3w}}$

Dengan mengganti $\beta\to\beta_1$ pada lampiran 4 diperoleh residu dari $f(k_4)$ pada *simple pole* $k_4=\sqrt{2}\sqrt{nw-\sqrt{m^2n^2-n^4+2n^3w}}$ adalah

$$-\frac{n\left(\sqrt{2}\sqrt{nw-\sqrt{m^2n^2-n^4+2n^3w}}\right)^{1+2l}\sqrt{\left(\sqrt{2}\sqrt{nw-\sqrt{m^2n^2-n^4+2n^3w}}\right)^2+m^2}}{\left(\left(\sqrt{2}\sqrt{nw-\sqrt{m^2n^2-n^4+2n^3w}}\right)^2+\beta_1^{\,2}\right)^2\left(n+\sqrt{\left(\sqrt{2}\sqrt{nw-\sqrt{m^2n^2-n^4+2n^3w}}\right)^2+m^2}\right)}$$

(3.2.3.1.6)

v. Akan dicari residu dari $f(k_5)$ pada *simple pole* $k_5 = -\sqrt{2}\sqrt{nw + \sqrt{m^2n^2 - n^4 + 2n^3w}}$

Dengan mengganti $\beta \to \beta_1$ pada lampiran 5 diperoleh residu dari $f(k_5)$ pada *simple pole* $k_5 = -\sqrt{2}\sqrt{nw + \sqrt{m^2n^2 - n^4 + 2n^3w}}$ adalah

$$\frac{n\left(\sqrt{2}\sqrt{nw+\sqrt{m^2n^2 - n^4 + 2n^3w}}\right)^{1+2l}\sqrt{\left(\sqrt{2}\sqrt{nw+\sqrt{m^2n^2 - n^4 + 2n^3w}}\right)^2 + m^2}}{\left(\left(\sqrt{2}\sqrt{nw+\sqrt{m^2n^2 - n^4 + 2n^3w}}\right)^2 + \beta_1^2\right)^2\left(n + \sqrt{\left(\sqrt{2}\sqrt{nw+\sqrt{m^2n^2 - n^4 + 2n^3w}}\right)^2 + m^2}\right)}$$

(3.2.3.1.7)

vi. Akan dicari residu dari $f(k_6)$ pada *simple pole* $k_6 = \sqrt{2}\sqrt{nw + \sqrt{m^2n^2 - n^4 + 2n^3w}}$

Dengan mengganti $\beta \to \beta_1$ pada lampiran 6 diperoleh residu dari $f(k_6)$ pada *simple pole* $k_6 = \sqrt{2}\sqrt{nw + \sqrt{m^2n^2 - n^4 + 2n^3w}}$ adalah

$$-\frac{n\left(\sqrt{2}\sqrt{nw+\sqrt{m^2n^2 - n^4 + 2n^3w}}\right)^{1+2l}\sqrt{\left(\sqrt{2}\sqrt{nw+\sqrt{m^2n^2 - n^4 + 2n^3w}}\right)^2 + m^2}}{\left(\left(\sqrt{2}\sqrt{nw+\sqrt{m^2n^2 - n^4 + 2n^3w}}\right)^2 + \beta_1^2\right)^2\left(n + \sqrt{\left(\sqrt{2}\sqrt{nw+\sqrt{m^2n^2 - n^4 + 2n^3w}}\right)^2 + m^2}\right)}$$

(3.2.3.1.8)

3.2.4. Pencarian nilai $X_{22}(w) = \int_0^\infty dkk^2 g_2(k) G_N(w,k) g_2(k)$

Dengan asumsi yang sama pada $X_{12}(w)$, yaitu

$$G_N(w;k) = \left[w + i\varepsilon - w_N(k)\right]^{-1}$$

$$w_N(k) = \sqrt{k^2 + m_K^{\ 2}} + \frac{k^2}{2m_N} + m_N.$$

$$g_n(k) = k^l \sum_{i=1}^{N} \frac{c_{n,i}}{k^2 + \beta_{n,i}^2}\ ;\ l = \text{momentum angular} = 0 \text{ atau } 1$$

$$N = 1 \Rightarrow g_n(k) = \frac{k^l c_n}{k^2 + \beta_n^2}; c_{n,1} = c_n, \beta_{n,1} = \beta_n$$

maka didapatkan :

$$X^l_{22}(w) = \int_0^\infty dk k^2 \left(\frac{k^l c_2}{k^2 + \beta_2^2}\right)\left(\frac{1}{w + i\varepsilon - \left(\sqrt{k^2 + m_K^2} + \frac{k^2}{2m_N} + m_N\right)}\right)\left(\frac{k^l c_2}{k^2 + \beta_2^2}\right)$$

$$= \int_0^\infty dk \frac{k^{2+2l} c_2^2}{\left(k^2 + \beta_2^2\right)^2 \left(w + i\varepsilon - \sqrt{k^2 + m_K^2} - \frac{k^2}{2m_N} - m_N\right)}$$

$$= c_2^2 \int_0^\infty dk \frac{k^{2+2l}}{\left(k^2 + \beta_2^2\right)^2 \left(w + i\varepsilon - \sqrt{k^2 + m_K^2} - \frac{k^2}{2m_N} - m_N\right)}$$

Akan diselesaikan dengan Teorema Residu :

$$\left(k^2 + \beta_2^2\right)^2 \left(w + i\varepsilon - \sqrt{k^2 + m_K^2} - \frac{k^2}{2m_N} - m_N\right) = 0$$

(i) $\left(k^2 + \beta_2^2\right)^2 = 0$

$$\Rightarrow k^2 + \beta_2^2 = 0$$
$$k^2 = -\beta_2^2$$
$$k = \pm\sqrt{-\beta_2^2}$$
$$k_{1,2} = \pm i\beta_2^2$$

(ii) $$\left(w + i\varepsilon - \sqrt{k^2 + m_K^2} - \frac{k^2}{2m_N} - m_N \right) = 0$$

Dengan program Mathematica didapat :

$$k_3 = -\sqrt{2}\sqrt{m_N w + m_N i\varepsilon - \sqrt{m_K^2 m_N^2 - m_N^4 + 2m_N^3 w + 2m_N^3 i\varepsilon}}$$
$$k_4 = \sqrt{2}\sqrt{m_N w + m_N i\varepsilon - \sqrt{m_K^2 m_N^2 - m_N^4 + 2m_N^3 w + 2m_N^3 i\varepsilon}}$$
$$k_5 = -\sqrt{2}\sqrt{m_N w + m_N i\varepsilon + \sqrt{m_K^2 m_N^2 - m_N^4 + 2m_N^3 w + 2m_N^3 i\varepsilon}}$$
$$k_6 = \sqrt{2}\sqrt{m_N w + m_N i\varepsilon + \sqrt{m_K^2 m_N^2 - m_N^4 + 2m_N^3 w + 2m_N^3 i\varepsilon}}$$

Dengan mengasumsikan $\varepsilon = 0$, maka akan diperoleh

$$k_3 = -\sqrt{2}\sqrt{m_N w - \sqrt{m_K^2 m_N^2 - m_N^4 + 2m_N^3 w}}$$
$$k_4 = \sqrt{2}\sqrt{m_N w - \sqrt{m_K^2 m_N^2 - m_N^4 + 2m_N^3 w}}$$
$$k_5 = -\sqrt{2}\sqrt{m_N w + \sqrt{m_K^2 m_N^2 - m_N^4 + 2m_N^3 w}}$$
$$k_6 = \sqrt{2}\sqrt{m_N w + \sqrt{m_K^2 m_N^2 - m_N^4 + 2m_N^3 w}}$$

Sehingga didapat:

$$k_{1,2} = \pm i\beta_2$$
$$k_{3,4,5,6} = \mp\sqrt{2}\sqrt{m_N w \mp \sqrt{m_K^2 m_N^2 - m_N^4 + 2m_N^3 w}}$$

dimana :

$k_{1,2}$ adalah *pole second order*

$k_{3,4,5,6}$ adalah *simple pole*

Nilai untuk setiap *k* dicari dengan cara serupa pada potensial separabel rank-1.

3.2.4.1. PERHITUNGAN NILAI RESIDU UNTUK TIAP NILAI *k*

Dengan memisalkan $m_K = m$ dan $m_N = n$, maka :

i. Akan dicari residu dari $f(k_1)$ pada *double pole* $k_1 = i\beta_2$

Dengan mengganti $\beta \rightarrow \beta_2$ pada lampiran 1 diperoleh residu dari $f(k_1)$ pada *double pole* $k_1 = i\beta_2$ adalah

$$= -\frac{(2+2l)(i\beta_2)^{1+2l}}{4\beta_2^2\left(w + i\beta_2 - \sqrt{m^2 - \beta_2^2} + \frac{\beta_2^2}{2n} - n\right)} + \frac{2(i\beta_2)^{2+2l}}{18\beta_2^3\left(w + i\beta_2 - \sqrt{m^2 - \beta_2^2} + \frac{\beta_2^2}{2n} - n\right)} - \frac{\left(\frac{i\beta_2}{\sqrt{m^2 - \beta_2^2}} + \frac{i\beta_2}{n}\right)(i\beta_2)^{2+2l}}{4\beta_2^2\left(w + i\beta_2 - \sqrt{m^2 - \beta_2^2} + \frac{\beta_2^2}{2n} - n\right)^2}. \quad (3.2.4.2)$$

ii. Akan dicari residu dari $f(k_2)$ pada *double pole* $k_2 = -i\beta_2$

Dengan mengganti $\beta \rightarrow \beta_2$ pada lampiran 2 diperoleh residu dari $f(k_2)$ pada *double pole* $k_2 = -i\beta_2$ adalah

$$= -\frac{(2+2l)(-i\beta_2)^{1+2l}}{4\beta_2{}^2\left(w-\sqrt{m^2-\beta_2{}^2}-\frac{\beta_2{}^2}{2n}-n\right)}+\frac{2(-i\beta_2)^{2+2l}}{18\beta_2{}^3\left(w-\sqrt{m^2-\beta_2{}^2}-\frac{\beta_2{}^2}{2n}-n\right)}$$

$$-\frac{\left(\frac{i\beta_2}{\sqrt{m^2-\beta_2{}^2}}+\frac{i\beta_2}{n}\right)(-i\beta_2)^{2+2l}}{4\beta_2{}^2\left(w-\sqrt{m^2-\beta_2{}^2}-\frac{\beta_2{}^2}{2n}-n\right)^2}. \quad (3.2.4.4)$$

iii. Akan dicari residu dari $f(k_3)$ pada *simple pole* $k_3 = -\sqrt{2}\sqrt{nw-\sqrt{m^2n^2-n^4+2n^3w}}$

Dengan mengganti $\beta \rightarrow \beta_2$ pada lampiran 3 diperoleh residu dari $f(k_3)$ pada *simple pole* $k_3 = -\sqrt{2}\sqrt{nw-\sqrt{m^2n^2-n^4+2n^3w}}$ adalah

$$\frac{n\left(\sqrt{2}\sqrt{nw-\sqrt{m^2n^2-n^4+2n^3w}}\right)^{1+2l}\sqrt{\left(\sqrt{2}\sqrt{nw-\sqrt{m^2n^2-n^4+2n^3w}}\right)^2+m^2}}{\left(\left(\sqrt{2}\sqrt{nw-\sqrt{m^2n^2-n^4+2n^3w}}\right)^2+\beta_2{}^2\right)^2\left(n+\sqrt{\left(\sqrt{2}\sqrt{nw-\sqrt{m^2n^2-n^4+2n^3w}}\right)^2+m^2}\right)}$$

(3.2.4.5)

iv. Akan dicari residu dari $f(k_4)$ pada *simple pole* $k_4 = \sqrt{2}\sqrt{nw-\sqrt{m^2n^2-n^4+2n^3w}}$

Dengan mengganti $\beta \rightarrow \beta_2$ pada lampiran 4 diperoleh residu dari $f(k_4)$ pada *simple pole* $k_4 = \sqrt{2}\sqrt{nw-\sqrt{m^2n^2-n^4+2n^3w}}$ adalah

$$-\frac{n\left(\sqrt{2}\sqrt{nw-\sqrt{m^2n^2-n^4+2n^3w}}\right)^{1+2l}\sqrt{\left(\sqrt{2}\sqrt{nw-\sqrt{m^2n^2-n^4+2n^3w}}\right)^2+m^2}}{\left(\left(\sqrt{2}\sqrt{nw-\sqrt{m^2n^2-n^4+2n^3w}}\right)^2+\beta_2{}^2\right)^2\left(n+\sqrt{\left(\sqrt{2}\sqrt{nw-\sqrt{m^2n^2-n^4+2n^3w}}\right)^2+m^2}\right)}$$

(3.2.4.6)

v. Akan dicari residu dari $f(k_5)$ pada *simple pole* $k_5 = -\sqrt{2}\sqrt{nw + \sqrt{m^2 n^2 - n^4 + 2n^3 w}}$

Dengan mengganti $\beta \to \beta_2$ pada lampiran 5 diperoleh residu dari $f(k_5)$ pada *simple pole* $k_5 = -\sqrt{2}\sqrt{nw + \sqrt{m^2 n^2 - n^4 + 2n^3 w}}$ adalah

$$\frac{n\left(\sqrt{2}\sqrt{nw+\sqrt{m^2n^2-n^4+2n^3w}}\right)^{1+2l}\sqrt{\left(\sqrt{2}\sqrt{nw+\sqrt{m^2n^2-n^4+2n^3w}}\right)^2+m^2}}{\left(\left(\sqrt{2}\sqrt{nw+\sqrt{m^2n^2-n^4+2n^3w}}\right)^2+\beta_2{}^2\right)^2\left(n+\sqrt{\left(\sqrt{2}\sqrt{nw+\sqrt{m^2n^2-n^4+2n^3w}}\right)^2+m^2}\right)}$$

(3.2.4.7)

vi. Akan dicari residu dari $f(k_6)$ pada *simple pole* $k_6 = \sqrt{2}\sqrt{nw + \sqrt{m^2 n^2 - n^4 + 2n^3 w}}$

Dengan mengganti $\beta \to \beta_2$ pada lampiran 6 diperoleh residu dari $f(k_6)$ pada *simple pole* $k_6 = \sqrt{2}\sqrt{nw + \sqrt{m^2 n^2 - n^4 + 2n^3 w}}$ adalah

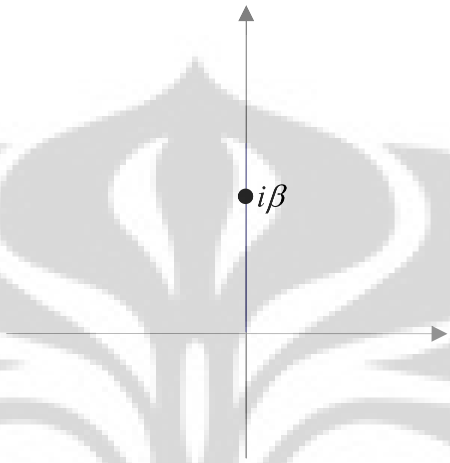
$$-\frac{n\left(\sqrt{2}\sqrt{nw+\sqrt{m^2n^2-n^4+2n^3w}}\right)^{1+2l}\sqrt{\left(\sqrt{2}\sqrt{nw+\sqrt{m^2n^2-n^4+2n^3w}}\right)^2+m^2}}{\left(\left(\sqrt{2}\sqrt{nw+\sqrt{m^2n^2-n^4+2n^3w}}\right)^2+\beta_2{}^2\right)^2\left(n+\sqrt{\left(\sqrt{2}\sqrt{nw+\sqrt{m^2n^2-n^4+2n^3w}}\right)^2+m^2}\right)}$$

(3.2.4.8)

4. HASIL PERHITUNGAN NUMERIK

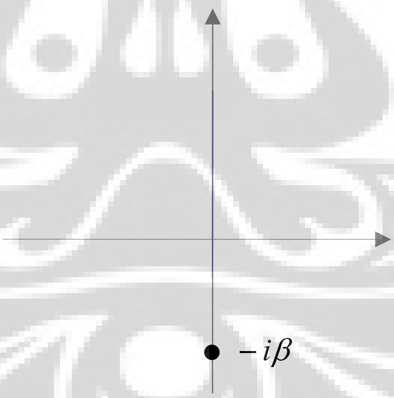
4.1. Penggambaran Pole Residu Pada Potensial Separabel Rank-1

4.1.1. Pole Residu $k_1 = i\beta$



Gambar 4.1.1 Pole residu di k_1 untuk rank-1

4.1.2. Pole Residu $k_2 = -i\beta$



Gambar 4.1.2 Pole residu di k_2 untuk rank-1

4.1.3. Pole Residu $k_3 = -\sqrt{2}\sqrt{m_N w - \sqrt{m_K^2 m_N^2 - m_N^4 + 2m_N^3 w}}$

Misalkan :

$$\left.\begin{aligned} m_N &= n \\ m_K &= m \end{aligned}\right\} \rightarrow k_3 = -\sqrt{2}\sqrt{nw - \sqrt{m^2 n^2 - n^4 + 2n^3 w}}$$

Misalkan pula :

$$\left.\begin{array}{l} a = nw \\ b = m^2 n^2 - n^4 + 2n^3 w \end{array}\right\} \to k_3 = -\sqrt{2}\sqrt{a - \sqrt{b}}$$

Maka ada dua kemungkinan, yaitu :

(i) $b > 0 \to B = \sqrt{b}, B = \mathrm{Re}(z_1)$

Sehingga,

$$(b > 0, a > B) \to A = \sqrt{a - B}, A = \mathrm{Re}(z) \quad (4.1.3.1)$$

$$(b > 0, a < B) \to A = i\sqrt{B - a}, A = \mathrm{Im}(z) \quad (4.1.3.2)$$

(ii) $b < 0 \to B = i\sqrt{-b}, B = \mathrm{Im}(z_1)$

Sehingga,

$$A = \sqrt{a - iB}\ , B = \sqrt{-b}$$

$$A = a^{\frac{1}{2}}\left(\left(1 + \frac{B^2}{8a^2}\right) - i\left(\frac{B}{2a} - \frac{B^3}{16a^3}\right)\right) \quad (4.1.3.3.a)$$

$$A = \left(a^{\frac{1}{2}} + \frac{1}{8}a^{-\frac{3}{2}}B^2\right) - i\left(\frac{1}{2}a^{-\frac{1}{2}}B - \frac{1}{16}a^{-\frac{5}{2}}B^3\right) \quad (4.1.3.3)$$

$$\therefore k_3 = -\sqrt{2}A$$

Bukti persamaan (4.1.3.3.a)

Akan dihitung $A = \sqrt{a - iB} = (a - iB)^{\frac{1}{2}}$

Rumus Binomial untuk $p \in Z^+ : (1 + x)^p = 1 + \binom{p}{1}x + \binom{p}{2}x^2 + \cdots + \binom{p}{p}x^p$

$$(a + b)^p = a^p + \binom{p}{1}a^{p-1}b + \binom{p}{2}a^{p-2}b^2 + \cdots + \binom{p}{p}b^p$$

Deret Binomial untuk $p \in R, |x| < 1 : (1 + x)^p = 1 + \binom{p}{1}x + \binom{p}{2}x^2 + \cdots$

Dimana $\binom{p}{k} = \dfrac{p(p-1)(p-2)\cdots(p-k+1)}{k!}$

Maka nilai A akan dicari dengan deret Binomial (ketelitian sampai orde ke-4)

$$A_1 = (a+iB)^{\frac{1}{2}}$$

$$= \left(a\left(1+i\frac{B}{a}\right)\right)^{\frac{1}{2}}$$

$$= a^{\frac{1}{2}}\left(1+i\frac{B}{a}\right)^{\frac{1}{2}}$$

$$= a^{\frac{1}{2}}\left(1+(i)\frac{1}{2}\frac{B}{a}+(-1)\frac{\left(\frac{1}{2}\right)\left(-\frac{1}{2}\right)}{2!}\frac{B^2}{a^2}+(-i)\frac{\left(\frac{1}{2}\right)\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)}{3!}\frac{B^3}{a^3}+\cdots\right)$$

$$= a^{\frac{1}{2}}\left(1+i\frac{B}{2a}+\frac{B^2}{8a^2}-i\frac{B^3}{16a^3}\right)$$

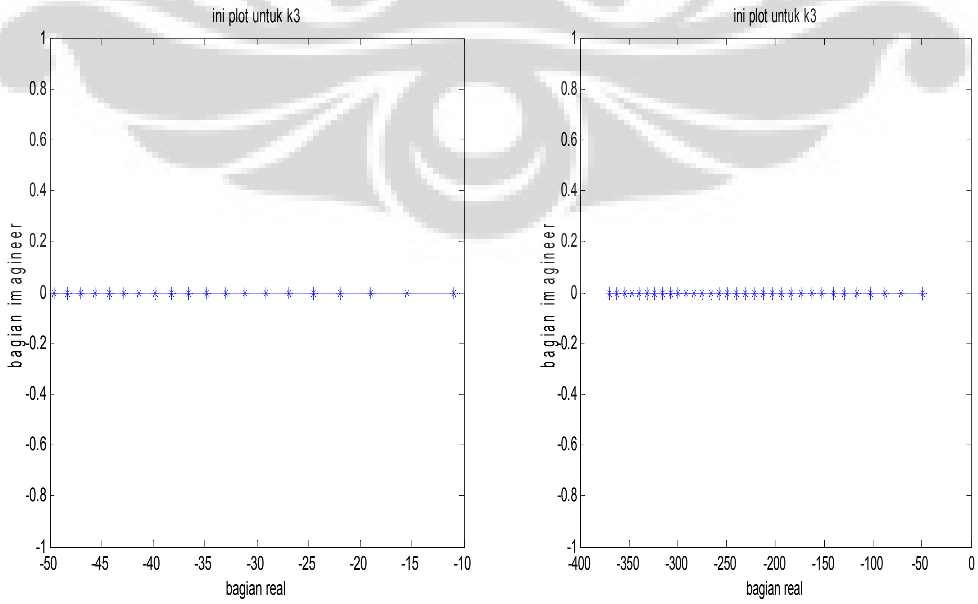
$$= a^{\frac{1}{2}}\left(\left(1+\frac{B^2}{8a^2}\right)+i\left(\frac{B}{2a}-\frac{B^3}{16a^3}\right)\right)$$

$$A = (a+i(-B))^{\frac{1}{2}} = (a-iB)^{\frac{1}{2}}$$

$$= a^{\frac{1}{2}}\left(\left(1+\frac{(-B)^2}{8a^2}\right)+i\left(\frac{(-B)}{2a}-\frac{(-B)^3}{16a^3}\right)\right)$$

$$= a^{\frac{1}{2}}\left(\left(1+\frac{B^2}{8a^2}\right)-i\left(\frac{B}{2a}-\frac{B^3}{16a^3}\right)\right)$$

Terbuktilah persamaan (4.1.3.3.a).



Gambar 4.1.3.a Pole residu di k_3 untuk $1076{,}27 \le w \le 1086$ jarak antar w adl 0,5

Gambar 4.1.3.b Pole residu di k_3 untuk $1076{,}27 \le w \le 1400$ jarak antar w adl 10

4.1.4. Pole Residu $k_4 = \sqrt{2}\sqrt{m_N w - \sqrt{m_K^2 m_N^2 - m_N^4 + 2m_N^3 w}}$

Misalkan :

$$\left.\begin{matrix} m_N = n \\ m_K = m \end{matrix}\right\} \rightarrow k_4 = \sqrt{2}\sqrt{nw - \sqrt{m^2 n^2 - n^4 + 2n^3 w}}$$

Misalkan pula :

$$\left.\begin{matrix} a = nw \\ b = m^2 n^2 - n^4 + 2n^3 w \end{matrix}\right\} \rightarrow k_4 = \sqrt{2}\sqrt{a - \sqrt{b}}$$

Maka ada dua kemungkinan, yaitu :

(i) $b > 0 \rightarrow B = \sqrt{b}, B = \mathrm{Re}(z_1)$

Sehingga,

$$(b > 0, a > B) \rightarrow A = \sqrt{a - B}, A = \mathrm{Re}(z) \qquad (4.1.4.1)$$

$$(b > 0, a < B) \rightarrow A = i\sqrt{B - a}, A = \mathrm{Im}(z) \qquad (4.1.4.2)$$

(ii) $b < 0 \rightarrow B = i\sqrt{-b}, B = \mathrm{Im}(z_1)$

Sehingga,

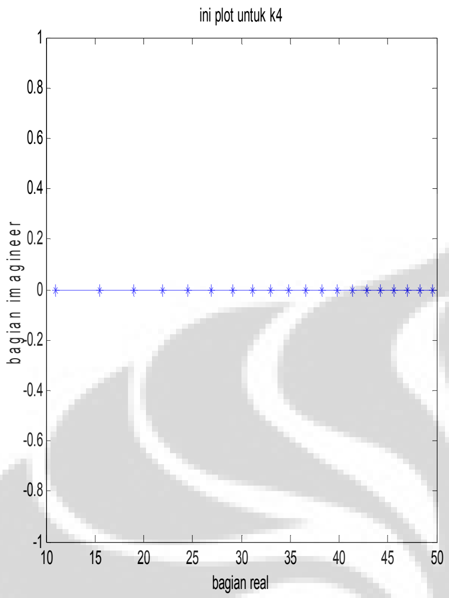
$$A = \sqrt{a - in}\ , n = \sqrt{-b}$$

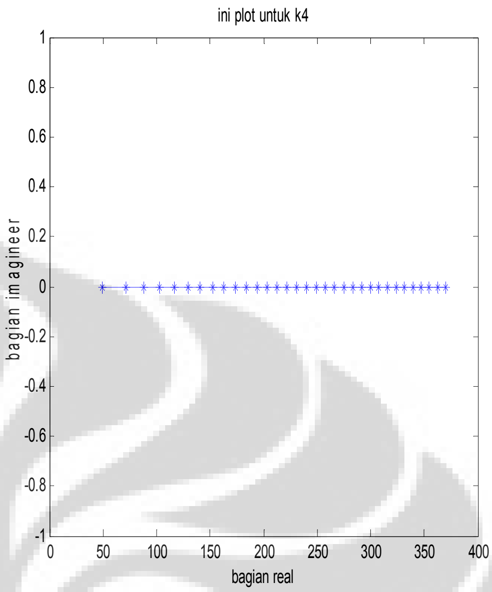
$$A = a^{\frac{1}{2}}\left(\left(1 + \frac{n^2}{8a^2}\right) - i\left(\frac{n}{2a} - \frac{n^3}{16a^3}\right)\right) \qquad (4.1.4.3.a)$$

$$A = \left(a^{\frac{1}{2}} + \frac{1}{8}a^{-\frac{3}{2}}n^2\right) - i\left(\frac{1}{2}a^{-\frac{1}{2}}n - \frac{1}{16}a^{-\frac{5}{2}}n^3\right) \qquad (4.1.4.3)$$

$$\therefore k_4 = \sqrt{2}A$$

Bukti persamaan (4.1.4.3.a) adalah serupa dengan bukti persamaan (4.1.3.3.a)





Gambar 4.1.4.a Pole residu di k_4 untuk $1076{,}27 \le w \le 1086$ jarak antar w adl 0,5

Gambar 4.1.4.b Pole residu di k_4 untuk $1076{,}27 \le w \le 1400$ jarak antar w adl 10

4.1.5. Pole Residu $k_5 = -\sqrt{2}\sqrt{m_N w + \sqrt{m_K^2 m_N^2 - m_N^4 + 2m_N^3 w}}$

Misalkan :

$$\left.\begin{aligned} m_N &= n \\ m_K &= m \end{aligned}\right\} \rightarrow k_5 = -\sqrt{2}\sqrt{nw + \sqrt{m^2 n^2 - n^4 + 2n^3 w}}$$

Misalkan pula :

$$\left.\begin{aligned} a &= nw \\ b &= m^2 n^2 - n^4 + 2n^3 w \end{aligned}\right\} \rightarrow k_5 = -\sqrt{2}\sqrt{a + \sqrt{b}}$$

Maka ada dua kemungkinan, yaitu :

(i) $b > 0 \rightarrow B = \sqrt{b}, B = \mathrm{Re}(z_1) \rightarrow A = \sqrt{a + B}$ (4.1.5.1)

(ii) $b < 0 \rightarrow B = i\sqrt{-b}, B = \mathrm{Im}(z_1)$

Sehingga,

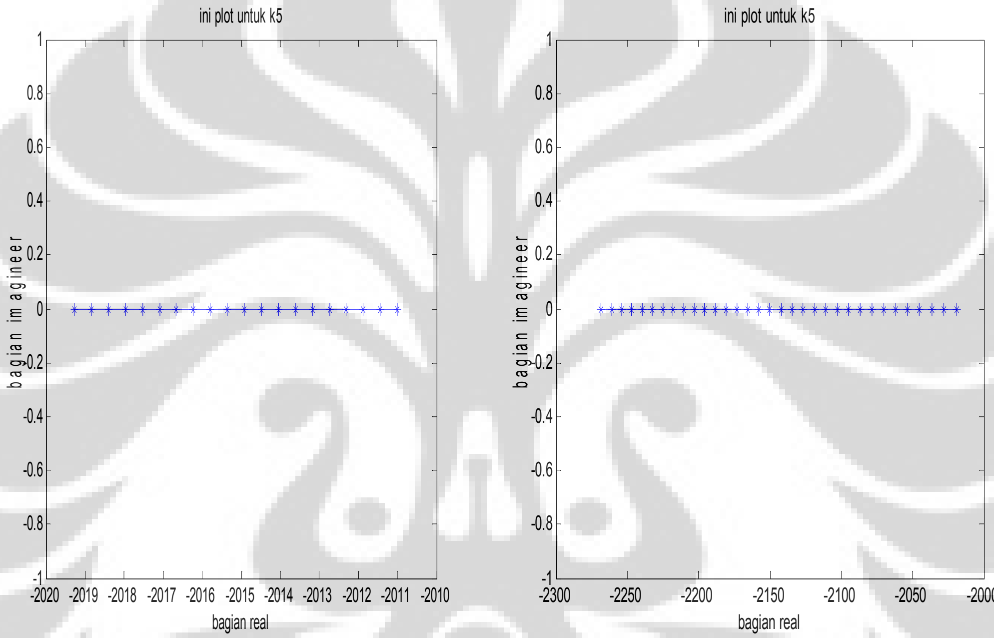
$A = \sqrt{a + in}\ , n = \sqrt{-b}$

$$A = a^{\frac{1}{2}}\left(\left(1+\frac{n^2}{8a^2}\right)-i\left(\frac{n}{2a}-\frac{n^3}{16a^3}\right)\right) \qquad (4.1.5.2.a)$$

$$A = \left(a^{\frac{1}{2}}+\frac{1}{8}a^{-\frac{3}{2}}n^2\right)+i\left(\frac{1}{2}a^{-\frac{1}{2}}n-\frac{1}{16}a^{-\frac{5}{2}}n^3\right) \qquad (4.1.5.2)$$

$$\therefore k_5 = -\sqrt{2}A$$

Bukti persamaan (4.1.4.5.a) adalah serupa dengan bukti persamaan (4.1.3.3.a)



Gambar 4.1.5.a Pole residu di k_5 untuk $1076{,}27 \le w \le 1086$ jarak antar w adl 0,5

Gambar 4.1.5.b Pole residu di k_5 untuk $1076{,}27 \le w \le 1400$ jarak antar w adl 10

4.1.6. Pole Residu $k_6 = \sqrt{2}\sqrt{m_N w + \sqrt{m_K^2 m_N^2 - m_N^4 + 2m_N^3 w}}$

Misalkan :

$$\left.\begin{aligned} m_N &= n \\ m_K &= m \end{aligned}\right\} \rightarrow k_6 = \sqrt{2}\sqrt{nw + \sqrt{m^2 n^2 - n^4 + 2n^3 w}}$$

Misalkan pula :

$$\left.\begin{aligned} a &= nw \\ b &= m^2 n^2 - n^4 + 2n^3 w \end{aligned}\right\} \rightarrow k_6 = \sqrt{2}\sqrt{a+\sqrt{b}}$$

Maka ada dua kemungkinan, yaitu :

(i) $b > 0 \rightarrow B = \sqrt{b}, B = \mathrm{Re}(z_1) \rightarrow A = \sqrt{a + B}$ (4.1.5.1)

(ii) $b < 0 \rightarrow B = i\sqrt{-b}, B = \mathrm{Im}(z_1)$

Sehingga,

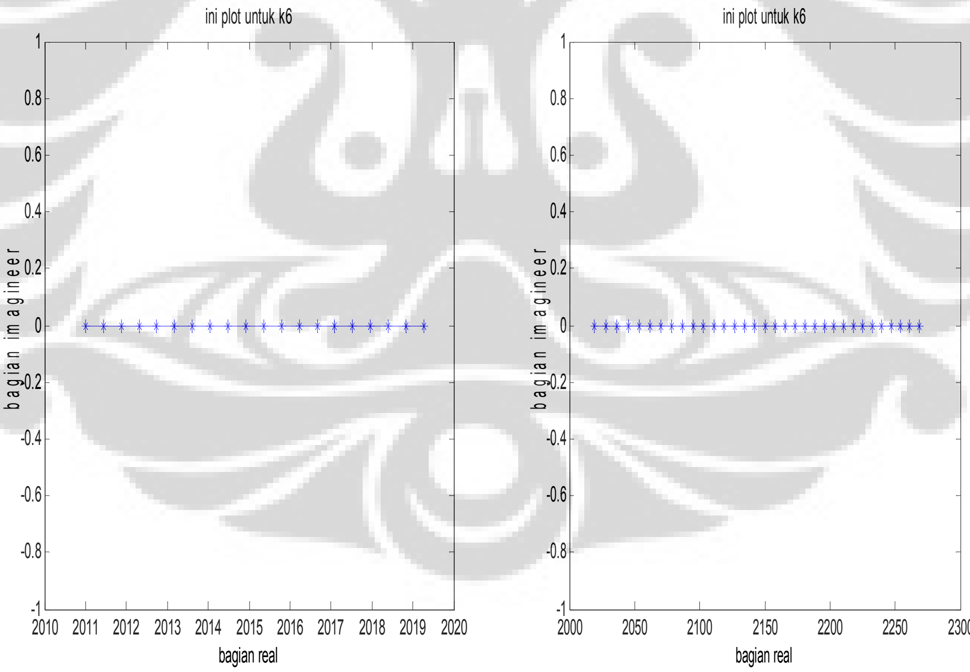
$A = \sqrt{a + in}\ , n = \sqrt{-b}$

$$A = a^{\frac{1}{2}}\left(\left(1 + \frac{n^2}{8a^2}\right) - i\left(\frac{n}{2a} - \frac{n^3}{16a^3}\right)\right) \qquad (4.1.6.2.a)$$

$$A = \left(a^{\frac{1}{2}} + \frac{1}{8}a^{-\frac{3}{2}}n^2\right) + i\left(\frac{1}{2}a^{-\frac{1}{2}}n - \frac{1}{16}a^{-\frac{5}{2}}n^3\right) \qquad (4.1.6.2)$$

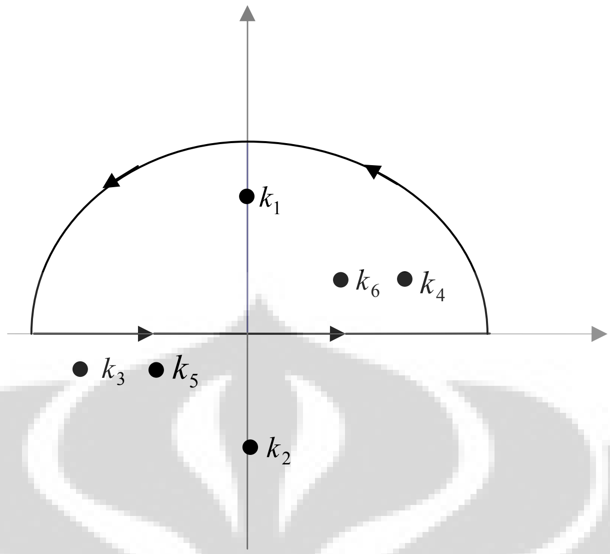
$\therefore k_6 = \sqrt{2}A$

Bukti persamaan (4.1.4.6.a) adalah serupa dengan bukti persamaan (4.1.3.3.a)



Gambar 4.1.6.a Pole residu di k_6 untuk $1076{,}27 \le w \le 1086$ jarak antar w adl 0,5

Gambar 4.1.6.b Pole residu di k_6 untuk $1076{,}27 \le w \le 1400$ jarak antar w adl 10



Gambar 4.1.7 Pole residu di $k_1, k_2, k_3, k_4, k_5, k_6$

Dari hasil penggambaran plot titik residu, maka dari (3.1.1.2), (3.1.1.4) hingga (3.1.1.9) dan menerapkan teorema Residu pada (3.1.5) kita dapatkan

$$A_l = 2c^2 \pi i \sum_{i=1}^{6} f(k_i)$$

$$A_l = 2c^2 \pi i \left(f(k_1) + f(k_4) + f(k_6) \right)$$

$$=2c^2\pi i\left(\left(-\frac{(2+2l)(i\beta)^{1+2l}}{4\beta^2\left(w-\sqrt{m^2-\beta^2}+\frac{\beta^2}{2n}-n\right)}+\frac{2(i\beta)^{2+2l}}{18\beta^3\left(w-\sqrt{m^2-\beta^2}+\frac{\beta^2}{2n}-n\right)}-\frac{\left(\frac{i\beta}{\sqrt{m^2-\beta^2}}+\frac{i\beta}{n}\right)(i\beta)^{2+2l}}{4\beta^2\left(w-\sqrt{m^2-\beta^2}+\frac{\beta^2}{2n}-n\right)^2}\right)-\frac{n\left(\sqrt{2}\sqrt{nw-\sqrt{m^2n^2-n^4+2n^3w}}\right)^{1+2l}\sqrt{\left(\sqrt{2}\sqrt{nw-\sqrt{m^2n^2-n^4+2n^3w}}\right)^2+m^2}}{\left(\left(\sqrt{2}\sqrt{nw-\sqrt{m^2n^2-n^4+2n^3w}}\right)^2+\beta^2\right)^2\left(n+\sqrt{\left(\sqrt{2}\sqrt{nw-\sqrt{m^2n^2-n^4+2n^3w}}\right)^2+m^2}\right)}-\frac{n\left(\sqrt{2}\sqrt{nw+\sqrt{m^2n^2-n^4+2n^3w}}\right)^{1+2l}\sqrt{\left(\sqrt{2}\sqrt{nw+\sqrt{m^2n^2-n^4+2n^3w}}\right)^2+m^2}}{\left(\left(\sqrt{2}\sqrt{nw+\sqrt{m^2n^2-n^4+2n^3w}}\right)^2+\beta^2\right)^2\left(n+\sqrt{\left(\sqrt{2}\sqrt{nw+\sqrt{m^2n^2-n^4+2n^3w}}\right)^2+m^2}\right)}\right)$$

$$=2c^2\pi i\left(\left(-\frac{(2+2l)(i\beta)^{1+2l}}{4\beta^2\left(w-\sqrt{m^2-\beta^2}+\frac{\beta^2}{2n}-n\right)}+\frac{2(i\beta)^{2+2l}}{18\beta^3\left(w-\sqrt{m^2-\beta^2}+\frac{\beta^2}{2n}-n\right)}-\frac{\left(\frac{1}{\sqrt{m^2-\beta^2}}+\frac{1}{n}\right)(i\beta)^{3+2l}}{4\beta^2\left(w-\sqrt{m^2-\beta^2}+\frac{\beta^2}{2n}-n\right)^2}\right)-\frac{n\left(\sqrt{2}\sqrt{nw-\sqrt{m^2n^2-n^4+2n^3w}}\right)^{1+2l}\sqrt{\left(\sqrt{2}\sqrt{nw-\sqrt{m^2n^2-n^4+2n^3w}}\right)^2+m^2}}{\left(\left(\sqrt{2}\sqrt{nw-\sqrt{m^2n^2-n^4+2n^3w}}\right)^2+\beta^2\right)^2\left(n+\sqrt{\left(\sqrt{2}\sqrt{nw-\sqrt{m^2n^2-n^4+2n^3w}}\right)^2+m^2}\right)}-\frac{n\left(\sqrt{2}\sqrt{nw+\sqrt{m^2n^2-n^4+2n^3w}}\right)^{1+2l}\sqrt{\left(\sqrt{2}\sqrt{nw+\sqrt{m^2n^2-n^4+2n^3w}}\right)^2+m^2}}{\left(\left(\sqrt{2}\sqrt{nw+\sqrt{m^2n^2-n^4+2n^3w}}\right)^2+\beta^2\right)^2\left(n+\sqrt{\left(\sqrt{2}\sqrt{nw+\sqrt{m^2n^2-n^4+2n^3w}}\right)^2+m^2}\right)}\right)$$

(4.1.6.3)

Dengan menggantikan $l = 0$ maka kita peroleh

$$A_0 = 2c^2\pi i\left(\left(-\frac{2(i\beta)}{4\beta^2\left(w-\sqrt{m^2-\beta^2}+\frac{\beta^2}{2n}-n\right)}+\frac{2(i\beta)^2}{18\beta^3\left(w-\sqrt{m^2-\beta^2}+\frac{\beta^2}{2n}-n\right)}-\frac{\left(\frac{1}{\sqrt{m^2-\beta^2}}+\frac{1}{n}\right)(i\beta)^3}{4\beta^2\left(w-\sqrt{m^2-\beta^2}+\frac{\beta^2}{2n}-n\right)^2}\right)-\frac{n\left(\sqrt{2}\sqrt{nw-\sqrt{m^2n^2-n^4+2n^3w}}\right)\sqrt{\left(\sqrt{2}\sqrt{nw-\sqrt{m^2n^2-n^4+2n^3w}}\right)^2+m^2}}{\left(\left(\sqrt{2}\sqrt{nw-\sqrt{m^2n^2-n^4+2n^3w}}\right)^2+\beta^2\right)^2\left(n+\sqrt{\left(\sqrt{2}\sqrt{nw-\sqrt{m^2n^2-n^4+2n^3w}}\right)^2+m^2}\right)}-\frac{n\left(\sqrt{2}\sqrt{nw+\sqrt{m^2n^2-n^4+2n^3w}}\right)\sqrt{\left(\sqrt{2}\sqrt{nw+\sqrt{m^2n^2-n^4+2n^3w}}\right)^2+m^2}}{\left(\left(\sqrt{2}\sqrt{nw+\sqrt{m^2n^2-n^4+2n^3w}}\right)^2+\beta^2\right)^2\left(n+\sqrt{\left(\sqrt{2}\sqrt{nw+\sqrt{m^2n^2-n^4+2n^3w}}\right)^2+m^2}\right)}\right)$$

$$A_0 = 2c^2\pi i\left(\left(\frac{-9i-2}{18\beta\left(w-\sqrt{m^2-\beta^2}+\frac{\beta^2}{2n}-n\right)}+\frac{\left(\frac{1}{\sqrt{m^2-\beta^2}}+\frac{1}{n}\right)i\beta}{4\left(w-\sqrt{m^2-\beta^2}+\frac{\beta^2}{2n}-n\right)^2}\right)-\frac{n\left(\sqrt{2}\sqrt{nw-\sqrt{m^2n^2-n^4+2n^3w}}\right)\sqrt{\left(\sqrt{2}\sqrt{nw-\sqrt{m^2n^2-n^4+2n^3w}}\right)^2+m^2}}{\left(\left(\sqrt{2}\sqrt{nw-\sqrt{m^2n^2-n^4+2n^3w}}\right)^2+\beta^2\right)^2\left(n+\sqrt{\left(\sqrt{2}\sqrt{nw-\sqrt{m^2n^2-n^4+2n^3w}}\right)^2+m^2}\right)}-\frac{n\left(\sqrt{2}\sqrt{nw+\sqrt{m^2n^2-n^4+2n^3w}}\right)\sqrt{\left(\sqrt{2}\sqrt{nw+\sqrt{m^2n^2-n^4+2n^3w}}\right)^2+m^2}}{\left(\left(\sqrt{2}\sqrt{nw+\sqrt{m^2n^2-n^4+2n^3w}}\right)^2+\beta^2\right)^2\left(n+\sqrt{\left(\sqrt{2}\sqrt{nw+\sqrt{m^2n^2-n^4+2n^3w}}\right)^2+m^2}\right)}\right)$$

Sehingga,

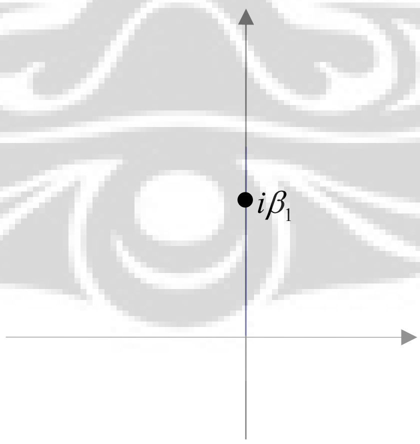
$$A_0 = 2c^2\pi i\left(\left(\frac{-2(9i+2)\left(w-\sqrt{m^2-\beta^2}+\frac{\beta^2}{2n}-n\right)+9\left(\frac{1}{\sqrt{m^2-\beta^2}}+\frac{1}{n}\right)i\beta^2}{36\beta\left(w-\sqrt{m^2-\beta^2}+\frac{\beta^2}{2n}-n\right)^2}\right) - \frac{n\left(\sqrt{2}\sqrt{nw-\sqrt{m^2n^2-n^4+2n^3w}}\right)\sqrt{\left(\sqrt{2}\sqrt{nw-\sqrt{m^2n^2-n^4+2n^3w}}\right)^2+m^2}}{\left(\left(\sqrt{2}\sqrt{nw-\sqrt{m^2n^2-n^4+2n^3w}}\right)^2+\beta^2\right)^2\left(n+\sqrt{\left(\sqrt{2}\sqrt{nw-\sqrt{m^2n^2-n^4+2n^3w}}\right)^2+m^2}\right)} - \frac{n\left(\sqrt{2}\sqrt{nw+\sqrt{m^2n^2-n^4+2n^3w}}\right)\sqrt{\left(\sqrt{2}\sqrt{nw+\sqrt{m^2n^2-n^4+2n^3w}}\right)^2+m^2}}{\left(\left(\sqrt{2}\sqrt{nw+\sqrt{m^2n^2-n^4+2n^3w}}\right)^2+\beta^2\right)^2\left(n+\sqrt{\left(\sqrt{2}\sqrt{nw+\sqrt{m^2n^2-n^4+2n^3w}}\right)^2+m^2}\right)}\right)$$

(4.1.6.4)

4.2. Penggambaran Pole Residu Pada Potensial Separabel Rank-2

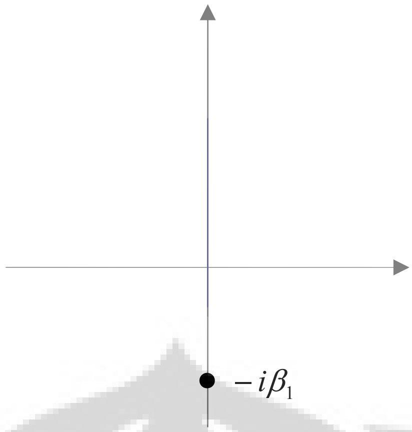
4.2.1. Pada nilai $X_{12}(w)$

4.2.1.1 Pole Residu $k_1 = i\beta_1$



Gambar 4.2.1.1 Pole residu di k_1 di rank-2

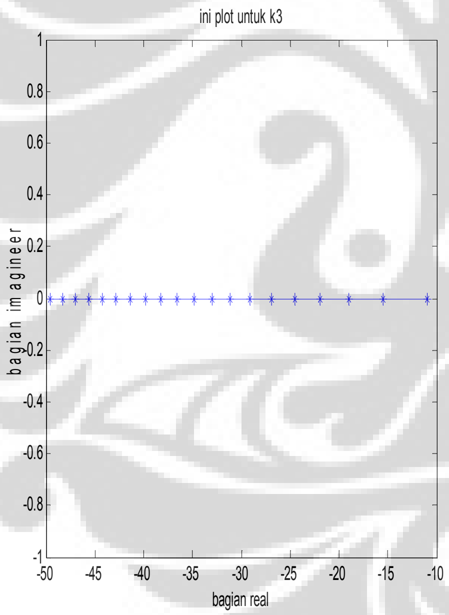
4.2.1.2 Pole Residu $k_2 = -i\beta_1$

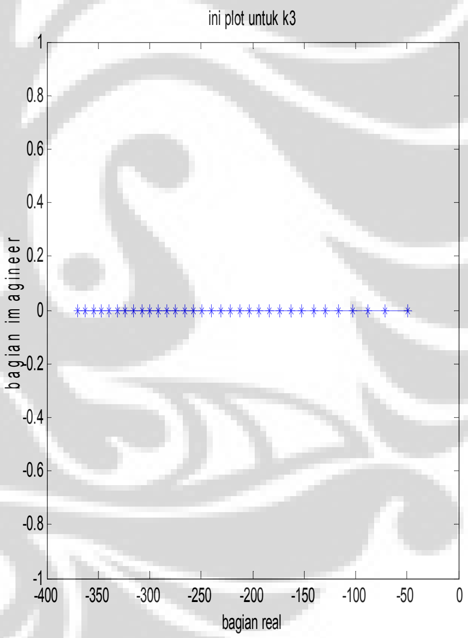


Gambar 4.2.1.2 Pole residu di k_2 di rank-2

4.2.1.3 Pole Residu $k_3 = -\sqrt{2}\sqrt{m_N w - \sqrt{m_K^2 m_N^2 - m_N^4 + 2m_N^3 w}}$

Dengan analisa seperti pada (4.1.3) maka



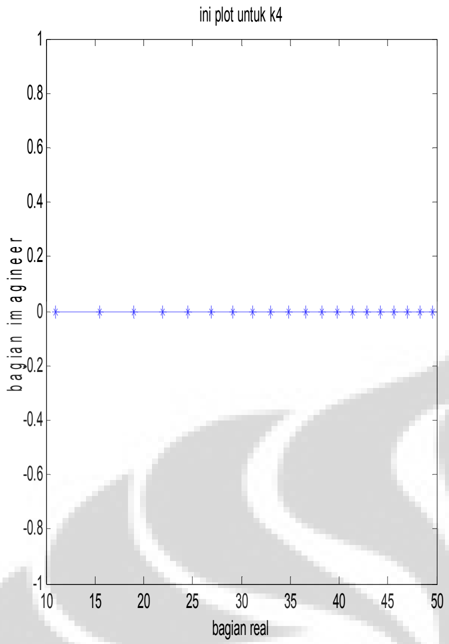


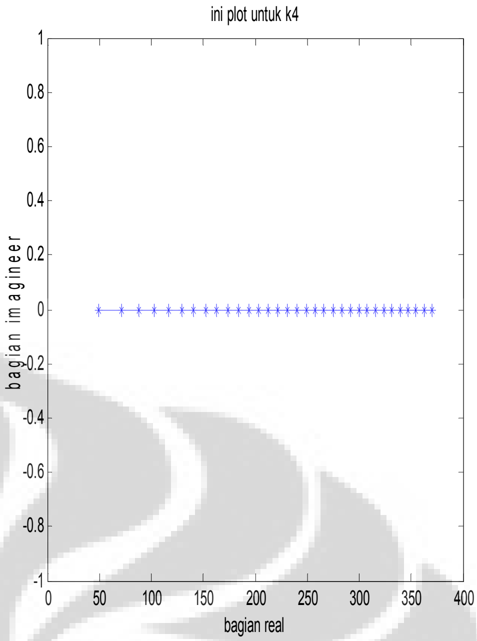
Gambar 4.2.1.3.a Pole residu di k_3 untuk $1076{,}27 \le w \le 1086$ jarak antar w adl 0,5

Gambar 4.2.1.3.b Pole residu di k_3 untuk $1076{,}27 \le w \le 1400$ jarak antar w adl 10

4.2.1.6 Pole Residu $k_4 = \sqrt{2}\sqrt{m_N w - \sqrt{m_K^2 m_N^2 - m_N^4 + 2m_N^3 w}}$

Dengan analisa seperti pada (4.1.4) maka



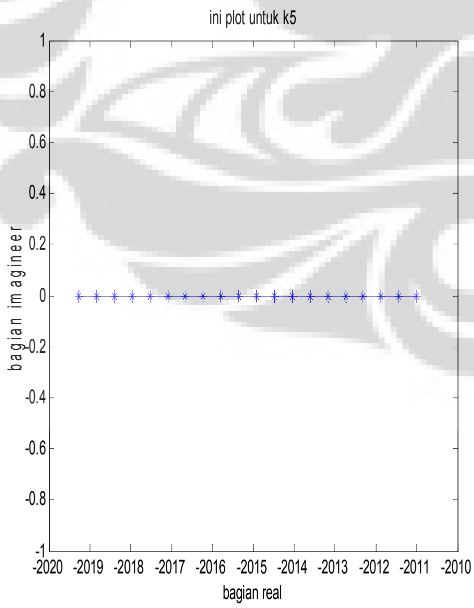


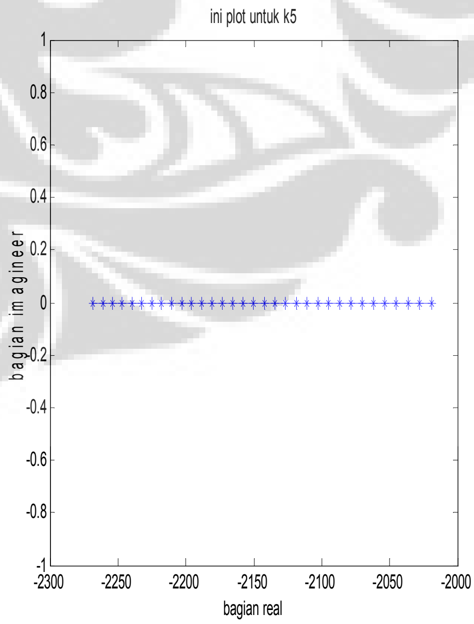
Gambar 4.2.1.4.a Pole residu di k_4 untuk $1076{,}27 \le w \le 1086$ jarak antar w adl 0,5

Gambar 4.2.1.4.b Pole residu di k_4 untuk $1076{,}27 \le w \le 1400$ jarak antar w adl 10

4.2.1.6 Pole Residu $k_5 = -\sqrt{2}\sqrt{m_N w + \sqrt{m_K^2 m_N^2 - m_N^4} + 2m_N^3 w}$

Dengan analisa seperti pada (4.1.5) maka



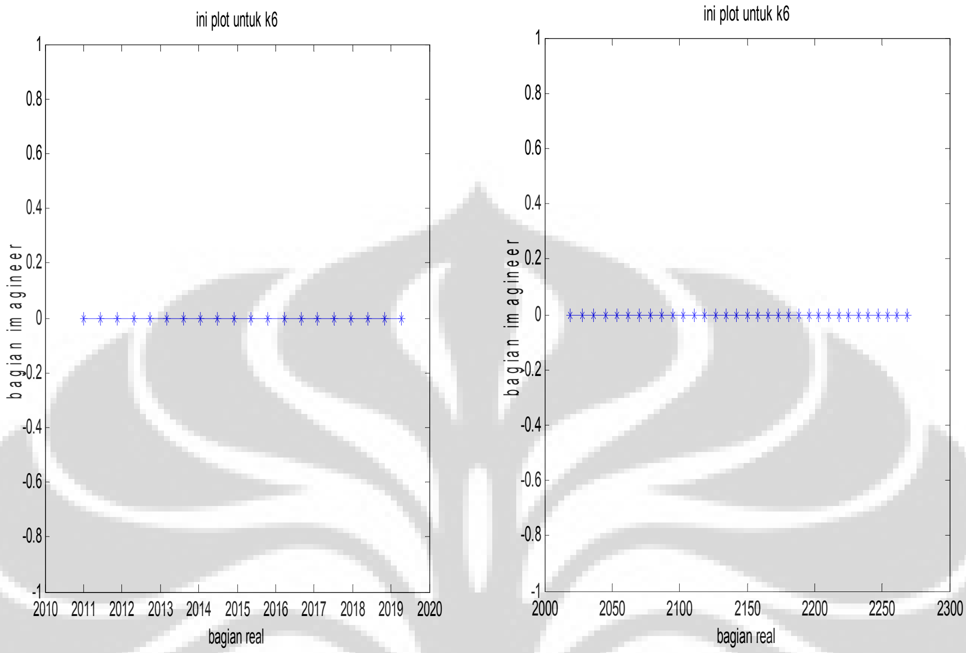


Gambar 4.2.1.5.aPole residu di k_5 untuk $1076{,}27 \le w \le 1086$ jarak antar w adl 0,5

Gambar 4.2.1.5.b Pole residu di k_5 untuk $1076{,}27 \le w \le 1400$ jarak antar w adl 10

4.2.1.6 Pole Residu $k_6 = \sqrt{2}\sqrt{m_N w + \sqrt{m_K^2 m_N^2 - m_N^4 + 2m_N^3 w}}$

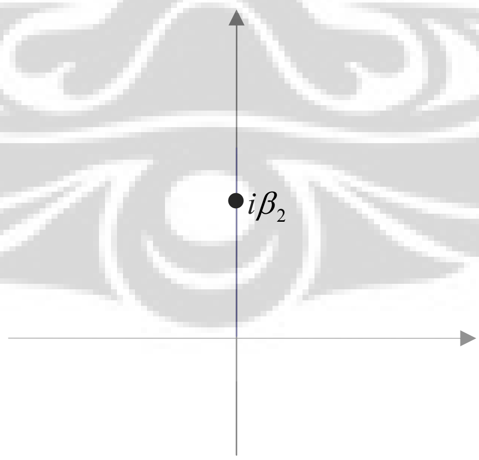
Dengan analisa seperti pada (4.1.6) maka



Gambar 4.2.1.6.a.Pole residu di k_6 untuk $1076{,}27 \le w \le 1086$ jarak antar w adl 0,5

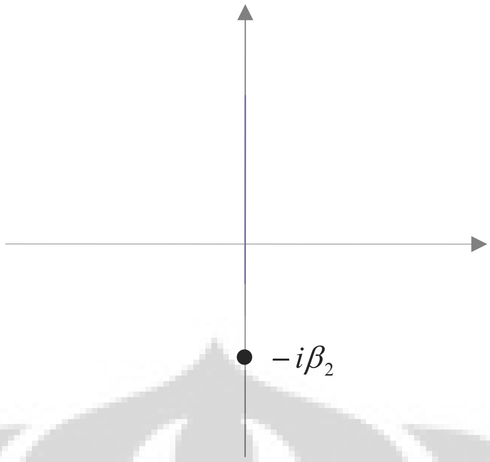
Gambar 4.2.1.6.b Pole residu di k_6 untuk $1076{,}27 \le w \le 1400$ jarak antar w adl 10

4.2.1.7 Pole Residu $k_7 = i\beta_2$

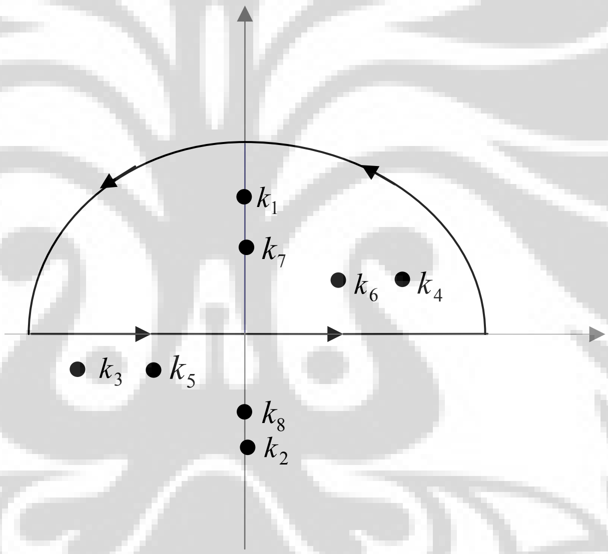


Gambar 4.2.1.7 Pole residu di k_7, rank-2

4.2.1.8 Pole Residu $k_8 = -i\beta_2$



Gambar 4.2.1.8 Pole residu di k_8, rank-2



Gambar 4.2.1.9 Pole residu di $k_1, k_2, k_3, k_4, k_5, k_6, k_7, k_8$

Dari (3.2.1.1.1.1) hingga (3.2.1.1.1.8) dan menerapkan teorema Residu ke (3.2.1.1) maka kita dapatkan

$$X^{l}_{12}(w) = 2c_1 c_2 \pi i \sum_{i=1}^{8} f(k_i)$$

$$= 2c_1 c_2 \pi i \left(f(k_1) + f(k_4) + f(k_6) + f(k_7)\right)$$

$$= 2c_1c_2\pi\, i\cdot\left(\frac{(i\beta_1)^{2+2l}}{(2i\beta_1)\left(-\beta_1{}^2+\beta_2^2\right)\left(w-\sqrt{-\beta_1{}^2+m_K^2}+\frac{\beta_1{}^2}{2m_N}-m_N\right)}+\frac{(i\beta_2)^{2+2l}}{(2i\beta_2)\left(\beta_1{}^2-\beta_2^2\right)\left(w-\sqrt{-\beta_2{}^2+m_K^2}+\frac{\beta_2{}^2}{2m_N}-m_N\right)}\right)$$

$$+2c_1c_2\pi i\cdot\left(-\frac{n\left(\sqrt{2}\sqrt{nw-\sqrt{m^2n^2-n^4+2n^3w}}\right)^{1+2l}\sqrt{\left(\sqrt{2}\sqrt{nw-\sqrt{m^2n^2-n^4+2n^3w}}\right)^2+m^2}}{\left(\left(\left(\sqrt{2}\sqrt{nw-\sqrt{m^2n^2-n^4+2n^3w}}\right)^2+\beta_1^2\right)\left(\left(\sqrt{2}\sqrt{nw-\sqrt{m^2n^2-n^4+2n^3w}}\right)^2+\beta_2^2\right)\left(n+\sqrt{\left(\sqrt{2}\sqrt{nw-\sqrt{m^2n^2-n^4+2n^3w}}\right)^2+m^2}\right)\right)}+\frac{n\left(\sqrt{2}\sqrt{nw+\sqrt{m^2n^2-n^4+2n^3w}}\right)^{1+2l}\sqrt{\left(\sqrt{2}\sqrt{nw+\sqrt{m^2n^2-n^4+2n^3w}}\right)^2+m^2}}{\left(\left(\left(\sqrt{2}\sqrt{nw+\sqrt{m^2n^2-n^4+2n^3w}}\right)^2+\beta_1^2\right)\left(\left(\sqrt{2}\sqrt{nw+\sqrt{m^2n^2-n^4+2n^3w}}\right)^2+\beta_2^2\right)\left(n+\sqrt{\left(\sqrt{2}\sqrt{nw+\sqrt{m^2n^2-n^4+2n^3w}}\right)^2+m^2}\right)\right)}\right)$$

Sehingga

$$X_{12}^{l}(w) = 2c_1c_2\pi\, i\cdot\left(\frac{(i\beta_1)^{2+2l}}{(2i\beta_1)\left(-\beta_1{}^2+\beta_2^2\right)\left(w-\sqrt{-\beta_1{}^2+m_K^2}+\frac{\beta_1{}^2}{2m_N}-m_N\right)}+\frac{(i\beta_2)^{2+2l}}{(2i\beta_2)\left(\beta_1{}^2-\beta_2^2\right)\left(w-\sqrt{-\beta_2{}^2+m_K^2}+\frac{\beta_2{}^2}{2m_N}-m_N\right)}\right)$$

$$+2c_1c_2\pi i\cdot\left(\begin{array}{l}-\dfrac{n\left(\sqrt{2}\sqrt{nw-\sqrt{m^2n^2-n^4+2n^3w}}\right)^{1+2l}\sqrt{\left(\sqrt{2}\sqrt{nw-\sqrt{m^2n^2-n^4+2n^3w}}\right)^2+m^2}}{\left(\begin{array}{l}\left(\left(\sqrt{2}\sqrt{nw-\sqrt{m^2n^2-n^4+2n^3w}}\right)^2+\beta_1^2\right)\\ \left(\left(\sqrt{2}\sqrt{nw-\sqrt{m^2n^2-n^4+2n^3w}}\right)^2+\beta_2^2\right)\\ \left(n+\sqrt{\left(\sqrt{2}\sqrt{nw-\sqrt{m^2n^2-n^4+2n^3w}}\right)^2+m^2}\right)\end{array}\right)}\\ -\dfrac{n\left(\sqrt{2}\sqrt{nw+\sqrt{m^2n^2-n^4+2n^3w}}\right)^{1+2l}\sqrt{\left(\sqrt{2}\sqrt{nw+\sqrt{m^2n^2-n^4+2n^3w}}\right)^2+m^2}}{\left(\begin{array}{l}\left(\left(\sqrt{2}\sqrt{nw+\sqrt{m^2n^2-n^4+2n^3w}}\right)^2+\beta_1^2\right)\\ \left(\left(\sqrt{2}\sqrt{nw+\sqrt{m^2n^2-n^4+2n^3w}}\right)^2+\beta_2^2\right)\\ \left(n+\sqrt{\left(\sqrt{2}\sqrt{nw+\sqrt{m^2n^2-n^4+2n^3w}}\right)^2+m^2}\right)\end{array}\right)}\end{array}\right)$$

(4.2.1.1)

$$X^0_{12}(w)=2c_1c_2\pi i\cdot\left(\begin{array}{l}\dfrac{(i\beta_1)^2}{(2i\beta_1)\left(-\beta_1{}^2+\beta_2^2\right)\left(w-\sqrt{-\beta_1{}^2+m_K^2}+\dfrac{\beta_1{}^2}{2m_N}-m_N\right)}\\ +\dfrac{(i\beta_2)^2}{(2i\beta_2)\left(\beta_1{}^2-\beta_2^2\right)\left(w-\sqrt{-\beta_2{}^2+m_K^2}+\dfrac{\beta_2{}^2}{2m_N}-m_N\right)}\end{array}\right)$$

$$+2c_1c_2\pi i\cdot\left(-\frac{n\left(\sqrt{2}\sqrt{nw-\sqrt{m^2n^2-n^4+2n^3w}}\right)\sqrt{\left(\sqrt{2}\sqrt{nw-\sqrt{m^2n^2-n^4+2n^3w}}\right)^2+m^2}}{\left(\left(\sqrt{2}\sqrt{nw-\sqrt{m^2n^2-n^4+2n^3w}}\right)^2+\beta_1^2\right)\left(\left(\sqrt{2}\sqrt{nw-\sqrt{m^2n^2-n^4+2n^3w}}\right)^2+\beta_2^2\right)\left(n+\sqrt{\left(\sqrt{2}\sqrt{nw-\sqrt{m^2n^2-n^4+2n^3w}}\right)^2+m^2}\right)}-\frac{n\left(\sqrt{2}\sqrt{nw+\sqrt{m^2n^2-n^4+2n^3w}}\right)\sqrt{\left(\sqrt{2}\sqrt{nw+\sqrt{m^2n^2-n^4+2n^3w}}\right)^2+m^2}}{\left(\left(\sqrt{2}\sqrt{nw+\sqrt{m^2n^2-n^4+2n^3w}}\right)^2+\beta_1^2\right)\left(\left(\sqrt{2}\sqrt{nw+\sqrt{m^2n^2-n^4+2n^3w}}\right)^2+\beta_2^2\right)\left(n+\sqrt{\left(\sqrt{2}\sqrt{nw+\sqrt{m^2n^2-n^4+2n^3w}}\right)^2+m^2}\right)}\right)$$

$$X^0_{12}(w)=2c_1c_2\pi i\cdot\left(\frac{i\beta_1}{2\left(-\beta_1^2+\beta_2^2\right)\left(w-\sqrt{-\beta_1^2+m_K^2}+\dfrac{\beta_1^2}{2m_N}-m_N\right)}-\frac{i\beta_2}{2\left(-\beta_1^2+\beta_2^2\right)\left(w-\sqrt{-\beta_2^2+m_K^2}+\dfrac{\beta_2^2}{2m_N}-m_N\right)}\right)$$

$$+2c_1c_2\pi i\cdot\left(\begin{array}{l}-\dfrac{n\left(\sqrt{2}\sqrt{nw-\sqrt{m^2n^2-n^4+2n^3w}}\right)\sqrt{\left(\sqrt{2}\sqrt{nw-\sqrt{m^2n^2-n^4+2n^3w}}\right)^2+m^2}}{\left(\begin{array}{l}\left(\left(\sqrt{2}\sqrt{nw-\sqrt{m^2n^2-n^4+2n^3w}}\right)^2+\beta_1^{\,2}\right)\\ \left(\left(\sqrt{2}\sqrt{nw-\sqrt{m^2n^2-n^4+2n^3w}}\right)^2+\beta_2^{\,2}\right)\\ \left(n+\sqrt{\left(\sqrt{2}\sqrt{nw-\sqrt{m^2n^2-n^4+2n^3w}}\right)^2+m^2}\right)\end{array}\right)}\\ -\dfrac{n\left(\sqrt{2}\sqrt{nw+\sqrt{m^2n^2-n^4+2n^3w}}\right)\sqrt{\left(\sqrt{2}\sqrt{nw+\sqrt{m^2n^2-n^4+2n^3w}}\right)^2+m^2}}{\left(\begin{array}{l}\left(\left(\sqrt{2}\sqrt{nw+\sqrt{m^2n^2-n^4+2n^3w}}\right)^2+\beta_1^{\,2}\right)\\ \left(\left(\sqrt{2}\sqrt{nw+\sqrt{m^2n^2-n^4+2n^3w}}\right)^2+\beta_2^{\,2}\right)\\ \left(n+\sqrt{\left(\sqrt{2}\sqrt{nw+\sqrt{m^2n^2-n^4+2n^3w}}\right)^2+m^2}\right)\end{array}\right)}\end{array}\right)$$

(4.2.1.2)

4.2.2. Pada nilai $X_{21}(w)$

Karena

$$X^0_{21}(w)=\int_0^\infty dkk^2 g_2(k)G_N(w,k)g_1(k)$$

$$=\int_0^\infty dkk^2 g_1(k)G_N(w,k)g_2(k)$$

$$=X^0_{12}(w)$$

Maka

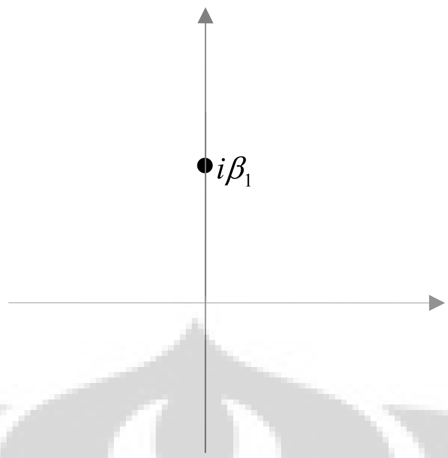
$$X^{0}_{21}(w) = 2c_1c_2\pi\, i\cdot\left(\begin{array}{l}\dfrac{i\beta_1}{2\left(-\beta_1{}^2+\beta_2^2\right)\left(w-\sqrt{-\beta_1{}^2+m_K^2}+\dfrac{\beta_1{}^2}{2m_N}-m_N\right)} \\ -\dfrac{i\beta_2}{2\left(-\beta_1{}^2+\beta_2^2\right)\left(w-\sqrt{-\beta_2{}^2+m_K^2}+\dfrac{\beta_2{}^2}{2m_N}-m_N\right)}\end{array}\right)$$

$$+2c_1c_2\pi i\cdot\left(\begin{array}{l}-\dfrac{n\left(\sqrt{2}\sqrt{nw-\sqrt{m^2n^2-n^4+2n^3w}}\right)\sqrt{\left(\sqrt{2}\sqrt{nw-\sqrt{m^2n^2-n^4+2n^3w}}\right)^2+m^2}}{\left(\begin{array}{l}\left(\left(\sqrt{2}\sqrt{nw-\sqrt{m^2n^2-n^4+2n^3w}}\right)^2+\beta_1^2\right)\\ \left(\left(\sqrt{2}\sqrt{nw-\sqrt{m^2n^2-n^4+2n^3w}}\right)^2+\beta_2{}^2\right)\\ \left(n+\sqrt{\left(\sqrt{2}\sqrt{nw-\sqrt{m^2n^2-n^4+2n^3w}}\right)^2+m^2}\right)\end{array}\right)} \\ -\dfrac{n\left(\sqrt{2}\sqrt{nw+\sqrt{m^2n^2-n^4+2n^3w}}\right)\sqrt{\left(\sqrt{2}\sqrt{nw+\sqrt{m^2n^2-n^4+2n^3w}}\right)^2+m^2}}{\left(\begin{array}{l}\left(\left(\sqrt{2}\sqrt{nw+\sqrt{m^2n^2-n^4+2n^3w}}\right)^2+\beta_1{}^2\right)\\ \left(\left(\sqrt{2}\sqrt{nw+\sqrt{m^2n^2-n^4+2n^3w}}\right)^2+\beta_2{}^2\right)\\ \left(n+\sqrt{\left(\sqrt{2}\sqrt{nw+\sqrt{m^2n^2-n^4+2n^3w}}\right)^2+m^2}\right)\end{array}\right)}\end{array}\right)$$

(4.2.2.1)

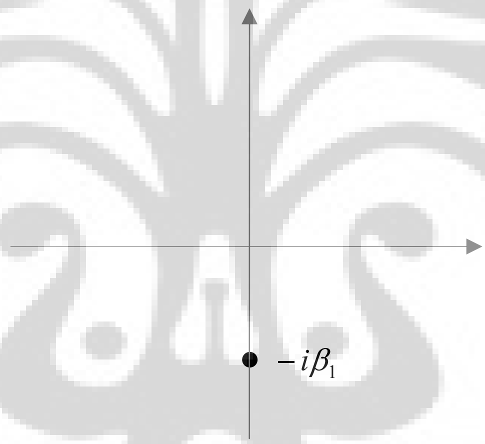
4.2.3. Pada nilai $X_{11}(w)$

4.2.3.1 Pole Residu $k_1 = i\beta_1$



Gambar 4.2.3.1 Pole residu di k_1

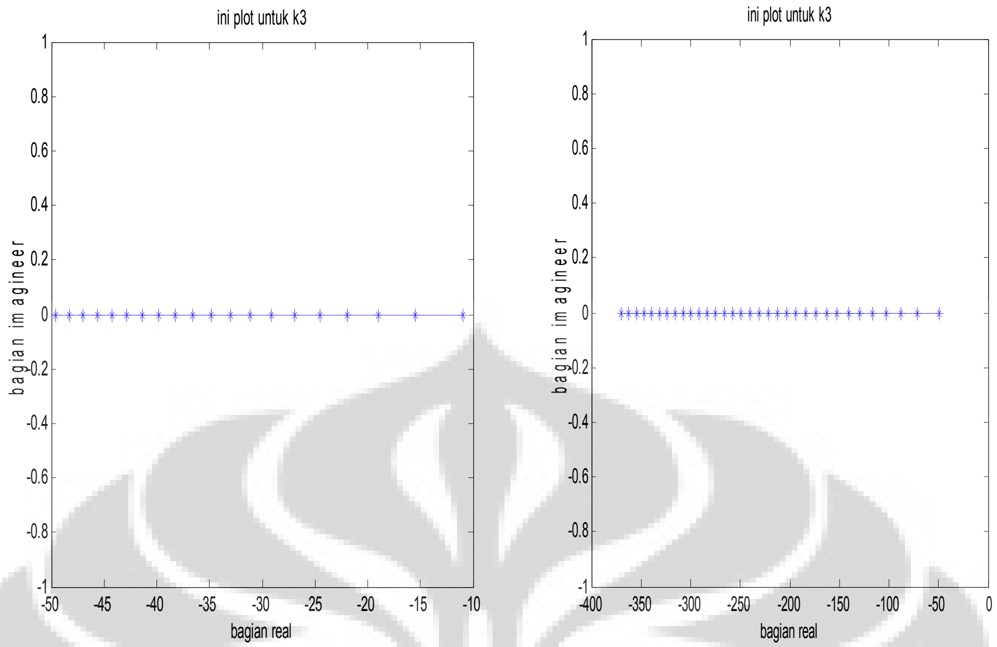
4.2.3.2 Pole Residu $k_2 = -i\beta_1$



Gambar 4.2.3.2 Pole residu di k_2

4.2.3.3 Pole Residu $k_3 = -\sqrt{2}\sqrt{m_N w - \sqrt{m_K^2 m_N^2 - m_N^4 + 2m_N^3 w}}$

Dengan analisa seperti pada (4.1.3) maka

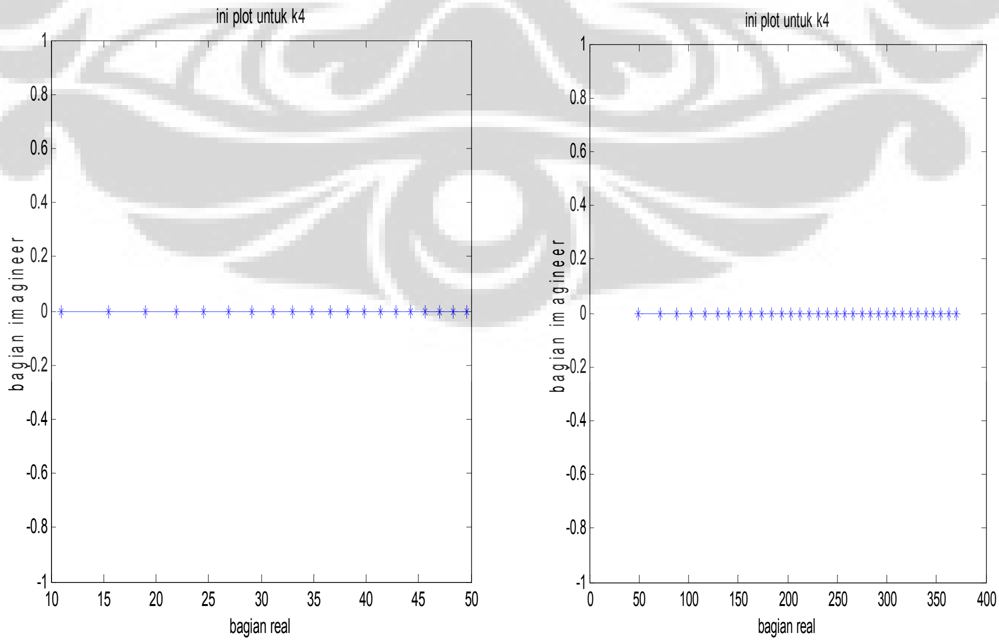


Gambar 4.2.3.3.a Pole residu di k_3 untuk $1076{,}27 \le w \le 1086$ jarak antar w adl 0,5

Gambar 4.2.3.3.b Pole residu di k_3 untuk $1076{,}27 \le w \le 1400$ jarak antar w adl 10

4.2.3.4 Pole Residu $k_4 = \sqrt{2}\sqrt{m_N w - \sqrt{m_K^2 m_N^2 - m_N^4 + 2m_N^3 w}}$

Dengan analisa seperti pada (4.1.4) maka

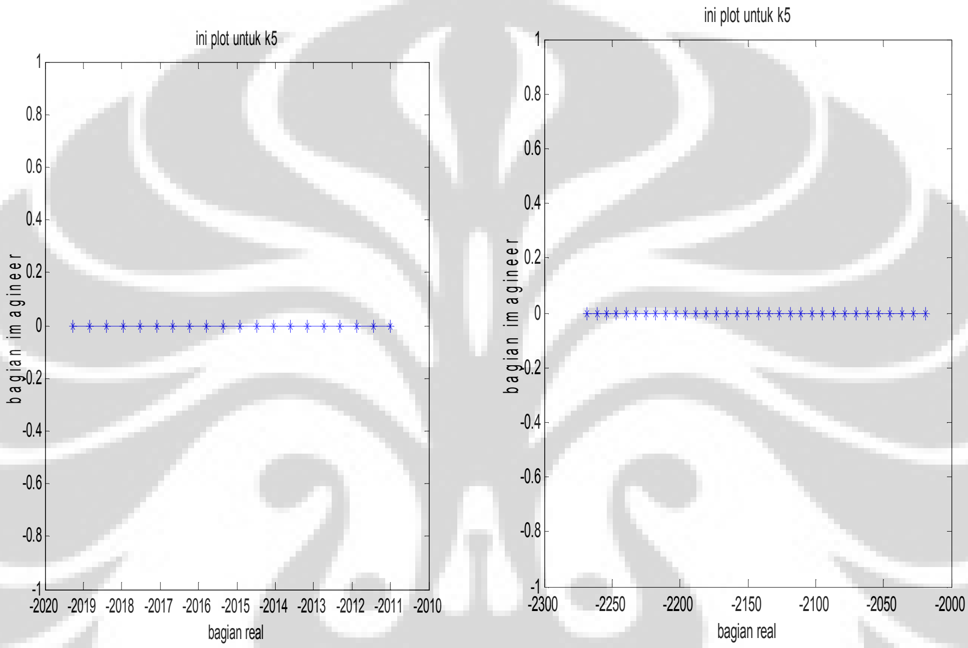


Gambar 4.2.3.4.a Pole residu di k_4 untuk $1076{,}27 \le w \le 1086$ jarak antar w adl 0,5

Gambar 4.2.3.4.b Pole residu di k_4 untuk $1076{,}27 \le w \le 1400$ jarak antar w adl 10

4.2.3.5 Pole Residu $k_5 = -\sqrt{2}\sqrt{m_N w + \sqrt{m_K^2 m_N^2 - m_N^4 + 2m_N^3 w}}$

Dengan analisa seperti pada (4.1.5) maka

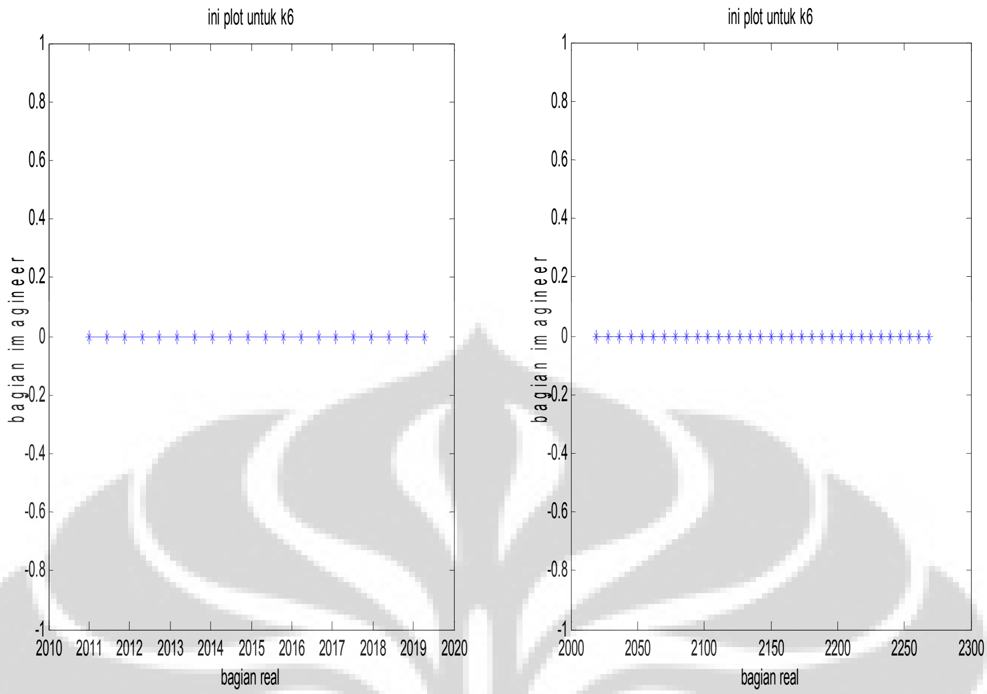


Gambar 4.2.3.5.aPole residu di k_5 untuk $1076{,}27 \le w \le 1086$ jarak antar w adl 0,5

Gambar 4.2.3.5.b Pole residu di k_5 untuk $1076{,}27 \le w \le 1400$ jarak antar w adl 10

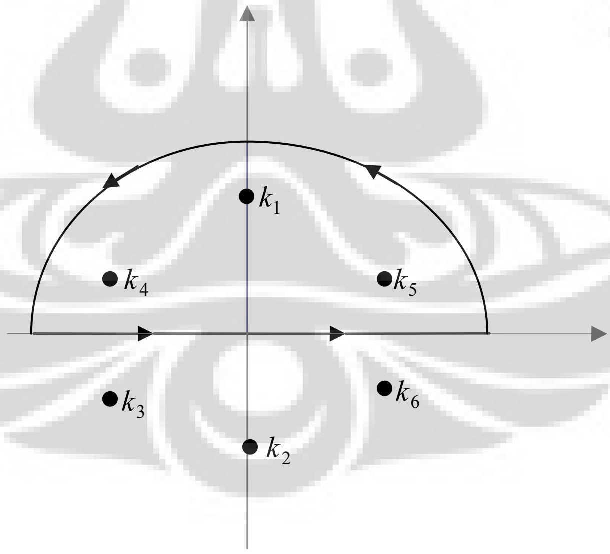
4.2.3.6 Pole Residu $k_6 = \sqrt{2}\sqrt{m_N w + \sqrt{m_K^2 m_N^2 - m_N^4 + 2m_N^3 w}}$

Dengan analisa seperti pada (4.1.6) maka



Gambar 4.2.3.6.aPole residu di k_6 untuk $1076{,}27 \le w \le 1086$ jarak antar w adl 0,5

Gambar 4.2.3.6.b Pole residu di k_6 untuk $1076{,}27 \le w \le 1400$ jarak antar w adl 10



Gambar 4.1.7 Pole residu di $k_1, k_2, k_3, k_4, k_5, k_6$

Dari (3.2.3.1.2), (3.2.3.1.4) hingga (3.2.3.1.8) maka kita dapatkan

$$X^{0}_{11} = 2c_1^{\,2}\pi\, i\sum_{i=1}^{6} f(k_i)$$

$$= 2c_1^{\,2}\pi\, i\big(f(k_1)+f(k_4)+f(k_5)\big)$$

$$=2c_1^2\pi i\left(\left(-\frac{(2+2l)(i\beta_1)^{1+2l}}{4\beta_1^2\left(w-\sqrt{m^2-\beta_1^2}+\frac{\beta_1^2}{2n}-n\right)}+\frac{2(i\beta_1)^{2+2l}}{18\beta_1^3\left(w-\sqrt{m^2-\beta_1^2}+\frac{\beta_1^2}{2n}-n\right)}-\frac{\left(\frac{i\beta_1}{\sqrt{m^2-\beta_1^2}}+\frac{i\beta_1}{n}\right)(i\beta_1)^{2+2l}}{4\beta_1^2\left(w-\sqrt{m^2-\beta_1^2}+\frac{\beta_1^2}{2n}-n\right)^2}\right)-\frac{n\left(\sqrt{2}\sqrt{nw-\sqrt{m^2n^2-n^4+2n^3w}}\right)^{1+2l}\sqrt{\left(\sqrt{2}\sqrt{nw-\sqrt{m^2n^2-n^4+2n^3w}}\right)^2+m^2}}{\left(\left(\sqrt{2}\sqrt{nw-\sqrt{m^2n^2-n^4+2n^3w}}\right)^2+\beta_1^2\right)^2\left(n+\sqrt{\left(\sqrt{2}\sqrt{nw-\sqrt{m^2n^2-n^4+2n^3w}}\right)^2+m^2}\right)}-\frac{n\left(\sqrt{2}\sqrt{nw+\sqrt{m^2n^2-n^4+2n^3w}}\right)^{1+2l}\sqrt{\left(\sqrt{2}\sqrt{nw+\sqrt{m^2n^2-n^4+2n^3w}}\right)^2+m^2}}{\left(\left(\sqrt{2}\sqrt{nw+\sqrt{m^2n^2-n^4+2n^3w}}\right)^2+\beta_1^2\right)^2\left(n+\sqrt{\left(\sqrt{2}\sqrt{nw+\sqrt{m^2n^2-n^4+2n^3w}}\right)^2+m^2}\right)}\right)$$

$$=2c_1^2\pi i\left(\left(-\frac{(2+2l)(i\beta_1)^{1+2l}}{4\beta_1^2\left(w-\sqrt{m^2-\beta_1^2}+\frac{\beta_1^2}{2n}-n\right)}+\frac{2(i\beta_1)^{2+2l}}{18\beta_1^3\left(w-\sqrt{m^2-\beta_1^2}+\frac{\beta_1^2}{2n}-n\right)}-\frac{\left(\frac{1}{\sqrt{m^2-\beta_1^2}}+\frac{1}{n}\right)(i\beta_1)^{3+2l}}{4\beta_1^2\left(w-\sqrt{m^2-\beta_1^2}+\frac{\beta_1^2}{2n}-n\right)^2}\right)-\frac{n\left(\sqrt{2}\sqrt{nw-\sqrt{m^2n^2-n^4+2n^3w}}\right)^{1+2l}\sqrt{\left(\sqrt{2}\sqrt{nw-\sqrt{m^2n^2-n^4+2n^3w}}\right)^2+m^2}}{\left(\left(\sqrt{2}\sqrt{nw-\sqrt{m^2n^2-n^4+2n^3w}}\right)^2+\beta_1^2\right)^2\left(n+\sqrt{\left(\sqrt{2}\sqrt{nw-\sqrt{m^2n^2-n^4+2n^3w}}\right)^2+m^2}\right)}-\frac{n\left(\sqrt{2}\sqrt{nw+\sqrt{m^2n^2-n^4+2n^3w}}\right)^{1+2l}\sqrt{\left(\sqrt{2}\sqrt{nw+\sqrt{m^2n^2-n^4+2n^3w}}\right)^2+m^2}}{\left(\left(\sqrt{2}\sqrt{nw+\sqrt{m^2n^2-n^4+2n^3w}}\right)^2+\beta_1^2\right)^2\left(n+\sqrt{\left(\sqrt{2}\sqrt{nw+\sqrt{m^2n^2-n^4+2n^3w}}\right)^2+m^2}\right)}\right)$$

Sehingga

$$X_{11}^{l}=2c_1^2\pi i\left(\begin{array}{l}\left(-\frac{(2+2l)(i\beta_1)^{1+2l}}{4\beta_1^2\left(w-\sqrt{m^2-\beta_1^2}+\frac{\beta_1^2}{2n}-n\right)}+\frac{2(i\beta_1)^{2+2l}}{18\beta_1^3\left(w-\sqrt{m^2-\beta_1^2}+\frac{\beta_1^2}{2n}-n\right)}-\frac{\left(\frac{1}{\sqrt{m^2-\beta_1^2}}+\frac{1}{n}\right)(i\beta_1)^{3+2l}}{4\beta_1^2\left(w-\sqrt{m^2-\beta_1^2}+\frac{\beta_1^2}{2n}-n\right)^2}\right)\\ -\frac{n\left(\sqrt{2}\sqrt{nw-\sqrt{m^2n^2-n^4+2n^3w}}\right)^{1+2l}\sqrt{\left(\sqrt{2}\sqrt{nw-\sqrt{m^2n^2-n^4+2n^3w}}\right)^2+m^2}}{\left(\left(\sqrt{2}\sqrt{nw-\sqrt{m^2n^2-n^4+2n^3w}}\right)^2+\beta_1^2\right)^2\left(n+\sqrt{\left(\sqrt{2}\sqrt{nw-\sqrt{m^2n^2-n^4+2n^3w}}\right)^2+m^2}\right)}\\ -\frac{n\left(\sqrt{2}\sqrt{nw+\sqrt{m^2n^2-n^4+2n^3w}}\right)^{1+2l}\sqrt{\left(\sqrt{2}\sqrt{nw+\sqrt{m^2n^2-n^4+2n^3w}}\right)^2+m^2}}{\left(\left(\sqrt{2}\sqrt{nw+\sqrt{m^2n^2-n^4+2n^3w}}\right)^2+\beta_1^2\right)^2\left(n+\sqrt{\left(\sqrt{2}\sqrt{nw+\sqrt{m^2n^2-n^4+2n^3w}}\right)^2+m^2}\right)}\end{array}\right)$$

(4.2.3.1)

Dengan menggantikan $l=0$ maka kita peroleh

$$X_{11}^{0}=2c_1^2\pi i\left(\begin{array}{l}\left(-\frac{2(i\beta)}{4\beta_1^2\left(w-\sqrt{m^2-\beta_1^2}+\frac{\beta_1^2}{2n}-n\right)}+\frac{2(i\beta_1)^{2}}{18\beta_1^3\left(w-\sqrt{m^2-\beta_1^2}+\frac{\beta_1^2}{2n}-n\right)}-\frac{\left(\frac{1}{\sqrt{m^2-\beta_1^2}}+\frac{1}{n}\right)(i\beta_1)^{3}}{4\beta_1^2\left(w-\sqrt{m^2-\beta_1^2}+\frac{\beta_1^2}{2n}-n\right)^2}\right)\\ -\frac{n\left(\sqrt{2}\sqrt{nw-\sqrt{m^2n^2-n^4+2n^3w}}\right)\sqrt{\left(\sqrt{2}\sqrt{nw-\sqrt{m^2n^2-n^4+2n^3w}}\right)^2+m^2}}{\left(\left(\sqrt{2}\sqrt{nw-\sqrt{m^2n^2-n^4+2n^3w}}\right)^2+\beta_1^2\right)^2\left(n+\sqrt{\left(\sqrt{2}\sqrt{nw-\sqrt{m^2n^2-n^4+2n^3w}}\right)^2+m^2}\right)}\\ -\frac{n\left(\sqrt{2}\sqrt{nw+\sqrt{m^2n^2-n^4+2n^3w}}\right)\sqrt{\left(\sqrt{2}\sqrt{nw+\sqrt{m^2n^2-n^4+2n^3w}}\right)^2+m^2}}{\left(\left(\sqrt{2}\sqrt{nw+\sqrt{m^2n^2-n^4+2n^3w}}\right)^2+\beta_1^2\right)^2\left(n+\sqrt{\left(\sqrt{2}\sqrt{nw+\sqrt{m^2n^2-n^4+2n^3w}}\right)^2+m^2}\right)}\end{array}\right)$$

$$X^0_{11}=2c_1^2\pi i\left(\begin{array}{l}\left(\dfrac{-9i-2}{18\beta_1\left(w-\sqrt{m^2-\beta_1^2}+\frac{\beta_1^2}{2n}-n\right)}+\dfrac{\left(\frac{1}{\sqrt{m^2-\beta_1^2}}+\frac{1}{n}\right)i\beta_1}{4\left(w-\sqrt{m^2-\beta_1^2}+\frac{\beta_1^2}{2n}-n\right)^2}\right)\\ -\dfrac{n\left(\sqrt{2}\sqrt{nw-\sqrt{m^2n^2-n^4+2n^3w}}\right)\sqrt{\left(\sqrt{2}\sqrt{nw-\sqrt{m^2n^2-n^4+2n^3w}}\right)^2+m^2}}{\left(\left(\sqrt{2}\sqrt{nw-\sqrt{m^2n^2-n^4+2n^3w}}\right)^2+\beta_1^2\right)^2\left(n+\sqrt{\left(\sqrt{2}\sqrt{nw-\sqrt{m^2n^2-n^4+2n^3w}}\right)^2+m^2}\right)}\\ -\dfrac{n\left(\sqrt{2}\sqrt{nw+\sqrt{m^2n^2-n^4+2n^3w}}\right)\sqrt{\left(\sqrt{2}\sqrt{nw+\sqrt{m^2n^2-n^4+2n^3w}}\right)^2+m^2}}{\left(\left(\sqrt{2}\sqrt{nw+\sqrt{m^2n^2-n^4+2n^3w}}\right)^2+\beta_1^2\right)^2\left(n+\sqrt{\left(\sqrt{2}\sqrt{nw+\sqrt{m^2n^2-n^4+2n^3w}}\right)^2+m^2}\right)}\end{array}\right)$$

Sehingga,

$$X^0_{11}=2c_1^2\pi i\left(\begin{array}{l}\left(\dfrac{-2(9i+2)\left(w-\sqrt{m^2-\beta_1^2}+\frac{\beta_1^2}{2n}-n\right)+9\left(\frac{1}{\sqrt{m^2-\beta_1^2}}+\frac{1}{n}\right)i\beta_1^2}{36\beta_1\left(w-\sqrt{m^2-\beta_1^2}+\frac{\beta_1^2}{2n}-n\right)^2}\right)\\ -\dfrac{n\left(\sqrt{2}\sqrt{nw-\sqrt{m^2n^2-n^4+2n^3w}}\right)\sqrt{\left(\sqrt{2}\sqrt{nw-\sqrt{m^2n^2-n^4+2n^3w}}\right)^2+m^2}}{\left(\left(\sqrt{2}\sqrt{nw-\sqrt{m^2n^2-n^4+2n^3w}}\right)^2+\beta_1^2\right)^2\left(n+\sqrt{\left(\sqrt{2}\sqrt{nw-\sqrt{m^2n^2-n^4+2n^3w}}\right)^2+m^2}\right)}\\ -\dfrac{n\left(\sqrt{2}\sqrt{nw+\sqrt{m^2n^2-n^4+2n^3w}}\right)\sqrt{\left(\sqrt{2}\sqrt{nw+\sqrt{m^2n^2-n^4+2n^3w}}\right)^2+m^2}}{\left(\left(\sqrt{2}\sqrt{nw+\sqrt{m^2n^2-n^4+2n^3w}}\right)^2+\beta_1^2\right)^2\left(n+\sqrt{\left(\sqrt{2}\sqrt{nw+\sqrt{m^2n^2-n^4+2n^3w}}\right)^2+m^2}\right)}\end{array}\right)$$

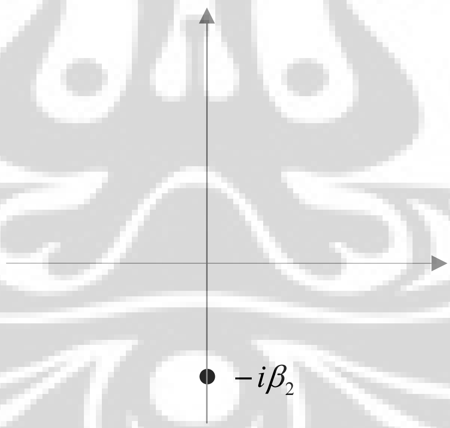
(4.2.3.2)

4.2.4. Pada nilai $X_{22}(w)$

4.2.4.1 Pole Residu $k_1 = i\beta_2$

Gambar 4.2.4.1 Pole residu di k_1

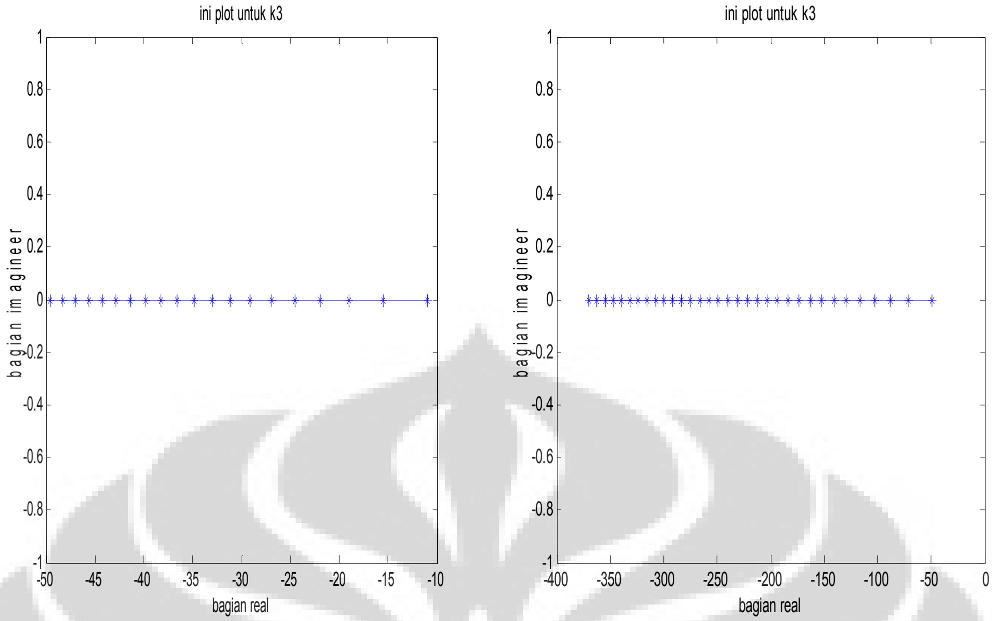
4.2.4.2 Pole Residu $k_2 = -i\beta_2$



Gambar 4.2.4.2 Pole residu di k_2

4.2.4.3 Pole Residu $k_3 = -\sqrt{2}\sqrt{m_N w - \sqrt{m_K^2 m_N^2 - m_N^4 + 2m_N^3 w}}$

Dengan analisa seperti pada (4.1.3) maka

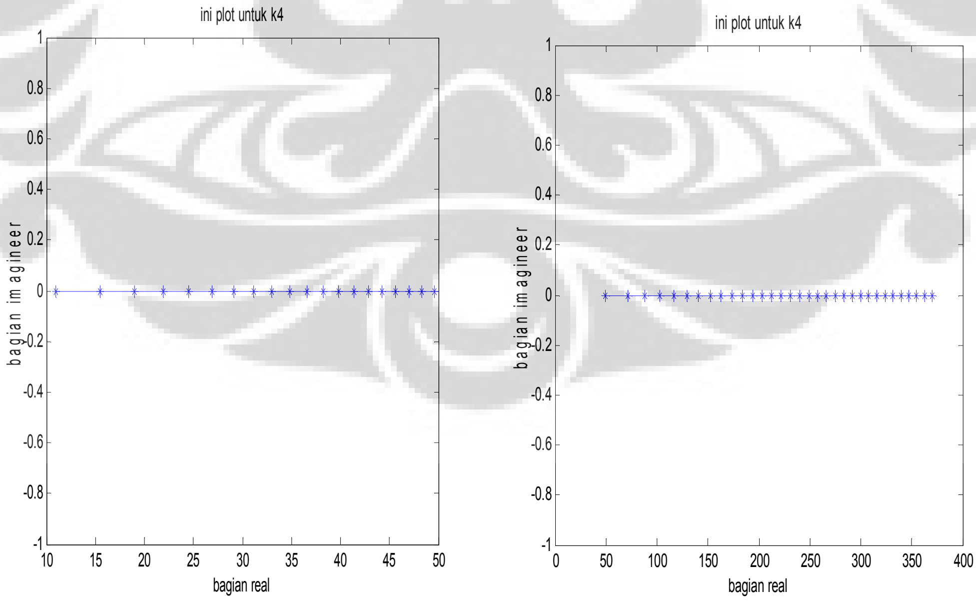


Gambar 4.2.4.3.a Pole residu di k_3 untuk $1076,27 \leq w \leq 1086$ jarak antar w adl 0,5

Gambar 4.2.4.3.b Pole residu di k_3 untuk $1076,27 \leq w \leq 1400$ jarak antar w adl 10

4.2.4.4 Pole Residu $k_4 = \sqrt{2}\sqrt{m_N w - \sqrt{m_K^2 m_N^2 - m_N^4 + 2m_N^3 w}}$

Dengan analisa seperti pada (4.1.4) maka

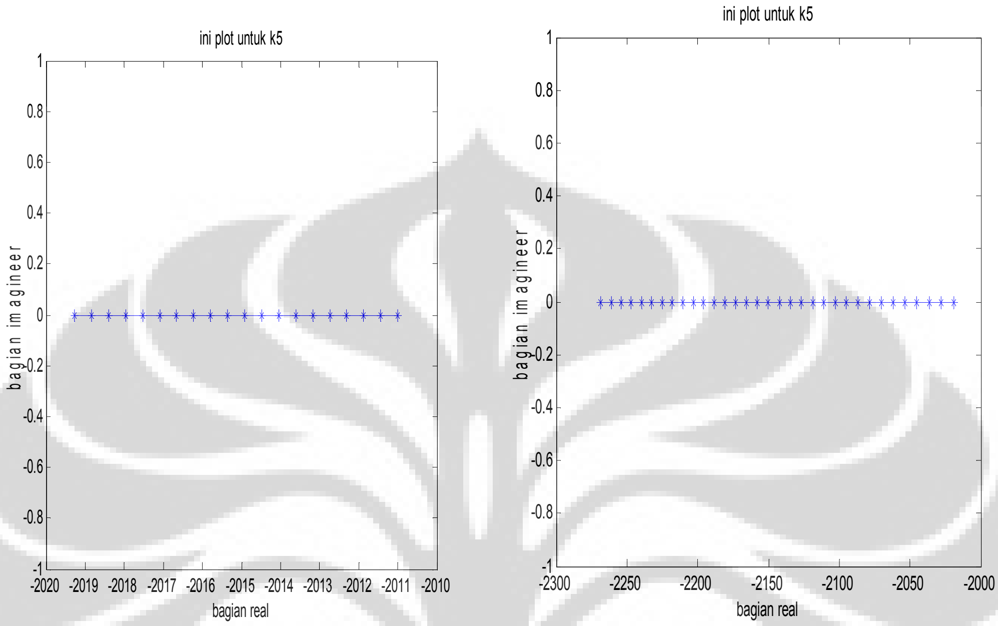


Gambar 4.2.4.4.a Pole residu di k_4 untuk $1076,27 \leq w \leq 1086$ jarak antar w adl 0,5

Gambar 4.2.4.4.b Pole residu di k_4 untuk $1076,27 \leq w \leq 1400$ jarak antar w adl 10

4.2.4.5 Pole Residu $k_5 = -\sqrt{2}\sqrt{m_N w + \sqrt{m_K^2 m_N^2 - m_N^4} + 2m_N^3 w}$

Dengan analisa seperti pada (4.1.5) maka

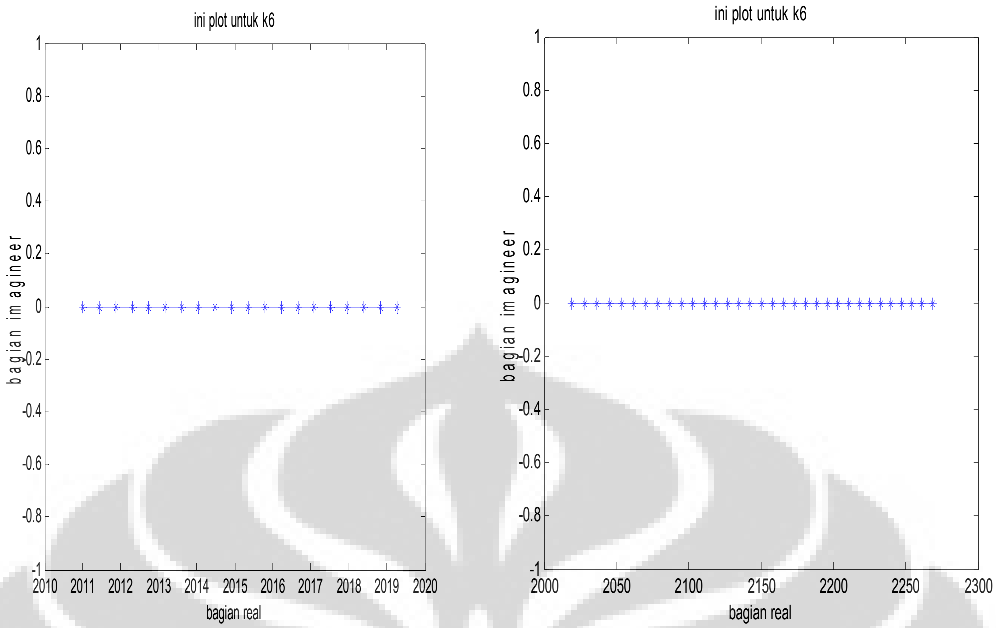


Gambar 4.2.4.5.aPole residu di k_5 untuk $1076{,}27 \le w \le 1086$ jarak antar w adl 0,5

Gambar 4.2.4.5.b Pole residu di k_5 untuk $1076{,}27 \le w \le 1400$ jarak antar w adl 10

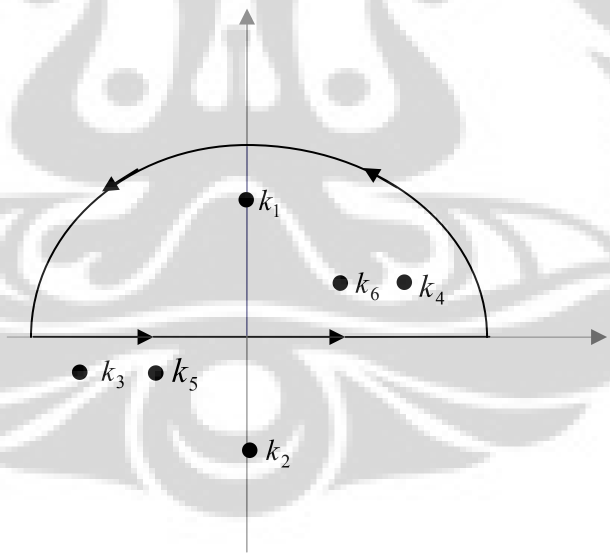
4.2.4.6 Pole Residu $k_6 = \sqrt{2}\sqrt{m_N w + \sqrt{m_K^2 m_N^2 - m_N^4} + 2m_N^3 w}$

Dengan analisa seperti pada (4.1.6) maka



Gambar 4.2.4.6.aPole residu di k_6 untuk $1076{,}27 \le w \le 1086$ jarak antar w adl 0,5

Gambar 4.2.4.6.b Pole residu di k_6 untuk $1076{,}27 \le w \le 1400$ jarak antar w adl 10



Gambar 4.2.4.7 Pole residu di $k_1, k_2, k_3, k_4, k_5, k_6$

Dari (3.2.4.2), (3.2.4.4) hingga (3.2.4.8) maka kita dapatkan

$$X_{22}^{l} = 2c_2{}^2 \pi\, i \sum_{i=1}^{6} f(k_i)$$

$$= 2c_2{}^2 \pi\, i \big(f(k_1) + f(k_4) + f(k_6) \big)$$

$$=2c_2^2\pi i\left(\begin{aligned}&\left(-\frac{(2+2l)(i\beta_2)^{1+2l}}{4\beta_2^2\left(w-\sqrt{m^2-\beta_2^2}+\frac{\beta_2^2}{2n}-n\right)}+\frac{2(i\beta_2)^{2+2l}}{18\beta_2^3\left(w-\sqrt{m^2-\beta_2^2}+\frac{\beta_2^2}{2n}-n\right)}-\frac{\left(\frac{i\beta_2}{\sqrt{m^2-\beta_2^2}}+\frac{i\beta_2}{n}\right)(i\beta_2)^{2+2l}}{4\beta_2^2\left(w-\sqrt{m^2-\beta_2^2}+\frac{\beta_2^2}{2n}-n\right)^2}\right)+\\&-\frac{n\left(\sqrt{2}\sqrt{nw-\sqrt{m^2n^2-n^4+2n^3w}}\right)^{1+2l}\sqrt{\left(\sqrt{2}\sqrt{nw-\sqrt{m^2n^2-n^4+2n^3w}}\right)^2+m^2}}{\left(\left(\sqrt{2}\sqrt{nw-\sqrt{m^2n^2-n^4+2n^3w}}\right)^2+\beta_2^2\right)^2\left(n+\sqrt{\left(\sqrt{2}\sqrt{nw-\sqrt{m^2n^2-n^4+2n^3w}}\right)^2+m^2}\right)}+\\&\frac{n\left(\sqrt{2}\sqrt{nw+\sqrt{m^2n^2-n^4+2n^3w}}\right)^{1+2l}\sqrt{\left(\sqrt{2}\sqrt{nw+\sqrt{m^2n^2-n^4+2n^3w}}\right)^2+m^2}}{\left(\left(\sqrt{2}\sqrt{nw+\sqrt{m^2n^2-n^4+2n^3w}}\right)^2+\beta_2^2\right)^2\left(n+\sqrt{\left(\sqrt{2}\sqrt{nw+\sqrt{m^2n^2-n^4+2n^3w}}\right)^2+m^2}\right)}\end{aligned}\right)$$

$$=2c_2^2\pi i\left(\begin{aligned}&\left(-\frac{(2+2l)(i\beta_2)^{1+2l}}{4\beta_2^2\left(w-\sqrt{m^2-\beta_2^2}+\frac{\beta_2^2}{2n}-n\right)}+\frac{2(i\beta_2)^{2+2l}}{18\beta_2^3\left(w-\sqrt{m^2-\beta_2^2}+\frac{\beta_2^2}{2n}-n\right)}-\frac{\left(\frac{1}{\sqrt{m^2-\beta_2^2}}+\frac{1}{n}\right)(i\beta_2)^{3+2l}}{4\beta_2^2\left(w-\sqrt{m^2-\beta_2^2}+\frac{\beta_2^2}{2n}-n\right)^2}\right)\\&-\frac{n\left(\sqrt{2}\sqrt{nw-\sqrt{m^2n^2-n^4+2n^3w}}\right)^{1+2l}\sqrt{\left(\sqrt{2}\sqrt{nw-\sqrt{m^2n^2-n^4+2n^3w}}\right)^2+m^2}}{\left(\left(\sqrt{2}\sqrt{nw-\sqrt{m^2n^2-n^4+2n^3w}}\right)^2+\beta_2^2\right)^2\left(n+\sqrt{\left(\sqrt{2}\sqrt{nw-\sqrt{m^2n^2-n^4+2n^3w}}\right)^2+m^2}\right)}\\&-\frac{n\left(\sqrt{2}\sqrt{nw+\sqrt{m^2n^2-n^4+2n^3w}}\right)^{1+2l}\sqrt{\left(\sqrt{2}\sqrt{nw+\sqrt{m^2n^2-n^4+2n^3w}}\right)^2+m^2}}{\left(\left(\sqrt{2}\sqrt{nw+\sqrt{m^2n^2-n^4+2n^3w}}\right)^2+\beta_2^2\right)^2\left(n+\sqrt{\left(\sqrt{2}\sqrt{nw+\sqrt{m^2n^2-n^4+2n^3w}}\right)^2+m^2}\right)}\end{aligned}\right)$$

Sehingga

$$X_{22}^{l}=2c_2^2\pi i\left(\left(-\frac{(2+2l)(i\beta_2)^{1+2l}}{4\beta_2^2\left(w-\sqrt{m^2-\beta_2^2}+\frac{\beta_2^2}{2n}-n\right)}+\frac{2(i\beta_2)^{2+2l}}{18\beta_2^3\left(w-\sqrt{m^2-\beta_2^2}+\frac{\beta_2^2}{2n}-n\right)}-\frac{\left(\frac{1}{\sqrt{m^2-\beta_2^2}}+\frac{1}{n}\right)(i\beta_2)^{3+2l}}{4\beta_2^2\left(w-\sqrt{m^2-\beta_2^2}+\frac{\beta_2^2}{2n}-n\right)^2}\right)\frac{n\left(\sqrt{2}\sqrt{nw-\sqrt{m^2n^2-n^4+2n^3w}}\right)^{1+2l}\sqrt{\left(\sqrt{2}\sqrt{nw-\sqrt{m^2n^2-n^4+2n^3w}}\right)^2+m^2}}{\left(\left(\sqrt{2}\sqrt{nw-\sqrt{m^2n^2-n^4+2n^3w}}\right)^2+\beta_2^2\right)^2\left(n+\sqrt{\left(\sqrt{2}\sqrt{nw-\sqrt{m^2n^2-n^4+2n^3w}}\right)^2+m^2}\right)}\;\frac{n\left(\sqrt{2}\sqrt{nw+\sqrt{m^2n^2-n^4+2n^3w}}\right)^{1+2l}\sqrt{\left(\sqrt{2}\sqrt{nw+\sqrt{m^2n^2-n^4+2n^3w}}\right)^2+m^2}}{\left(\left(\sqrt{2}\sqrt{nw+\sqrt{m^2n^2-n^4+2n^3w}}\right)^2+\beta_2^2\right)^2\left(n+\sqrt{\left(\sqrt{2}\sqrt{nw+\sqrt{m^2n^2-n^4+2n^3w}}\right)^2+m^2}\right)}\right)$$

(4.2.4.1)

Dengan menggantikan $l=0$ maka kita peroleh

$$X_{22}^{0}=2c_2^2\pi i\left(\left(-\frac{2(i\beta_2)}{4\beta_2^2\left(w-\sqrt{m^2-\beta_2^2}+\frac{\beta_2^2}{2n}-n\right)}+\frac{2(i\beta_2)^{2}}{18\beta_2^3\left(w-\sqrt{m^2-\beta_2^2}+\frac{\beta_2^2}{2n}-n\right)}-\frac{\left(\frac{1}{\sqrt{m^2-\beta_2^2}}+\frac{1}{n}\right)(i\beta_2)^{3}}{4\beta_2^2\left(w-\sqrt{m^2-\beta_2^2}+\frac{\beta_2^2}{2n}-n\right)^2}\right)\frac{n\left(\sqrt{2}\sqrt{nw-\sqrt{m^2n^2-n^4+2n^3w}}\right)\sqrt{\left(\sqrt{2}\sqrt{nw-\sqrt{m^2n^2-n^4+2n^3w}}\right)^2+m^2}}{\left(\left(\sqrt{2}\sqrt{nw-\sqrt{m^2n^2-n^4+2n^3w}}\right)^2+\beta_2^2\right)^2\left(n+\sqrt{\left(\sqrt{2}\sqrt{nw-\sqrt{m^2n^2-n^4+2n^3w}}\right)^2+m^2}\right)}\;\frac{n\left(\sqrt{2}\sqrt{nw+\sqrt{m^2n^2-n^4+2n^3w}}\right)\sqrt{\left(\sqrt{2}\sqrt{nw+\sqrt{m^2n^2-n^4+2n^3w}}\right)^2+m^2}}{\left(\left(\sqrt{2}\sqrt{nw+\sqrt{m^2n^2-n^4+2n^3w}}\right)^2+\beta_2^2\right)^2\left(n+\sqrt{\left(\sqrt{2}\sqrt{nw+\sqrt{m^2n^2-n^4+2n^3w}}\right)^2+m^2}\right)}\right)$$

$$X^{0}_{22}=2c_2^{\,2}\pi i\left(\left(\frac{-9i-2}{18\beta_2\left(w-\sqrt{m^2-\beta_2^{\,2}}+\frac{\beta_2^2}{2n}-n\right)}+\frac{\left(\frac{1}{\sqrt{m^2-\beta_2^{\,2}}}+\frac{1}{n}\right)i\beta_2}{4\left(w-\sqrt{m^2-\beta_2^{\,2}}+\frac{\beta_2^2}{2n}-n\right)^2}\right)-\frac{n\left(\sqrt{2}\sqrt{nw-\sqrt{m^2n^2-n^4+2n^3w}}\right)\sqrt{\left(\sqrt{2}\sqrt{nw-\sqrt{m^2n^2-n^4+2n^3w}}\right)^2+m^2}}{\left(\left(\sqrt{2}\sqrt{nw-\sqrt{m^2n^2-n^4+2n^3w}}\right)^2+\beta_2^{\,2}\right)^2\left(n+\sqrt{\left(\sqrt{2}\sqrt{nw-\sqrt{m^2n^2-n^4+2n^3w}}\right)^2+m^2}\right)}-\frac{n\left(\sqrt{2}\sqrt{nw+\sqrt{m^2n^2-n^4+2n^3w}}\right)\sqrt{\left(\sqrt{2}\sqrt{nw+\sqrt{m^2n^2-n^4+2n^3w}}\right)^2+m^2}}{\left(\left(\sqrt{2}\sqrt{nw+\sqrt{m^2n^2-n^4+2n^3w}}\right)^2+\beta_2^{\,2}\right)^2\left(n+\sqrt{\left(\sqrt{2}\sqrt{nw+\sqrt{m^2n^2-n^4+2n^3w}}\right)^2+m^2}\right)}\right)$$

Sehingga,

$$X^{0}_{22}=2c_2^{\,2}\pi i\left(\left(\frac{-2(9i+2)\left(w-\sqrt{m^2-\beta_2^{\,2}}+\frac{\beta_2^2}{2n}-n\right)+9\left(\frac{1}{\sqrt{m^2-\beta_2^{\,2}}}+\frac{1}{n}\right)i\beta_2^{\,2}}{36\beta_2\left(w-\sqrt{m^2-\beta_2^{\,2}}+\frac{\beta_2^2}{2n}-n\right)^2}\right)\frac{n\left(\sqrt{2}\sqrt{nw-\sqrt{m^2n^2-n^4+2n^3w}}\right)\sqrt{\left(\sqrt{2}\sqrt{nw-\sqrt{m^2n^2-n^4+2n^3w}}\right)^2+m^2}}{\left(\left(\sqrt{2}\sqrt{nw-\sqrt{m^2n^2-n^4+2n^3w}}\right)^2+\beta_2^{\,2}\right)^2\left(n+\sqrt{\left(\sqrt{2}\sqrt{nw-\sqrt{m^2n^2-n^4+2n^3w}}\right)^2+m^2}\right)}-\frac{n\left(\sqrt{2}\sqrt{nw+\sqrt{m^2n^2-n^4+2n^3w}}\right)\sqrt{\left(\sqrt{2}\sqrt{nw+\sqrt{m^2n^2-n^4+2n^3w}}\right)^2+m^2}}{\left(\left(\sqrt{2}\sqrt{nw+\sqrt{m^2n^2-n^4+2n^3w}}\right)^2+\beta_2^{\,2}\right)^2\left(n+\sqrt{\left(\sqrt{2}\sqrt{nw+\sqrt{m^2n^2-n^4+2n^3w}}\right)^2+m^2}\right)}\right)$$

(4.2.4.2)

5. KESIMPULAN DAN SARAN

Telah dilakukan perhitungan amplitudo transisi hamburan kaon-nukleon dengan menggunakan potensial separabel rank-1 dan rank-2 untuk sembarang gelombang parsial. Perhitungan tersebut menunjukkan bahwa penggunakan potensial separabel membuat amplitudo transisi dapat dihitung secara sederhana dan analitik. Meskipun demikian, perhitungan tersebut tetap menyisakan beberapa parameter pada amplitudo transisi yang harus ditentukan melalui *fitting* dengan data eksperimen.

Amplitudo transisi dari potensial separabel rank-1 memiliki 2 parameter dan 6 *pole*. Sedangkan amplitudo transisi dari rank-2 memiliki 4 parameter dan 6 serta 8 *pole*. Jika dikaitkan dengan diagram-diagram Feynman, *pole-pole* tersebut dapat diinterpretasikan sebagai propagator-propagator pada diagram-diagram tersebut. Hal itu sesuai dengan asal *pole* tersebut yang bermula dari fungsi Green pada persamaan Lippmann-Schwinger. Parameter-parameter yang disebutkan tadi berasal dari faktor-*g* pada bentuk potensial separabel. Faktor ini bersesuaian dengan faktor bentuk yang biasa disisipkan pada verteks suatu diagram Feynman, yang biasanya berbentuk dipol.

Perhitungan ini perlu dilanjutkan dengan melakukan *fitting* untuk menentukan nilai parameter-parameter yang ada pada amplitudo transisi. *Fitting* bisa dilakukan dengan menggunakan data pergeseran fase dari hamburan kaon-nukleon ataupun observabel lain, seperti penampang lintang sebagian, total, dan polarisasi. Setelah nilai-nilai tersebut diketahui, amplitudo tersebut dapat digunakan untuk menghitung sektor kaon-nukleon pada suatu reaksi lain yang memerlukannya.

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Lampiran 1

Akan dicari residu dari $f(k_1)$ pada *double pole* $k_1 = i\beta$

$$\frac{1}{(2-1)!}\left\{\frac{d^{2-1}}{dk^{2-1}}\left((k-i\beta)^2 \cdot \frac{k^{2+2l}}{(k^2+\beta^2)^2\left(w-\sqrt{k^2+m^2}-\frac{k^2}{2n}-n\right)}\right)\right\}_{k=i\beta}$$

$$=\frac{d}{dk}\left\{(k-i\beta)^2 \cdot \frac{k^{2+2l}}{(k^2+\beta^2)^2\left(w-\sqrt{k^2+m^2}-\frac{k^2}{2n}-n\right)}\right\}_{k=i\beta} \qquad (3.1.1.1)$$

Karena

$$\frac{(k-i\beta)^2}{(k^2+\beta^2)^2}=\frac{(k-i\beta)^2}{[(k+i\beta)(k-i\beta)]^2}=\frac{1}{(k+i\beta)^2}$$

Maka persamaan (3.1.1.1) menjadi

$$=\frac{d}{dk}\left\{\frac{k^{2+2l}}{(k+i\beta)^2\left(w-\sqrt{k^2+m^2}-\frac{k^2}{2n}-n\right)}\right\}_{k=i\beta}$$

Akan diselesaikan dengan diferensiasi logaritmik, sehingga dengan memisalkan

$$y=\frac{k^{2+2l}}{(k+i\beta)^2\left(w-\sqrt{k^2+m^2}-\frac{k^2}{2n}-n\right)}$$

diperoleh

$$\frac{1}{y}y' = \frac{1}{k^{2+2l}}(2+2l)k^{1+2l} - \frac{2}{k+i\beta} - \frac{-\frac{1}{2}\left(k^2+m^2\right)^{-\frac{1}{2}}2k - \frac{2k}{2n}}{w - \sqrt{k^2+m^2} - \frac{k^2}{2n} - n}$$

$$\frac{1}{y}y' = \frac{2+2l}{k} - \frac{2}{k+i\beta} - \frac{-\frac{k}{\sqrt{k^2+m^2}} - \frac{k}{n}}{w - \sqrt{k^2+m^2} - \frac{k^2}{2n} - n}$$

$$y' = \left(\frac{2+2l}{k} - \frac{2}{k+i\beta} + \frac{\frac{k}{\sqrt{k^2+m^2}} + \frac{k}{n}}{w - \sqrt{k^2+m^2} - \frac{k^2}{2n} - n}\right)\frac{k^{2+2l}}{(k+i\beta)^2\left(w - \sqrt{k^2+m^2} - \frac{k^2}{2n} - n\right)}$$

$$y' = \frac{(2+2l)k^{1+2l}}{(k+i\beta)^2\left(w - \sqrt{k^2+m^2} - \frac{k^2}{2n} - n\right)} - \frac{2k^{2+2l}}{(k+i\beta)^3\left(w - \sqrt{k^2+m^2} - \frac{k^2}{2n} - n\right)}$$
$$+ \frac{\left(\frac{k}{\sqrt{k^2+m^2}} + \frac{k}{n}\right)k^{2+2l}}{(k+i\beta)^2\left(w - \sqrt{k^2+m^2} - \frac{k^2}{2n} - n\right)^2}$$

$$y'\big|_{k=i\beta} = \frac{(2+2l)(i\beta)^{1+2l}}{-4\beta^2\left(w - \sqrt{m^2-\beta^2} + \frac{\beta^2}{2n} - n\right)} - \frac{2(i\beta)^{2+2l}}{-18\beta^3\left(w - \sqrt{m^2-\beta^2} + \frac{\beta^2}{2n} - n\right)}$$
$$+ \frac{\left(\frac{i\beta}{\sqrt{m^2-\beta^2}} + \frac{i\beta}{n}\right)(i\beta)^{2+2l}}{-4\beta^2\left(w - \sqrt{m^2-\beta^2} + \frac{\beta^2}{2n} - n\right)^2}.$$

Sehingga residu dari $f(k_1)$ pada *double pole* $k_1 = i\beta$ adalah

$$= -\frac{(2+2l)(i\beta)^{1+2l}}{4\beta^2\left(w - \sqrt{m^2-\beta^2} + \frac{\beta^2}{2n} - n\right)} + \frac{2(i\beta)^{2+2l}}{18\beta^3\left(w - \sqrt{m^2-\beta^2} + \frac{\beta^2}{2n} - n\right)}$$
$$- \frac{\left(\frac{i\beta}{\sqrt{m^2-\beta^2}} + \frac{i\beta}{n}\right)(i\beta)^{2+2l}}{4\beta^2\left(w - \sqrt{m^2-\beta^2} + \frac{\beta^2}{2n} - n\right)^2}. \quad (3.1.1.3)$$

Lampiran 2.
Akan dicari residu dari $f(k_2)$ pada *double pole* $k_2 = -i\beta$

$$\frac{1}{(2-1)!}\left\{\frac{d^{2-1}}{dk^{2-1}}\left((k+i\beta)^2 \cdot \frac{k^{2+2l}}{(k^2+\beta^2)^2\left(w-\sqrt{k^2+m^2}-\frac{k^2}{2n}-n\right)}\right)\right\}_{k=-i\beta}$$

$$=\frac{d}{dk}\left\{(k+i\beta)^2 \cdot \frac{k^{2+2l}}{(k^2+\beta^2)^2\left(w-\sqrt{k^2+m^2}-\frac{k^2}{2n}-n\right)}\right\}_{k=-i\beta} \qquad (3.1.1.4)$$

Karena

$$\frac{(k+i\beta)^2}{(k^2+\beta^2)^2} = \frac{(k+i\beta)^2}{[(k+i\beta)(k-i\beta)]^2} = \frac{1}{(k-i\beta)^2}$$

maka persamaan (3.1.1.4) menjadi

$$=\frac{d}{dk}\left\{\frac{k^{2+2l}}{(k-i\beta)^2\left(w-\sqrt{k^2+m^2}-\frac{k^2}{2n}-n\right)}\right\}_{k=-i\beta}$$

Akan diselesaikan dengan diferensiasi logaritmik, sehingga dengan memisalkan

$$y=\frac{k^{2+2l}}{(k-i\beta)^2\left(w-\sqrt{k^2+m^2}-\frac{k^2}{2n}-n\right)}$$

diperoleh

$$\frac{1}{y}y' = \frac{1}{k^{2+2l}}(2+2l)k^{1+2l} - \frac{2}{k-i\beta} - \frac{-\frac{1}{2}\left(k^2+m^2\right)^{-\frac{1}{2}}2k - \frac{2k}{2n}}{w-\sqrt{k^2+m^2}-\frac{k^2}{2n}-n}$$

$$\frac{1}{y}y' = \frac{2+2l}{k} - \frac{2}{k-i\beta} - \frac{-\frac{k}{\sqrt{k^2+m^2}}-\frac{k}{n}}{w-\sqrt{k^2+m^2}-\frac{k^2}{2n}-n}$$

$$y' = \left(\frac{2+2l}{k} - \frac{2}{k-i\beta} + \frac{\frac{k}{\sqrt{k^2+m^2}}+\frac{k}{n}}{w-\sqrt{k^2+m^2}-\frac{k^2}{2n}-n}\right)\frac{k^{2+2l}}{(k-i\beta)^2\left(w-\sqrt{k^2+m^2}-\frac{k^2}{2n}-n\right)}$$

$$y' = \frac{(2+2l)k^{1+2l}}{(k-i\beta)^2\left(w-\sqrt{k^2+m^2}-\frac{k^2}{2n}-n\right)} - \frac{2k^{2+2l}}{(k-i\beta)^3\left(w-\sqrt{k^2+m^2}-\frac{k^2}{2n}-n\right)}$$
$$+\frac{\left(\frac{k}{\sqrt{k^2+m^2}}+\frac{k}{n}\right)k^{2+2l}}{(k-i\beta)^2\left(w-\sqrt{k^2+m^2}-\frac{k^2}{2n}-n\right)^2}$$

$$y'\big|_{k=-i\beta} = \frac{(2+2l)(-i\beta)^{1+2l}}{-4\beta^2\left(w-\sqrt{m^2-\beta^2}-\frac{\beta^2}{2n}-n\right)} - \frac{2(-i\beta)^{2+2l}}{-18\beta^3\left(w-\sqrt{m^2-\beta^2}-\frac{\beta^2}{2n}-n\right)}$$
$$+\frac{\left(+\frac{i\beta}{\sqrt{m^2-\beta^2}}+\frac{i\beta}{n}\right)(-i\beta)^{2+2l}}{-4\beta^2\left(w-\sqrt{m^2-\beta^2}-\frac{\beta^2}{2n}-n\right)^2}.$$

$$= -\frac{(2+2l)(-i\beta)^{1+2l}}{4\beta^2\left(w-\sqrt{m^2-\beta^2}+\frac{\beta^2}{2n}-n\right)} + \frac{2(-i\beta)^{2+2l}}{18\beta^3\left(w-\sqrt{m^2-\beta^2}+\frac{\beta^2}{2n}-n\right)}$$
$$-\frac{\left(\frac{i\beta}{\sqrt{m^2-\beta^2}}+\frac{i\beta}{n}\right)(-i\beta)^{2+2l}}{4\beta^2\left(w-\sqrt{m^2-\beta^2}+\frac{\beta^2}{2n}-n\right)^2}.$$

Sehingga residu dari $f(k_2)$ pada *double pole* $k=-i\beta$ adalah

$$= -\frac{(2+2l)(-i\beta)^{1+2l}}{4\beta^2\left(w-\sqrt{m^2-\beta^2}+\frac{\beta^2}{2n}-n\right)} + \frac{2(-i\beta)^{2+2l}}{18\beta^3\left(w-\sqrt{m^2-\beta^2}+\frac{\beta^2}{2n}-n\right)}$$

$$-\frac{\left(\frac{i\beta}{\sqrt{m^2-\beta^2}}+\frac{i\beta}{n}\right)(-i\beta)^{2+2l}}{4\beta^2\left(w-\sqrt{m^2-\beta^2}+\frac{\beta^2}{2n}-n\right)^2}. \quad (3.1.1.5)$$

Lampiran 3.

Akan dicari residu dari $f(k_3)$ pada *simple pole* $k_3 = -\sqrt{2}\sqrt{nw - \sqrt{m^2n^2 - n^4 + 2n^3w}}$

$$\text{Res } f(k) = \lim_{k \to -\sqrt{2}\sqrt{nw - \sqrt{m^2n^2 - n^4 + 2n^3w}}} \left(k + \sqrt{2}\sqrt{nw - \sqrt{m^2n^2 - n^4 + 2n^3w}} \right) \cdot \frac{k^{2+2l}}{\left(k^2+\beta^2\right)^2 \left(w - \sqrt{k^2+m^2} - \frac{k^2}{2n} - n \right)}$$

$$= \lim_{k \to -\sqrt{2}\sqrt{nw - \sqrt{m^2n^2 - n^4 + 2n^3w}}} \frac{k^{3+2l} + k^{2+2l}\sqrt{2}\sqrt{nw - \sqrt{m^2n^2 - n^4 + 2n^3w}}}{\left(k^2+\beta^2\right)^2 \left(w - \sqrt{k^2+m^2} - \frac{k^2}{2n} - n \right)}$$

Dengan menggunakan kaidah L'Hospital, maka dengan memisalkan

$$f(x) = k^{2+2l}\left(k + \sqrt{2}\sqrt{nw - \sqrt{m^2n^2 - n^4 + 2n^3w}} \right)$$

$$g(x) = \left(k^2+\beta^2\right)^2 \left(w - \sqrt{k^2+m^2} - \frac{k^2}{2n} - n \right)$$

Maka

$$f'(x) = (2+2l)k^{1+2l}\left(k + \sqrt{2}\sqrt{nw - \sqrt{m^2n^2 - n^4 + 2n^3w}} \right) + k^{2+2l}$$

$$g'(x) = 2\left(k^2+\beta^2\right)2k\left(w - \sqrt{k^2+m^2} - \frac{k^2}{2n} - n \right) + \left(k^2+\beta^2\right)^2\left(-\frac{1}{2}\frac{1}{\sqrt{k^2+m^2}}2k - \frac{2k}{2n} \right)$$

$$g'(x) = 4k\left(k^2+\beta^2\right)\left(w - \sqrt{k^2+m^2} - \frac{k^2}{2n} - n \right) - \left(k^2+\beta^2\right)^2\left(\frac{k}{\sqrt{k^2+m^2}} + \frac{k}{n} \right)$$

Sehingga

$$\textbf{Res}\ f(k)=\lim_{k\to-\sqrt{2}\sqrt{nw-\sqrt{m^2n^2-n^4+2n^3w}}}\frac{\left((2+2l)k^{1+2l}\left(k+\sqrt{2}\sqrt{nw-\sqrt{m^2n^2-n^4+2n^3w}}\right)+k^{2+2l}\right)}{\left(4k\left(k^2+\beta^2\right)\left(w-\sqrt{k^2+m^2}-\frac{k^2}{2n}-n\right)-\left(k^2+\beta^2\right)^2\left(\frac{k}{\sqrt{k^2+m^2}}+\frac{k}{n}\right)\right)}$$

$$=\frac{(2+2l)\left(-\sqrt{2}\sqrt{nw-\sqrt{m^2n^2-n^4+2n^3w}}\right)^{1+2l}(0)+\left(-\sqrt{2}\sqrt{nw-\sqrt{m^2n^2-n^4+2n^3w}}\right)^{2+2l}}{\left(\begin{array}{l}4\left(-\sqrt{2}\sqrt{nw-\sqrt{m^2n^2-n^4+2n^3w}}\right)\left(\left(-\sqrt{2}\sqrt{nw-\sqrt{m^2n^2-n^4+2n^3w}}\right)^2+\beta^2\right)(0)-\\ \left(\left(-\sqrt{2}\sqrt{nw-\sqrt{m^2n^2-n^4+2n^3w}}\right)\left(-\sqrt{2}\sqrt{nw-\sqrt{m^2n^2-n^4+2n^3w}}\right)^2+\beta^2\right)^2\\ \left(\frac{1}{\sqrt{\left(-\sqrt{2}\sqrt{nw-\sqrt{m^2n^2-n^4+2n^3w}}\right)^2+m^2}}+\frac{1}{n}\right)\end{array}\right)}$$

$$=\frac{\left(-\sqrt{2}\sqrt{nw-\sqrt{m^2n^2-n^4+2n^3w}}\right)^{2+2l}}{\left(\begin{array}{l}-\left(\left(-\sqrt{2}\sqrt{nw-\sqrt{m^2n^2-n^4+2n^3w}}\right)\left(-\sqrt{2}\sqrt{nw-\sqrt{m^2n^2-n^4+2n^3w}}\right)^2+\beta^2\right)^2\\ \left(\frac{n+\sqrt{\left(-\sqrt{2}\sqrt{nw-\sqrt{m^2n^2-n^4+2n^3w}}\right)^2+m^2}}{n\sqrt{\left(-\sqrt{2}\sqrt{nw-\sqrt{m^2n^2-n^4+2n^3w}}\right)^2+m^2}}\right)\end{array}\right)}$$

$$=-\frac{n\left(-\sqrt{2}\sqrt{nw-\sqrt{m^2n^2-n^4+2n^3w}}\right)^{1+2l}\sqrt{\left(-\sqrt{2}\sqrt{nw-\sqrt{m^2n^2-n^4+2n^3w}}\right)^2+m^2}}{\left(\left(-\sqrt{2}\sqrt{nw-\sqrt{m^2n^2-n^4+2n^3w}}\right)^2+\beta^2\right)^2\left(n+\sqrt{\left(-\sqrt{2}\sqrt{nw-\sqrt{m^2n^2-n^4+2n^3w}}\right)^2+m^2}\right)}$$

$$= \frac{n\left(\sqrt{2}\sqrt{nw-\sqrt{m^2n^2-n^4+2n^3w}}\right)^{1+2l}\sqrt{\left(\sqrt{2}\sqrt{nw-\sqrt{m^2n^2-n^4+2n^3w}}\right)^2+m^2}}{\left(\left(\sqrt{2}\sqrt{nw-\sqrt{m^2n^2-n^4+2n^3w}}\right)^2+\beta^2\right)^2\left(n+\sqrt{\left(\sqrt{2}\sqrt{nw-\sqrt{m^2n^2-n^4+2n^3w}}\right)^2+m^2}\right)}$$

Sehingga residu dari $f(k_3)$ pada *simple pole* $k_3 = -\sqrt{2}\sqrt{nw-\sqrt{m^2n^2-n^4+2n^3w}}$ adalah

$$\frac{n\left(\sqrt{2}\sqrt{nw-\sqrt{m^2n^2-n^4+2n^3w}}\right)^{1+2l}\sqrt{\left(\sqrt{2}\sqrt{nw-\sqrt{m^2n^2-n^4+2n^3w}}\right)^2+m^2}}{\left(\left(\sqrt{2}\sqrt{nw-\sqrt{m^2n^2-n^4+2n^3w}}\right)^2+\beta^2\right)^2\left(n+\sqrt{\left(\sqrt{2}\sqrt{nw-\sqrt{m^2n^2-n^4+2n^3w}}\right)^2+m^2}\right)} \quad (3.1.1.6)$$

Lampiran 4.

Akan dicari residu dari $f(k_4)$ pada *simple pole* $k_4 = \sqrt{2}\sqrt{nw - \sqrt{m^2 n^2 - n^4 + 2n^3 w}}$

$$\textbf{Res}\ f(k) = \lim_{k \to \sqrt{2}\sqrt{nw - \sqrt{m^2 n^2 - n^4 + 2n^3 w}}} \left(k - \sqrt{2}\sqrt{nw - \sqrt{m^2 n^2 - n^4 + 2n^3 w}} \right) \cdot \frac{k^{2+2l}}{\left(k^2 + \beta^2\right)^2 \left(w - \sqrt{k^2 + m^2} - \frac{k^2}{2n} - n \right)}$$

$$= \lim_{k \to \sqrt{2}\sqrt{nw - \sqrt{m^2 n^2 - n^4 + 2n^3 w}}} \frac{k^{3+2l} - k^{2+2l}\sqrt{2}\sqrt{nw - \sqrt{m^2 n^2 - n^4 + 2n^3 w}}}{\left(k^2 + \beta^2\right)^2 \left(w - \sqrt{k^2 + m^2} - \frac{k^2}{2n} - n \right)}$$

Dengan menggunakan kaidah L'Hospital, maka dengan memisalkan

$$f(x) = k^{2+2l}\left(k - \sqrt{2}\sqrt{nw - \sqrt{m^2 n^2 - n^4 + 2n^3 w}} \right)$$

$$g(x) = \left(k^2 + \beta^2\right)^2 \left(w - \sqrt{k^2 + m^2} - \frac{k^2}{2n} - n \right)$$

Maka

$$f'(x) = (2 + 2l)k^{1+2l}\left(k - \sqrt{2}\sqrt{nw - \sqrt{m^2 n^2 - n^4 + 2n^3 w}} \right) + k^{2+2l}$$

$$g'(x) = 2\left(k^2 + \beta^2\right)2k\left(w - \sqrt{k^2 + m^2} - \frac{k^2}{2n} - n \right) + \left(k^2 + \beta^2\right)^2 \left(-\frac{1}{2}\frac{1}{\sqrt{k^2 + m^2}}2k - \frac{2k}{2n} \right)$$

$$g'(x) = 4k\left(k^2 + \beta^2\right)\left(w - \sqrt{k^2 + m^2} - \frac{k^2}{2n} - n \right) - \left(k^2 + \beta^2\right)^2 \left(\frac{k}{\sqrt{k^2 + m^2}} + \frac{k}{n} \right)$$

Sehingga

$$\mathbf{Res}\,f(k)=\lim_{k\to\sqrt{2}\sqrt{nw-\sqrt{m^2n^2-n^4+2n^3w}}}\frac{\left((2+2l)k^{1+2l}\left(k-\sqrt{2}\sqrt{nw-\sqrt{m^2n^2-n^4+2n^3w}}\right)+k^{2+2l}\right)}{\left(4k\left(k^2+\beta^2\right)\left(w-\sqrt{k^2+m^2}-\frac{k^2}{2n}-n\right)-\left(k^2+\beta^2\right)^2\left(\frac{k}{\sqrt{k^2+m^2}}+\frac{k}{n}\right)\right)}$$

$$=\frac{(2+2l)\left(\sqrt{2}\sqrt{nw-\sqrt{m^2n^2-n^4+2n^3w}}\right)^{1+2l}(0)+\left(\sqrt{2}\sqrt{nw-\sqrt{m^2n^2-n^4+2n^3w}}\right)^{2+2l}}{\left(\begin{array}{l}4\left(\sqrt{2}\sqrt{nw-\sqrt{m^2n^2-n^4+2n^3w}}\right)\left(\left(\sqrt{2}\sqrt{nw-\sqrt{m^2n^2-n^4+2n^3w}}\right)^2+\beta^2\right)(0)-\\ \left(\left(\sqrt{2}\sqrt{nw-\sqrt{m^2n^2-n^4+2n^3w}}\right)\left(\sqrt{2}\sqrt{nw-\sqrt{m^2n^2-n^4+2n^3w}}\right)^2+\beta^2\right)^2\\ \left(\frac{1}{\sqrt{\left(\sqrt{2}\sqrt{nw-\sqrt{m^2n^2-n^4+2n^3w}}\right)^2+m^2}}+\frac{1}{n}\right)\end{array}\right)}$$

$$=\frac{\left(\sqrt{2}\sqrt{nw-\sqrt{m^2n^2-n^4+2n^3w}}\right)^{2+2l}}{\left(\begin{array}{l}-\left(\left(\sqrt{2}\sqrt{nw-\sqrt{m^2n^2-n^4+2n^3w}}\right)\left(\sqrt{2}\sqrt{nw-\sqrt{m^2n^2-n^4+2n^3w}}\right)^2+\beta^2\right)^2\\ \left(\frac{n+\sqrt{\left(\sqrt{2}\sqrt{nw-\sqrt{m^2n^2-n^4+2n^3w}}\right)^2+m^2}}{n\sqrt{\left(\sqrt{2}\sqrt{nw-\sqrt{m^2n^2-n^4+2n^3w}}\right)^2+m^2}}\right)\end{array}\right)}$$

$$= -\frac{n\left(\sqrt{2}\sqrt{nw-\sqrt{m^2n^2-n^4+2n^3w}}\right)^{1+2l}\sqrt{\left(\sqrt{2}\sqrt{nw-\sqrt{m^2n^2-n^4+2n^3w}}\right)^2+m^2}}{\left(\left(\sqrt{2}\sqrt{nw-\sqrt{m^2n^2-n^4+2n^3w}}\right)^2+\beta^2\right)^2\left(n+\sqrt{\left(\sqrt{2}\sqrt{nw-\sqrt{m^2n^2-n^4+2n^3w}}\right)^2+m^2}\right)}$$

$$= -\frac{n\left(\sqrt{2}\sqrt{nw-\sqrt{m^2n^2-n^4+2n^3w}}\right)^{1+2l}\sqrt{\left(\sqrt{2}\sqrt{nw-\sqrt{m^2n^2-n^4+2n^3w}}\right)^2+m^2}}{\left(\left(\sqrt{2}\sqrt{nw-\sqrt{m^2n^2-n^4+2n^3w}}\right)^2+\beta^2\right)^2\left(n+\sqrt{\left(\sqrt{2}\sqrt{nw-\sqrt{m^2n^2-n^4+2n^3w}}\right)^2+m^2}\right)}$$

Sehingga residu dari $f(k_4)$ pada *simple pole* $k_4 = \sqrt{2}\sqrt{nw-\sqrt{m^2n^2-n^4+2n^3w}}$ adalah

$$-\frac{n\left(\sqrt{2}\sqrt{nw-\sqrt{m^2n^2-n^4+2n^3w}}\right)^{1+2l}\sqrt{\left(\sqrt{2}\sqrt{nw-\sqrt{m^2n^2-n^4+2n^3w}}\right)^2+m^2}}{\left(\left(\sqrt{2}\sqrt{nw-\sqrt{m^2n^2-n^4+2n^3w}}\right)^2+\beta^2\right)^2\left(n+\sqrt{\left(\sqrt{2}\sqrt{nw-\sqrt{m^2n^2-n^4+2n^3w}}\right)^2+m^2}\right)} \tag{3.1.1.7}$$

Lampiran 5.

Akan dicari residu dari $f(k_5)$ pada *simple pole*

$$k_5 = -\sqrt{2}\sqrt{nw + \sqrt{m^2n^2 - n^4 + 2n^3w}}$$

$$\textbf{Res}\ f(k) = \lim_{k \to -\sqrt{2}\sqrt{nw + \sqrt{m^2n^2 - n^4 + 2n^3w}}} \left(k + \sqrt{2}\sqrt{nw + \sqrt{m^2n^2 - n^4 + 2n^3w}}\right)$$

$$\cdot \frac{k^{2+2l}}{\left(k^2 + \beta^2\right)^2 \left(w - \sqrt{k^2 + m^2} - \frac{k^2}{2n} - n\right)}$$

$$= \lim_{k \to -\sqrt{2}\sqrt{nw + \sqrt{m^2n^2 - n^4 + 2n^3w}}} \frac{k^{3+2l} + k^{2+2l}\sqrt{2}\sqrt{nw + \sqrt{m^2n^2 - n^4 + 2n^3w}}}{\left(k^2 + \beta^2\right)^2 \left(w - \sqrt{k^2 + m^2} - \frac{k^2}{2n} - n\right)}$$

Dengan menggunakan kaidah L'Hospital, maka dengan memisalkan

$$f(x) = k^{2+2l}\left(k + \sqrt{2}\sqrt{nw + \sqrt{m^2n^2 - n^4 + 2n^3w}}\right)$$

$$g(x) = \left(k^2 + \beta^2\right)^2 \left(w - \sqrt{k^2 + m^2} - \frac{k^2}{2n} - n\right)$$

Maka

$$f'(x) = (2 + 2l)k^{1+2l}\left(k + \sqrt{2}\sqrt{nw + \sqrt{m^2n^2 - n^4 + 2n^3w}}\right) + k^{2+2l}$$

$$g'(x) = 2\left(k^2 + \beta^2\right)2k\left(w - \sqrt{k^2 + m^2} - \frac{k^2}{2n} - n\right) + \left(k^2 + \beta^2\right)^2\left(-\frac{1}{2}\frac{1}{\sqrt{k^2 + m^2}}2k - \frac{2k}{2n}\right)$$

$$g'(x) = 4k\left(k^2 + \beta^2\right)\left(w - \sqrt{k^2 + m^2} - \frac{k^2}{2n} - n\right) - \left(k^2 + \beta^2\right)^2\left(\frac{k}{\sqrt{k^2 + m^2}} + \frac{k}{n}\right)$$

Sehingga

$$\mathbf{Res}f(k)=\lim_{k\to-\sqrt{2}\sqrt{nw+\sqrt{m^2n^2-n^4+2n^3w}}}\frac{\left((2+2l)k^{1+2l}\left(k+\sqrt{2}\sqrt{nw+\sqrt{m^2n^2-n^4+2n^3w}}\right)+k^{2+2l}\right)}{\left(4k\left(k^2+\beta^2\right)\left(w-\sqrt{k^2+m^2}-\frac{k^2}{2n}-n\right)-\left(k^2+\beta^2\right)^2\left(\frac{k}{\sqrt{k^2+m^2}}+\frac{k}{n}\right)\right)}$$

$$\mathrm{Res}f(k)=\frac{\left((2+2l)\left(-\sqrt{2}\sqrt{nw+\sqrt{m^2n^2-n^4+2n^3w}}\right)^{1+2l}(0)+\left(-\sqrt{2}\sqrt{nw+\sqrt{m^2n^2-n^4+2n^3w}}\right)^{2+2l}\right)}{\left(4k\left(k^2+\beta^2\right)(0)-\left(\left(-\sqrt{2}\sqrt{nw+\sqrt{m^2n^2-n^4+2n^3w}}\right)\left(-\sqrt{2}\sqrt{nw+\sqrt{m^2n^2-n^4+2n^3w}}\right)^2+\beta^2\right)^2\left(\frac{1}{\sqrt{\left(-\sqrt{2}\sqrt{nw+\sqrt{m^2n^2-n^4+2n^3w}}\right)^2+m^2}}+\frac{1}{n}\right)\right)}$$

$$=\frac{\left(-\sqrt{2}\sqrt{nw+\sqrt{m^2n^2-n^4+2n^3w}}\right)^{2+2l}}{\left(-\left(\left(-\sqrt{2}\sqrt{nw+\sqrt{m^2n^2-n^4+2n^3w}}\right)\left(-\sqrt{2}\sqrt{nw+\sqrt{m^2n^2-n^4+2n^3w}}\right)^2+\beta^2\right)^2\left(\frac{n+\sqrt{\left(-\sqrt{2}\sqrt{nw+\sqrt{m^2n^2-n^4+2n^3w}}\right)^2+m^2}}{n\sqrt{\left(-\sqrt{2}\sqrt{nw+\sqrt{m^2n^2-n^4+2n^3w}}\right)^2+m^2}}\right)\right)}$$

$$=-\frac{n\left(-\sqrt{2}\sqrt{nw+\sqrt{m^2n^2-n^4+2n^3w}}\right)^{1+2l}\sqrt{\left(-\sqrt{2}\sqrt{nw+\sqrt{m^2n^2-n^4+2n^3w}}\right)^2+m^2}}{\left(\left(-\sqrt{2}\sqrt{nw+\sqrt{m^2n^2-n^4+2n^3w}}\right)^2+\beta^2\right)^2\left(n+\sqrt{\left(-\sqrt{2}\sqrt{nw+\sqrt{m^2n^2-n^4+2n^3w}}\right)^2+m^2}\right)}$$

$$=\frac{n\left(\sqrt{2}\sqrt{nw+\sqrt{m^2n^2-n^4+2n^3w}}\right)^{1+2l}\sqrt{\left(\sqrt{2}\sqrt{nw+\sqrt{m^2n^2-n^4+2n^3w}}\right)^2+m^2}}{\left(\left(\sqrt{2}\sqrt{nw+\sqrt{m^2n^2-n^4+2n^3w}}\right)^2+\beta^2\right)^2\left(n+\sqrt{\left(\sqrt{2}\sqrt{nw+\sqrt{m^2n^2-n^4+2n^3w}}\right)^2+m^2}\right)}$$

Sehingga residu dari $f(k_5)$ pada *simple pole* $k_5=-\sqrt{2}\sqrt{nw+\sqrt{m^2n^2-n^4+2n^3w}}$ adalah

$$\frac{n\left(\sqrt{2}\sqrt{nw+\sqrt{m^2n^2-n^4+2n^3w}}\right)^{1+2l}\sqrt{\left(\sqrt{2}\sqrt{nw+\sqrt{m^2n^2-n^4+2n^3w}}\right)^2+m^2}}{\left(\left(\sqrt{2}\sqrt{nw+\sqrt{m^2n^2-n^4+2n^3w}}\right)^2+\beta^2\right)^2\left(n+\sqrt{\left(\sqrt{2}\sqrt{nw+\sqrt{m^2n^2-n^4+2n^3w}}\right)^2+m^2}\right)}$$

(3.1.1.8)

Lampiran 6.

Akan dicari residu dari $f(k_6)$ pada *simple pole* $k_6 = \sqrt{2}\sqrt{nw + \sqrt{m^2n^2 - n^4 + 2n^3w}}$

$$\textbf{Res}\, f(k) = \lim_{k \to \sqrt{2}\sqrt{nw + \sqrt{m^2n^2 - n^4 + 2n^3w}}} \left(k - \sqrt{2}\sqrt{nw + \sqrt{m^2n^2 - n^4 + 2n^3w}} \right) \cdot \frac{k^{2+2l}}{(k^2+\beta^2)^2 \left(w - \sqrt{k^2+m^2} - \frac{k^2}{2n} - n \right)}$$

$$= \lim_{k \to \sqrt{2}\sqrt{nw + \sqrt{m^2n^2 - n^4 + 2n^3w}}} \frac{k^{3+2l} - k^{2+2l}\sqrt{2}\sqrt{nw + \sqrt{m^2n^2 - n^4 + 2n^3w}}}{(k^2+\beta^2)^2 \left(w - \sqrt{k^2+m^2} - \frac{k^2}{2n} - n \right)}$$

Dengan menggunakan kaidah L'Hospital, maka dengan memisalkan

$$f(x) = k^{2+2l}\left(k - \sqrt{2}\sqrt{nw + \sqrt{m^2n^2 - n^4 + 2n^3w}} \right)$$

$$g(x) = (k^2+\beta^2)^2 \left(w - \sqrt{k^2+m^2} - \frac{k^2}{2n} - n \right)$$

Maka

$$f'(x) = (2+2l)k^{1+2l}\left(k - \sqrt{2}\sqrt{nw + \sqrt{m^2n^2 - n^4 + 2n^3w}} \right) + k^{2+2l}$$

$$g'(x) = 2(k^2+\beta^2)2k\left(w - \sqrt{k^2+m^2} - \frac{k^2}{2n} - n \right) + (k^2+\beta^2)^2\left(-\frac{1}{2}\frac{1}{\sqrt{k^2+m^2}}2k - \frac{2k}{2n} \right)$$

$$g'(x) = 4k(k^2+\beta^2)\left(w - \sqrt{k^2+m^2} - \frac{k^2}{2n} - n \right) - (k^2+\beta^2)^2\left(\frac{k}{\sqrt{k^2+m^2}} + \frac{k}{n} \right)$$

Sehingga

$$\mathbf{Res}\,f(k)=\lim_{k\to\sqrt{2}\sqrt{nw+\sqrt{m^2n^2-n^4+2n^3w}}}\frac{\left((2+2l)k^{1+2l}\left(k-\sqrt{2}\sqrt{nw+\sqrt{m^2n^2-n^4+2n^3w}}\right)+k^{2+2l}\right)}{\left(4k\left(k^2+\beta^2\right)\left(w-\sqrt{k^2+m^2}-\frac{k^2}{2n}-n\right)-\left(k^2+\beta^2\right)^2\left(\frac{k}{\sqrt{k^2+m^2}}+\frac{k}{n}\right)\right)}$$

$$=\frac{(2+2l)\left(\sqrt{2}\sqrt{nw+\sqrt{m^2n^2-n^4+2n^3w}}\right)^{1+2l}(0)+\left(\sqrt{2}\sqrt{nw+\sqrt{m^2n^2-n^4+2n^3w}}\right)^{2+2l}}{\left(\begin{array}{l}4\left(\sqrt{2}\sqrt{nw+\sqrt{m^2n^2-n^4+2n^3w}}\right)\left(\left(\sqrt{2}\sqrt{nw+\sqrt{m^2n^2-n^4+2n^3w}}\right)^2+\beta^2\right)(0)-\\ \left(\left(\sqrt{2}\sqrt{nw+\sqrt{m^2n^2-n^4+2n^3w}}\right)\left(\sqrt{2}\sqrt{nw+\sqrt{m^2n^2-n^4+2n^3w}}\right)^2+\beta^2\right)^2\\ \left(\frac{1}{\sqrt{\left(\sqrt{2}\sqrt{nw+\sqrt{m^2n^2-n^4+2n^3w}}\right)^2+m^2}}+\frac{1}{n}\right)\end{array}\right)}$$

$$=\frac{\left(\sqrt{2}\sqrt{nw+\sqrt{m^2n^2-n^4+2n^3w}}\right)^{2+2l}}{\left(\begin{array}{l}-\left(\left(\sqrt{2}\sqrt{nw+\sqrt{m^2n^2-n^4+2n^3w}}\right)\left(\sqrt{2}\sqrt{nw+\sqrt{m^2n^2-n^4+2n^3w}}\right)^2+\beta^2\right)^2\\ \left(\frac{n+\sqrt{\left(\sqrt{2}\sqrt{nw+\sqrt{m^2n^2-n^4+2n^3w}}\right)^2+m^2}}{n\sqrt{\left(\sqrt{2}\sqrt{nw+\sqrt{m^2n^2-n^4+2n^3w}}\right)^2+m^2}}\right)\end{array}\right)}$$

$$=-\frac{n\left(\sqrt{2}\sqrt{nw+\sqrt{m^2n^2-n^4+2n^3w}}\right)^{1+2l}\sqrt{\left(\sqrt{2}\sqrt{nw+\sqrt{m^2n^2-n^4+2n^3w}}\right)^2+m^2}}{\left(\left(\sqrt{2}\sqrt{nw+\sqrt{m^2n^2-n^4+2n^3w}}\right)^2+\beta^2\right)^2\left(n+\sqrt{\left(\sqrt{2}\sqrt{nw+\sqrt{m^2n^2-n^4+2n^3w}}\right)^2+m^2}\right)}$$

$$=-\frac{n\left(\sqrt{2}\sqrt{nw+\sqrt{m^2n^2-n^4+2n^3w}}\right)^{1+2l}\sqrt{\left(\sqrt{2}\sqrt{nw+\sqrt{m^2n^2-n^4+2n^3w}}\right)^2+m^2}}{\left(\left(\sqrt{2}\sqrt{nw+\sqrt{m^2n^2-n^4+2n^3w}}\right)^2+\beta^2\right)^2\left(n+\sqrt{\left(\sqrt{2}\sqrt{nw+\sqrt{m^2n^2-n^4+2n^3w}}\right)^2+m^2}\right)}$$

Sehingga residu dari $f(k_6)$ pada *simple pole* $k_6=\sqrt{2}\sqrt{nw+\sqrt{m^2n^2-n^4+2n^3w}}$ adalah

$$-\frac{n\left(\sqrt{2}\sqrt{nw+\sqrt{m^2n^2-n^4+2n^3w}}\right)^{1+2l}\sqrt{\left(\sqrt{2}\sqrt{nw+\sqrt{m^2n^2-n^4+2n^3w}}\right)^2+m^2}}{\left(\left(\sqrt{2}\sqrt{nw+\sqrt{m^2n^2-n^4+2n^3w}}\right)^2+\beta^2\right)^2\left(n+\sqrt{\left(\sqrt{2}\sqrt{nw+\sqrt{m^2n^2-n^4+2n^3w}}\right)^2+m^2}\right)} \quad (3.1.1.9)$$

Lampiran 7.

Akan dicari residu dari $f(k_1)$ pada *simple pole* $k_1 = i\beta_1$

$$\text{Res } f(k) = \lim_{k \to i\beta_1} (k - i\beta_1) \cdot \frac{k^{2+2l}}{(k^2+\beta_1^2)(k^2+\beta_2^2)\left(w - \sqrt{k^2+m_K^2} - \frac{k^2}{2m_N} - m_N\right)}$$

Karena

$$\frac{(k - i\beta_1)}{(k^2+\beta_1^2)} = \frac{(k - i\beta_1)}{(k + i\beta_1)(k - i\beta_1)} = \frac{1}{(k + i\beta_1)}$$

Maka

$$\textbf{Res } f(k) = \lim_{k \to i\beta_1} \frac{k^{2+2l}}{(k + i\beta_1)(k^2+\beta_2^2)\left(w - \sqrt{k^2+m_K^2} - \frac{k^2}{2m_N} - m_N\right)}$$

$$= \frac{(i\beta_1)^{2+2l}}{(2i\beta_1)(-\beta_1^2+\beta_2^2)\left(w - \sqrt{-\beta_1^2+m_K^2} - \frac{-\beta_1^2}{2m_N} - m_N\right)}$$

Sehingga residu dari $f(k_1)$ pada *simple pole* $k_1 = i\beta_1$ adalah

$$= \frac{(i\beta_1)^{2+2l}}{(2i\beta_1)(-\beta_1^2+\beta_2^2)\left(w - \sqrt{-\beta_1^2+m_K^2} - \frac{-\beta_1^2}{2m_N} - m_N\right)} \qquad (3.2.1.1.1.1)$$

Lampiran 8.

Akan dicari residu dari $f(k_2)$ pada *simple pole* $k_2 = -i\beta_1$

$$\textbf{Res}\ f(k) = \lim_{k \to -i\beta_1} (k + i\beta_1) \cdot \frac{k^{2+2l}}{(k^2+\beta_1^2)(k^2+\beta_2^2)\left(w - \sqrt{k^2+m_K^2} - \frac{k^2}{2m_N} - m_N\right)}$$

Karena

$$\frac{(k+i\beta_1)}{(k^2+\beta_1{}^2)} = \frac{(k+i\beta_1)}{(k+i\beta_1)(k-i\beta_1)} = \frac{1}{(k-i\beta_1)}$$

maka persamaan menjadi

$$\textbf{Res}\ f(k) = \lim_{k \to -i\beta_1} \frac{k^{2+2l}}{(k-i\beta_1)(k^2+\beta_2^2)\left(w - \sqrt{k^2+m_K^2} - \frac{k^2}{2m_N} - m_N\right)}$$

$$= \frac{(-i\beta_1)^{2+2l}}{(-2i\beta_1)(-\beta_1{}^2+\beta_2^2)\left(w - \sqrt{-\beta_1{}^2+m_K^2} - \frac{-\beta_1{}^2}{2m_N} - m_N\right)}$$

Sehingga residu dari $f(k_2)$ pada *simple pole* $k_2 = -i\beta_1$ adalah

$$= \frac{(-i\beta_1)^{2+2l}}{(-2i\beta_1)(-\beta_1{}^2+\beta_2^2)\left(w - \sqrt{-\beta_1{}^2+m_K^2} + \frac{\beta_1{}^2}{2m_N} - m_N\right)} \qquad (3.2.1.1.1.2)$$

Lampiran 9.

Akan dicari residu dari $f(k_3)$ pada *simple pole* $k_3 = -\sqrt{2}\sqrt{nw - \sqrt{m^2n^2 - n^4 + 2n^3w}}$

Dengan memisalkan $m_K = m$ dan $m_N = n$, maka :

$$\textbf{Res } f(k) = \lim_{k \to -\sqrt{2}\sqrt{nw - \sqrt{m^2n^2 - n^4 + 2n^3w}}} \left(k + \sqrt{2}\sqrt{nw - \sqrt{m^2n^2 - n^4 + 2n^3w}} \right)$$

$$\cdot \frac{k^{2+2l}}{\left(k^2 + \beta_1^{\,2}\right)\left(k^2 + \beta_2^{\,2}\right)\left(w - \sqrt{k^2 + m^2} - \frac{k^2}{2n} - n \right)}$$

$$= \lim_{k \to -\sqrt{2}\sqrt{nw - \sqrt{m^2n^2 - n^4 + 2n^3w}}} \frac{k^{3+2l} + k^{2+2l}\sqrt{2}\sqrt{nw - \sqrt{m^2n^2 - n^4 + 2n^3w}}}{\left(k^2 + \beta_1^{\,2}\right)\left(k^2 + \beta_2^{\,2}\right)\left(w - \sqrt{k^2 + m^2} - \frac{k^2}{2n} - n \right)}$$

Dengan menggunakan kaidah L'Hospital, maka dengan memisalkan

$$f(x) = k^{2+2l}\left(k + \sqrt{2}\sqrt{nw - \sqrt{m^2n^2 - n^4 + 2n^3w}} \right)$$

$$g(x) = \left(k^2 + \beta_1^{\,2}\right)\left(k^2 + \beta_2^{\,2}\right)\left(w - \sqrt{k^2 + m^2} - \frac{k^2}{2n} - n \right)$$

Maka

$$f'(x) = (2 + 2l)k^{1+2l}\left(k + \sqrt{2}\sqrt{nw - \sqrt{m^2n^2 - n^4 + 2n^3w}} \right) + k^{2+2l}$$

$$g'(x) = 2k\left(k^2 + \beta_2^{\,2}\right)\left(w - \sqrt{k^2 + m^2} - \frac{k^2}{2n} - n \right)$$

$$+ \left(k^2 + \beta_1^{\,2}\right)\left(\begin{array}{l} 2k\left(w - \sqrt{k^2 + m^2} - \frac{k^2}{2n} - n \right) + \\ \left(k^2 + \beta_2^{\,2}\right)\left(-\frac{1}{2}\frac{1}{\sqrt{k^2 + m^2}}2k - \frac{2k}{2n} \right) \end{array} \right)$$

$$g'(x) = 2k\left(k^2+\beta_2{}^2\right)\left(w-\sqrt{k^2+m^2}-\frac{k^2}{2n}-n\right)$$
$$+2k\left(k^2+\beta_1{}^2\right)\left(w-\sqrt{k^2+m^2}-\frac{k^2}{2n}-n\right)-k\left(k^2+\beta_1{}^2\right)\left(k^2+\beta_2{}^2\right)\left(\frac{1}{\sqrt{k^2+m^2}}+\frac{1}{n}\right)$$

Sehingga

$$\mathbf{Res}f(k)=\lim_{k\to-\sqrt{2}\sqrt{nw-\sqrt{m^2n^2-n^4+2n^3w}}}\frac{\begin{pmatrix}(2+2l)k^{1+2l}\left(k+\sqrt{2}\sqrt{nw-\sqrt{m^2n^2-n^4+2n^3w}}\right)+\\ k^{2+2l}\end{pmatrix}}{\begin{pmatrix}2k\left(k^2+\beta_2{}^2\right)\left(w-\sqrt{k^2+m^2}-\frac{k^2}{2n}-n\right)+\\ 2k\left(k^2+\beta_1{}^2\right)\left(w-\sqrt{k^2+m^2}-\frac{k^2}{2n}-n\right)-\\ k\left(k^2+\beta_1{}^2\right)\left(k^2+\beta_2{}^2\right)\left(\frac{1}{\sqrt{k^2+m^2}}+\frac{1}{n}\right)\end{pmatrix}}$$

$$=\frac{(2+2l)\left(-\sqrt{2}\sqrt{nw-\sqrt{m^2n^2-n^4+2n^3w}}\right)^{1+2l}(0)+\left(-\sqrt{2}\sqrt{nw-\sqrt{m^2n^2-n^4+2n^3w}}\right)^{2+2l}}{\begin{pmatrix}2\left(-\sqrt{2}\sqrt{nw-\sqrt{m^2n^2-n^4+2n^3w}}\right)\left(\left(-\sqrt{2}\sqrt{nw-\sqrt{m^2n^2-n^4+2n^3w}}\right)^2+\beta_2{}^2\right)(0)+\\ 2\left(-\sqrt{2}\sqrt{nw-\sqrt{m^2n^2-n^4+2n^3w}}\right)\left(\left(-\sqrt{2}\sqrt{nw-\sqrt{m^2n^2-n^4+2n^3w}}\right)^2+\beta_1{}^2\right)(0)-\\ \left(-\sqrt{2}\sqrt{nw-\sqrt{m^2n^2-n^4+2n^3w}}\right)\left(\left(-\sqrt{2}\sqrt{nw-\sqrt{m^2n^2-n^4+2n^3w}}\right)^2+\beta_1{}^2\right)\\ \left(\left(-\sqrt{2}\sqrt{nw-\sqrt{m^2n^2-n^4+2n^3w}}\right)^2+\beta_2{}^2\right)\left(\frac{1}{\sqrt{\left(-\sqrt{2}\sqrt{nw-\sqrt{m^2n^2-n^4+2n^3w}}\right)^2+m^2}}+\frac{1}{n}\right)\end{pmatrix}}$$

$$=\frac{\left(-\sqrt{2}\sqrt{nw-\sqrt{m^2n^2-n^4+2n^3w}}\right)^{2+2l}}{\left(\begin{array}{l}-\left(-\sqrt{2}\sqrt{nw-\sqrt{m^2n^2-n^4+2n^3w}}\right)\left(\left(-\sqrt{2}\sqrt{nw-\sqrt{m^2n^2-n^4+2n^3w}}\right)^2+\beta_1^2\right)\\ \left(\left(-\sqrt{2}\sqrt{nw-\sqrt{m^2n^2-n^4+2n^3w}}\right)^2+\beta_2^2\right)\left(\dfrac{n+\sqrt{\left(-\sqrt{2}\sqrt{nw-\sqrt{m^2n^2-n^4+2n^3w}}\right)^2+m^2}}{n\sqrt{\left(-\sqrt{2}\sqrt{nw-\sqrt{m^2n^2-n^4+2n^3w}}\right)^2+m^2}}\right)\end{array}\right)}$$

$$=-\frac{n\left(-\sqrt{2}\sqrt{nw-\sqrt{m^2n^2-n^4+2n^3w}}\right)^{1+2l}\sqrt{\left(-\sqrt{2}\sqrt{nw-\sqrt{m^2n^2-n^4+2n^3w}}\right)^2+m^2}}{\left(\begin{array}{l}\left(\left(-\sqrt{2}\sqrt{nw-\sqrt{m^2n^2-n^4+2n^3w}}\right)^2+\beta_1^2\right)\\ \left(\left(-\sqrt{2}\sqrt{nw-\sqrt{m^2n^2-n^4+2n^3w}}\right)^2+\beta_2^2\right)\left(n+\sqrt{\left(-\sqrt{2}\sqrt{nw-\sqrt{m^2n^2-n^4+2n^3w}}\right)^2+m^2}\right)\end{array}\right)}$$

$$=\frac{n\left(\sqrt{2}\sqrt{nw-\sqrt{m^2n^2-n^4+2n^3w}}\right)^{1+2l}\sqrt{\left(\sqrt{2}\sqrt{nw-\sqrt{m^2n^2-n^4+2n^3w}}\right)^2+m^2}}{\left(\begin{array}{l}\left(\left(\sqrt{2}\sqrt{nw-\sqrt{m^2n^2-n^4+2n^3w}}\right)^2+\beta_1^2\right)\left(\left(\sqrt{2}\sqrt{nw-\sqrt{m^2n^2-n^4+2n^3w}}\right)^2+\beta_2^2\right)\\ \left(n+\sqrt{\left(\sqrt{2}\sqrt{nw-\sqrt{m^2n^2-n^4+2n^3w}}\right)^2+m^2}\right)\end{array}\right)}$$

Sehingga residu dari $f(k_3)$ pada *simple pole* $k_3=-\sqrt{2}\sqrt{nw-\sqrt{m^2n^2-n^4+2n^3w}}$ adalah

$$\frac{n\left(\sqrt{2}\sqrt{nw-\sqrt{m^2n^2-n^4+2n^3w}}\right)^{1+2l}\sqrt{\left(\sqrt{2}\sqrt{nw-\sqrt{m^2n^2-n^4+2n^3w}}\right)^2+m^2}}{\left(\begin{array}{l}\left(\left(\sqrt{2}\sqrt{nw-\sqrt{m^2n^2-n^4+2n^3w}}\right)^2+\beta_1{}^2\right)\left(\left(\sqrt{2}\sqrt{nw-\sqrt{m^2n^2-n^4+2n^3w}}\right)^2+\beta_2{}^2\right)\\ \left(n+\sqrt{\left(\sqrt{2}\sqrt{nw-\sqrt{m^2n^2-n^4+2n^3w}}\right)^2+m^2}\right)\end{array}\right)}$$

(3.2.1.1.1.3)

Lampiran 10

Akan dicari residu dari $f(k_4)$ pada *simple pole* $k_4 = \sqrt{2}\sqrt{nw - \sqrt{m^2n^2 - n^4 + 2n^3w}}$

$$\textbf{Res } f(k) = \lim_{k \to \sqrt{2}\sqrt{nw - \sqrt{m^2n^2 - n^4 + 2n^3w}}} \left(k - \sqrt{2}\sqrt{nw - \sqrt{m^2n^2 - n^4 + 2n^3w}} \right)$$

$$\cdot \frac{k^{2+2l}}{\left(k^2 + \beta_1^{\,2}\right)\left(k^2 + \beta_2^{\,2}\right)\left(w - \sqrt{k^2 + m^2} - \frac{k^2}{2n} - n \right)}$$

$$= \lim_{k \to \sqrt{2}\sqrt{nw - \sqrt{m^2n^2 - n^4 + 2n^3w}}} \frac{k^{3+2l} - k^{2+2l}\sqrt{2}\sqrt{nw - \sqrt{m^2n^2 - n^4 + 2n^3w}}}{\left(k^2 + \beta_1^{\,2}\right)\left(k^2 + \beta_1^{\,2}\right)\left(w - \sqrt{k^2 + m^2} - \frac{k^2}{2n} - n \right)}$$

Dengan menggunakan kaidah L'Hospital, maka dengan memisalkan

$$f(x) = k^{2+2l}\left(k - \sqrt{2}\sqrt{nw - \sqrt{m^2n^2 - n^4 + 2n^3w}} \right)$$

$$g(x) = \left(k^2 + \beta_1^{\,2}\right)\left(k^2 + \beta_2^{\,2}\right)\left(w - \sqrt{k^2 + m^2} - \frac{k^2}{2n} - n \right)$$

Maka

$$f'(x) = (2 + 2l)k^{1+2l}\left(k - \sqrt{2}\sqrt{nw - \sqrt{m^2n^2 - n^4 + 2n^3w}} \right) + k^{2+2l}$$

$$g'(x) = 2k\left(k^2 + \beta_2^{\,2}\right)\left(w - \sqrt{k^2 + m^2} - \frac{k^2}{2n} - n \right) + \left(k^2 + \beta_1^{\,2}\right)\left(\begin{array}{l} 2k\left(w - \sqrt{k^2 + m^2} - \frac{k^2}{2n} - n \right) + \\ \left(k^2 + \beta_2^{\,2}\right)\left(-\frac{1}{2}\frac{1}{\sqrt{k^2 + m^2}}2k - \frac{2k}{2n} \right) \end{array} \right)$$

$$g'(x) = 2k\left(k^2+\beta_2{}^2\right)\left(w-\sqrt{k^2+m^2}-\frac{k^2}{2n}-n\right)+2k\left(k^2+\beta_1{}^2\right)\left(w-\sqrt{k^2+m^2}-\frac{k^2}{2n}-n\right)-$$
$$k\left(k^2+\beta_1{}^2\right)\left(k^2+\beta_2{}^2\right)\left(\frac{1}{\sqrt{k^2+m^2}}+\frac{1}{n}\right)$$

Sehingga

$$\mathbf{Res}\, f(k) = \lim_{k\to\sqrt{2}\sqrt{nw-\sqrt{m^2n^2-n^4+2n^3w}}} \frac{\left((2+2l)k^{1+2l}\left(k-\sqrt{2}\sqrt{nw-\sqrt{m^2n^2-n^4+2n^3w}}\right)+ k^{2+2l}\right)}{\left(\begin{array}{l} 2k\left(k^2+\beta_2{}^2\right)\left(w-\sqrt{k^2+m^2}-\frac{k^2}{2n}-n\right)+ \\ 2k\left(k^2+\beta_1{}^2\right)\left(w-\sqrt{k^2+m^2}-\frac{k^2}{2n}-n\right)- \\ k\left(k^2+\beta_1{}^2\right)\left(k^2+\beta_2{}^2\right)\left(\frac{1}{\sqrt{k^2+m^2}}+\frac{1}{n}\right)\end{array}\right)}$$

$$= \frac{(2+2l)\left(\sqrt{2}\sqrt{nw-\sqrt{m^2n^2-n^4+2n^3w}}\right)^{1+2l}(0)+\left(\sqrt{2}\sqrt{nw-\sqrt{m^2n^2-n^4+2n^3w}}\right)^{2+2l}}{\left(\begin{array}{l} 2\left(\sqrt{2}\sqrt{nw-\sqrt{m^2n^2-n^4+2n^3w}}\right)\left(\left(\sqrt{2}\sqrt{nw-\sqrt{m^2n^2-n^4+2n^3w}}\right)^2+\beta_2{}^2\right)(0)+ \\ 2\left(\sqrt{2}\sqrt{nw-\sqrt{m^2n^2-n^4+2n^3w}}\right)\left(\left(\sqrt{2}\sqrt{nw-\sqrt{m^2n^2-n^4+2n^3w}}\right)^2+\beta_1{}^2\right)(0)- \\ \left(\sqrt{2}\sqrt{nw-\sqrt{m^2n^2-n^4+2n^3w}}\right)\left(\left(\sqrt{2}\sqrt{nw-\sqrt{m^2n^2-n^4+2n^3w}}\right)^2+\beta_1{}^2\right) \\ \left(\left(\sqrt{2}\sqrt{nw-\sqrt{m^2n^2-n^4+2n^3w}}\right)^2+\beta_2{}^2\right)\left(\frac{1}{\sqrt{\left(\sqrt{2}\sqrt{nw-\sqrt{m^2n^2-n^4+2n^3w}}\right)^2+m^2}}+\frac{1}{n}\right)\end{array}\right)}$$

$$=\frac{\left(\sqrt{2}\sqrt{nw-\sqrt{m^2n^2-n^4+2n^3w}}\right)^{2+2l}}{\left(\begin{array}{l}-\left(\sqrt{2}\sqrt{nw-\sqrt{m^2n^2-n^4+2n^3w}}\right)\left(\left(\sqrt{2}\sqrt{nw-\sqrt{m^2n^2-n^4+2n^3w}}\right)^2+\beta_1{}^2\right)\\ \left(\left(\sqrt{2}\sqrt{nw-\sqrt{m^2n^2-n^4+2n^3w}}\right)^2+\beta_2{}^2\right)\left(\frac{n+\sqrt{\left(\sqrt{2}\sqrt{nw-\sqrt{m^2n^2-n^4+2n^3w}}\right)^2+m^2}}{n\sqrt{\left(\sqrt{2}\sqrt{nw-\sqrt{m^2n^2-n^4+2n^3w}}\right)^2+m^2}}\right)\end{array}\right)}$$

$$=-\frac{n\left(\sqrt{2}\sqrt{nw-\sqrt{m^2n^2-n^4+2n^3w}}\right)^{1+2l}\sqrt{\left(\sqrt{2}\sqrt{nw-\sqrt{m^2n^2-n^4+2n^3w}}\right)^2+m^2}}{\left(\begin{array}{l}\left(\left(\sqrt{2}\sqrt{nw-\sqrt{m^2n^2-n^4+2n^3w}}\right)^2+\beta_1{}^2\right)\\ \left(\left(\sqrt{2}\sqrt{nw-\sqrt{m^2n^2-n^4+2n^3w}}\right)^2+\beta_2{}^2\right)\left(n+\sqrt{\left(\sqrt{2}\sqrt{nw-\sqrt{m^2n^2-n^4+2n^3w}}\right)^2+m^2}\right)\end{array}\right)}$$

$$=-\frac{n\left(\sqrt{2}\sqrt{nw-\sqrt{m^2n^2-n^4+2n^3w}}\right)^{1+2l}\sqrt{\left(\sqrt{2}\sqrt{nw-\sqrt{m^2n^2-n^4+2n^3w}}\right)^2+m^2}}{\left(\begin{array}{l}\left(\left(\sqrt{2}\sqrt{nw-\sqrt{m^2n^2-n^4+2n^3w}}\right)^2+\beta_1{}^2\right)\left(\left(\sqrt{2}\sqrt{nw-\sqrt{m^2n^2-n^4+2n^3w}}\right)^2+\beta_2{}^2\right)\\ \left(n+\sqrt{\left(\sqrt{2}\sqrt{nw-\sqrt{m^2n^2-n^4+2n^3w}}\right)^2+m^2}\right)\end{array}\right)}$$

Sehingga residu dari $f(k_4)$ pada *simple pole* $k_4=\sqrt{2}\sqrt{nw-\sqrt{m^2n^2-n^4+2n^3w}}$ adalah

$$-\frac{n\left(\sqrt{2}\sqrt{nw-\sqrt{m^2n^2-n^4+2n^3w}}\right)^{1+2l}\sqrt{\left(\sqrt{2}\sqrt{nw-\sqrt{m^2n^2-n^4+2n^3w}}\right)^2+m^2}}{\left(\left(\left(\sqrt{2}\sqrt{nw-\sqrt{m^2n^2-n^4+2n^3w}}\right)^2+\beta_1{}^2\right)\left(\left(\sqrt{2}\sqrt{nw-\sqrt{m^2n^2-n^4+2n^3w}}\right)^2+\beta_2{}^2\right)\left(n+\sqrt{\left(\sqrt{2}\sqrt{nw-\sqrt{m^2n^2-n^4+2n^3w}}\right)^2+m^2}\right)\right)}$$

(3.2.1.1.1.4)

Lampiran 11.

Akan dicari residu dari $f(k_5)$ pada *simple pole* $k_5 = -\sqrt{2}\sqrt{nw + \sqrt{m^2n^2 - n^4 + 2n^3w}}$

$$\textbf{Res } f(k) = \lim_{k \to -\sqrt{2}\sqrt{nw + \sqrt{m^2n^2 - n^4 + 2n^3w}}} \left(k + \sqrt{2}\sqrt{nw + \sqrt{m^2n^2 - n^4 + 2n^3w}} \right)$$

$$\cdot \frac{k^{2+2l}}{\left(k^2 + \beta_1^2\right)\left(k^2 + \beta_2^2\right)\left(w - \sqrt{k^2 + m^2} - \frac{k^2}{2n} - n \right)}$$

$$= \lim_{k \to -\sqrt{2}\sqrt{nw + \sqrt{m^2n^2 - n^4 + 2n^3w}}} \frac{k^{3+2l} + k^{2+2l}\sqrt{2}\sqrt{nw + \sqrt{m^2n^2 - n^4 + 2n^3w}}}{\left(k^2 + \beta_1^2\right)\left(k^2 + \beta_2^2\right)\left(w - \sqrt{k^2 + m^2} - \frac{k^2}{2n} - n \right)}$$

Dengan menggunakan kaidah L'Hospital, maka dengan memisalkan

$$f(x) = k^{2+2l}\left(k + \sqrt{2}\sqrt{nw + \sqrt{m^2n^2 - n^4 + 2n^3w}} \right)$$

$$g(x) = \left(k^2 + \beta_1^2\right)\left(k^2 + \beta_2^2\right)\left(w - \sqrt{k^2 + m^2} - \frac{k^2}{2n} - n \right)$$

Maka

$$f'(x) = (2 + 2l)k^{1+2l}\left(k + \sqrt{2}\sqrt{nw + \sqrt{m^2n^2 - n^4 + 2n^3w}} \right) + k^{2+2l}$$

$$g'(x) = 2k\left(k^2 + \beta_2^2\right)\left(w - \sqrt{k^2 + m^2} - \frac{k^2}{2n} - n \right) + \left(k^2 + \beta_1^2\right)\left(\begin{array}{l} 2k\left(w - \sqrt{k^2 + m^2} - \frac{k^2}{2n} - n \right) + \\ \left(k^2 + \beta_2^2\right)\left(-\frac{1}{2}\frac{1}{\sqrt{k^2 + m^2}}2k - \frac{2k}{2n} \right) \end{array} \right)$$

$$g'(x) = 2k\left(k^2+\beta_2{}^2\right)\left(w-\sqrt{k^2+m^2}-\frac{k^2}{2n}-n\right)+2k\left(k^2+\beta_1{}^2\right)\left(w-\sqrt{k^2+m^2}-\frac{k^2}{2n}-n\right)-$$
$$k\left(k^2+\beta_1{}^2\right)\left(k^2+\beta_2{}^2\right)\left(\frac{1}{\sqrt{k^2+m^2}}+\frac{1}{n}\right)$$

Sehingga

$$\mathbf{Res} f(k) = \lim_{k\to -\sqrt{2}\sqrt{nw+\sqrt{m^2n^2-n^4+2n^3w}}} \frac{\left((2+2l)k^{1+2l}\left(k+\sqrt{2}\sqrt{nw+\sqrt{m^2n^2-n^4+2n^3w}}\right)+ k^{2+2l}\right)}{\left(\begin{array}{l} 2k\left(k^2+\beta_2{}^2\right)\left(w-\sqrt{k^2+m^2}-\frac{k^2}{2n}-n\right)+ \\ 2k\left(k^2+\beta_1{}^2\right)\left(w-\sqrt{k^2+m^2}-\frac{k^2}{2n}-n\right)- \\ k\left(k^2+\beta_1{}^2\right)\left(k^2+\beta_2{}^2\right)\left(\frac{1}{\sqrt{k^2+m^2}}+\frac{1}{n}\right)\end{array}\right)}$$

$$= \frac{(2+2l)\left(-\sqrt{2}\sqrt{nw+\sqrt{m^2n^2-n^4+2n^3w}}\right)^{1+2l}(0)+\left(-\sqrt{2}\sqrt{nw+\sqrt{m^2n^2-n^4+2n^3w}}\right)^{2+2l}}{\left(\begin{array}{l} 2\left(-\sqrt{2}\sqrt{nw+\sqrt{m^2n^2-n^4+2n^3w}}\right)\left(\left(-\sqrt{2}\sqrt{nw+\sqrt{m^2n^2-n^4+2n^3w}}\right)^2+\beta_2{}^2\right)(0)+ \\ 2\left(-\sqrt{2}\sqrt{nw+\sqrt{m^2n^2-n^4+2n^3w}}\right)\left(\left(-\sqrt{2}\sqrt{nw+\sqrt{m^2n^2-n^4+2n^3w}}\right)^2+\beta_1{}^2\right)(0)- \\ \left(-\sqrt{2}\sqrt{nw+\sqrt{m^2n^2-n^4+2n^3w}}\right)\left(\left(-\sqrt{2}\sqrt{nw+\sqrt{m^2n^2-n^4+2n^3w}}\right)^2+\beta_1{}^2\right) \\ \left(\left(-\sqrt{2}\sqrt{nw+\sqrt{m^2n^2-n^4+2n^3w}}\right)^2+\beta_2{}^2\right)\left(\frac{1}{\sqrt{\left(-\sqrt{2}\sqrt{nw+\sqrt{m^2n^2-n^4+2n^3w}}\right)^2+m^2}}+\frac{1}{n}\right)\end{array}\right)}$$

$$=\frac{\left(-\sqrt{2}\sqrt{nw+\sqrt{m^2n^2-n^4+2n^3w}}\right)^{2+2l}}{\left(\begin{array}{l}-\left(-\sqrt{2}\sqrt{nw+\sqrt{m^2n^2-n^4+2n^3w}}\right)\left(\left(-\sqrt{2}\sqrt{nw+\sqrt{m^2n^2-n^4+2n^3w}}\right)^2+\beta_1{}^2\right)\\ \left(\left(-\sqrt{2}\sqrt{nw+\sqrt{m^2n^2-n^4+2n^3w}}\right)^2+\beta_2{}^2\right)\left(\dfrac{n+\sqrt{\left(-\sqrt{2}\sqrt{nw+\sqrt{m^2n^2-n^4+2n^3w}}\right)^2+m^2}}{n\sqrt{\left(-\sqrt{2}\sqrt{nw+\sqrt{m^2n^2-n^4+2n^3w}}\right)^2+m^2}}\right)\end{array}\right)}$$

$$=-\frac{n\left(-\sqrt{2}\sqrt{nw+\sqrt{m^2n^2-n^4+2n^3w}}\right)^{1+2l}\sqrt{\left(-\sqrt{2}\sqrt{nw+\sqrt{m^2n^2-n^4+2n^3w}}\right)^2+m^2}}{\left(\begin{array}{l}\left(\left(-\sqrt{2}\sqrt{nw+\sqrt{m^2n^2-n^4+2n^3w}}\right)^2+\beta_1{}^2\right)\left(\left(-\sqrt{2}\sqrt{nw+\sqrt{m^2n^2-n^4+2n^3w}}\right)^2+\beta_2{}^2\right)\\ \left(n+\sqrt{\left(-\sqrt{2}\sqrt{nw+\sqrt{m^2n^2-n^4+2n^3w}}\right)^2+m^2}\right)\end{array}\right)}$$

$$=\frac{n\left(\sqrt{2}\sqrt{nw+\sqrt{m^2n^2-n^4+2n^3w}}\right)^{1+2l}\sqrt{\left(\sqrt{2}\sqrt{nw+\sqrt{m^2n^2-n^4+2n^3w}}\right)^2+m^2}}{\left(\begin{array}{l}\left(\left(\sqrt{2}\sqrt{nw+\sqrt{m^2n^2-n^4+2n^3w}}\right)^2+\beta_1{}^2\right)\left(\left(\sqrt{2}\sqrt{nw+\sqrt{m^2n^2-n^4+2n^3w}}\right)^2+\beta_2{}^2\right)\\ \left(n+\sqrt{\left(\sqrt{2}\sqrt{nw+\sqrt{m^2n^2-n^4+2n^3w}}\right)^2+m^2}\right)\end{array}\right)}$$

Sehingga residu dari $f(k_5)$ pada *simple pole* $k_5=-\sqrt{2}\sqrt{nw+\sqrt{m^2n^2-n^4+2n^3w}}$ adalah

$$\frac{n\left(\sqrt{2}\sqrt{nw+\sqrt{m^2n^2-n^4+2n^3w}}\right)^{1+2l}\sqrt{\left(\sqrt{2}\sqrt{nw+\sqrt{m^2n^2-n^4+2n^3w}}\right)^2+m^2}}{\left(\left(\left(\sqrt{2}\sqrt{nw+\sqrt{m^2n^2-n^4+2n^3w}}\right)^2+\beta_1{}^2\right)\left(\left(\sqrt{2}\sqrt{nw+\sqrt{m^2n^2-n^4+2n^3w}}\right)^2+\beta_2{}^2\right)\left(n+\sqrt{\left(\sqrt{2}\sqrt{nw+\sqrt{m^2n^2-n^4+2n^3w}}\right)^2+m^2}\right)\right)}$$

(3.2.1.1.1.5)

Lampiran 12.

Akan dicari residu dari $f(k_6)$ pada *simple pole* $k_6 = \sqrt{2}\sqrt{nw + \sqrt{m^2n^2 - n^4 + 2n^3w}}$

$$\textbf{Res}\ f(k) = \lim_{k \to \sqrt{2}\sqrt{nw + \sqrt{m^2n^2 - n^4 + 2n^3w}}} \left(k - \sqrt{2}\sqrt{nw + \sqrt{m^2n^2 - n^4 + 2n^3w}} \right)$$

$$\cdot \frac{k^{2+2l}}{\left(k^2 + \beta_1^{\,2}\right)\left(k^2 + \beta_2^{\,2}\right)\left(w - \sqrt{k^2 + m^2} - \frac{k^2}{2n} - n \right)}$$

$$= \lim_{k \to \sqrt{2}\sqrt{nw + \sqrt{m^2n^2 - n^4 + 2n^3w}}} \frac{k^{3+2l} - k^{2+2l}\sqrt{2}\sqrt{nw + \sqrt{m^2n^2 - n^4 + 2n^3w}}}{\left(k^2 + \beta_1^{\,2}\right)\left(k^2 + \beta_2^{\,2}\right)\left(w - \sqrt{k^2 + m^2} - \frac{k^2}{2n} - n \right)}$$

Dengan menggunakan kaidah L'Hospital, maka dengan memisalkan

$$f(x) = k^{2+2l}\left(k - \sqrt{2}\sqrt{nw + \sqrt{m^2n^2 - n^4 + 2n^3w}} \right)$$

$$g(x) = \left(k^2 + \beta_1^{\,2}\right)\left(k^2 + \beta_2^{\,2}\right)\left(w - \sqrt{k^2 + m^2} - \frac{k^2}{2n} - n \right)$$

Maka

$$f'(x) = (2 + 2l)k^{1+2l}\left(k - \sqrt{2}\sqrt{nw + \sqrt{m^2n^2 - n^4 + 2n^3w}} \right) + k^{2+2l}$$

$$g'(x) = 2k\left(k^2 + \beta_2^{\,2}\right)\left(w - \sqrt{k^2 + m^2} - \frac{k^2}{2n} - n \right) + \left(k^2 + \beta_1^{\,2}\right)\left(\begin{array}{l} 2k\left(w - \sqrt{k^2 + m^2} - \frac{k^2}{2n} - n \right) + \\ \left(k^2 + \beta_2^{\,2}\right)\left(-\frac{1}{2}\frac{1}{\sqrt{k^2 + m^2}}2k - \frac{2k}{2n} \right) \end{array} \right)$$

$$g'(x)=2k\left(k^2+\beta_2^{\,2}\right)\left(w-\sqrt{k^2+m^2}-\frac{k^2}{2n}-n\right)+2k\left(k^2+\beta_1^{\,2}\right)\left(w-\sqrt{k^2+m^2}-\frac{k^2}{2n}-n\right)-$$
$$k\left(k^2+\beta_1^{\,2}\right)\left(k^2+\beta_2^{\,2}\right)\left(\frac{1}{\sqrt{k^2+m^2}}+\frac{1}{n}\right)$$

Sehingga

$$\mathbf{Res}\,f(k)=\lim_{k\to\sqrt{2}\sqrt{nw+\sqrt{m^2n^2-n^4+2n^3w}}}\frac{\left((2+2l)k^{1+2l}\left(k-\sqrt{2}\sqrt{nw+\sqrt{m^2n^2-n^4+2n^3w}}\right)+k^{2+2l}\right)}{\left(\begin{array}{l}2k\left(k^2+\beta_2^{\,2}\right)\left(w-\sqrt{k^2+m^2}-\frac{k^2}{2n}-n\right)+\\ 2k\left(k^2+\beta_1^{\,2}\right)\left(w-\sqrt{k^2+m^2}-\frac{k^2}{2n}-n\right)-\\ k\left(k^2+\beta_1^{\,2}\right)\left(k^2+\beta_2^{\,2}\right)\left(\frac{1}{\sqrt{k^2+m^2}}+\frac{1}{n}\right)\end{array}\right)}$$

$$=\frac{(2+2l)\left(\sqrt{2}\sqrt{nw+\sqrt{m^2n^2-n^4+2n^3w}}\right)^{1+2l}(0)+\left(\sqrt{2}\sqrt{nw+\sqrt{m^2n^2-n^4+2n^3w}}\right)^{2+2l}}{\left(\begin{array}{l}2\left(\sqrt{2}\sqrt{nw+\sqrt{m^2n^2-n^4+2n^3w}}\right)\left(\left(\sqrt{2}\sqrt{nw+\sqrt{m^2n^2-n^4+2n^3w}}\right)^2+\beta_2^{\,2}\right)(0)+\\ 2\left(\sqrt{2}\sqrt{nw+\sqrt{m^2n^2-n^4+2n^3w}}\right)\left(\left(\sqrt{2}\sqrt{nw+\sqrt{m^2n^2-n^4+2n^3w}}\right)^2+\beta_1^{\,2}\right)(0)-\\ \left(\sqrt{2}\sqrt{nw+\sqrt{m^2n^2-n^4+2n^3w}}\right)\left(\left(\sqrt{2}\sqrt{nw+\sqrt{m^2n^2-n^4+2n^3w}}\right)^2+\beta_1^{\,2}\right)\\ \left(\left(\sqrt{2}\sqrt{nw+\sqrt{m^2n^2-n^4+2n^3w}}\right)^2+\beta_2^{\,2}\right)\left(\frac{1}{\sqrt{\left(\sqrt{2}\sqrt{nw+\sqrt{m^2n^2-n^4+2n^3w}}\right)^2+m^2}}+\frac{1}{n}\right)\end{array}\right)}$$

$$=\frac{\left(\sqrt{2}\sqrt{nw-\sqrt{m^2n^2-n^4+2n^3w}}\right)^{2+2l}}{\left(\begin{array}{l}-\left(\sqrt{2}\sqrt{nw+\sqrt{m^2n^2-n^4+2n^3w}}\right)\left(\left(\sqrt{2}\sqrt{nw+\sqrt{m^2n^2-n^4+2n^3w}}\right)^2+\beta_1{}^2\right)\\ \left(\left(\sqrt{2}\sqrt{nw+\sqrt{m^2n^2-n^4+2n^3w}}\right)^2+\beta_2{}^2\right)\left(\dfrac{n+\sqrt{\left(\sqrt{2}\sqrt{nw+\sqrt{m^2n^2-n^4+2n^3w}}\right)^2+m^2}}{n\sqrt{\left(\sqrt{2}\sqrt{nw+\sqrt{m^2n^2-n^4+2n^3w}}\right)^2+m^2}}\right)\end{array}\right)}$$

$$=-\frac{n\left(\sqrt{2}\sqrt{nw+\sqrt{m^2n^2-n^4+2n^3w}}\right)^{1+2l}\sqrt{\left(\sqrt{2}\sqrt{nw+\sqrt{m^2n^2-n^4+2n^3w}}\right)^2+m^2}}{\left(\begin{array}{l}\left(\left(\sqrt{2}\sqrt{nw+\sqrt{m^2n^2-n^4+2n^3w}}\right)^2+\beta_1{}^2\right)\\ \left(\left(\sqrt{2}\sqrt{nw+\sqrt{m^2n^2-n^4+2n^3w}}\right)^2+\beta_2{}^2\right)\left(n+\sqrt{\left(\sqrt{2}\sqrt{nw+\sqrt{m^2n^2-n^4+2n^3w}}\right)^2+m^2}\right)\end{array}\right)}$$

$$=-\frac{n\left(\sqrt{2}\sqrt{nw+\sqrt{m^2n^2-n^4+2n^3w}}\right)^{1+2l}\sqrt{\left(\sqrt{2}\sqrt{nw+\sqrt{m^2n^2-n^4+2n^3w}}\right)^2+m^2}}{\left(\begin{array}{l}\left(\left(\sqrt{2}\sqrt{nw+\sqrt{m^2n^2-n^4+2n^3w}}\right)^2+\beta_1{}^2\right)\left(\left(\sqrt{2}\sqrt{nw+\sqrt{m^2n^2-n^4+2n^3w}}\right)^2+\beta_2{}^2\right)\\ \left(n+\sqrt{\left(\sqrt{2}\sqrt{nw+\sqrt{m^2n^2-n^4+2n^3w}}\right)^2+m^2}\right)\end{array}\right)}$$

Sehingga residu dari $f(k_6)$ pada *simple pole* $k_6=\sqrt{2}\sqrt{nw+\sqrt{m^2n^2-n^4+2n^3w}}$ adalah

$$-\frac{n\left(\sqrt{2}\sqrt{nw+\sqrt{m^2n^2-n^4+2n^3w}}\right)^{1+2l}\sqrt{\left(\sqrt{2}\sqrt{nw+\sqrt{m^2n^2-n^4+2n^3w}}\right)^2+m^2}}{\left(\left(\left(\sqrt{2}\sqrt{nw+\sqrt{m^2n^2-n^4+2n^3w}}\right)^2+\beta_1^{\,2}\right)\left(\left(\sqrt{2}\sqrt{nw+\sqrt{m^2n^2-n^4+2n^3w}}\right)^2+\beta_2^{\,2}\right)\left(n+\sqrt{\left(\sqrt{2}\sqrt{nw+\sqrt{m^2n^2-n^4+2n^3w}}\right)^2+m^2}\right)\right)}$$

(3.2.1.1.1.6)

Lampiran 13.

Akan dicari residu dari $f(k_7)$ pada *simple pole* $k_7 = i\beta_2$

$$\textbf{Res } f(k) = \lim_{k \to i\beta_2} (k - i\beta_2) \cdot \frac{k^{2+2l}}{(k^2 + \beta_1^2)(k^2 + \beta_2^2)\left(w - \sqrt{k^2 + m_K^2} - \frac{k^2}{2m_N} - m_N\right)}$$

Karena

$$\frac{(k - i\beta_2)}{(k^2 + \beta_2^{\,2})} = \frac{(k - i\beta_2)}{(k + i\beta_2)(k - i\beta_2)} = \frac{1}{(k + i\beta_2)}$$

Maka

$$\textbf{Res } f(k) = \lim_{k \to i\beta_2} \frac{k^{2+2l}}{(k + i\beta_2)(k^2 + \beta_1^2)\left(w - \sqrt{k^2 + m_K^2} - \frac{k^2}{2m_N} - m_N\right)}$$

$$= \frac{(i\beta_2)^{2+2l}}{(2i\beta_2)(\beta_1^{\,2} - \beta_2^2)\left(w - \sqrt{-\beta_2^{\,2} + m_K^2} - \frac{-\beta_2^{\,2}}{2m_N} - m_N\right)}$$

Sehingga residu dari $f(k_7)$ pada *simple pole* $k_7 = i\beta_2$ adalah

$$= \frac{(i\beta_2)^{2+2l}}{(2i\beta_2)(\beta_1^{\,2} - \beta_2^2)\left(w - \sqrt{-\beta_2^{\,2} + m_K^2} + \frac{\beta_2^{\,2}}{2m_N} - m_N\right)} \qquad (3.2.1.1.1.7)$$

Lampiran 14.

Akan dicari residu dari $f(k_8)$ pada *simple pole* $k_8 = -i\beta_2$

$$\textbf{Res } f(k) = \lim_{k \to -i\beta_2} (k + i\beta_2) \cdot \frac{k^{2+2l}}{(k^2+\beta_1^2)(k^2+\beta_2^2)\left(w - \sqrt{k^2+m_K^2} - \frac{k^2}{2m_N} - m_N\right)}$$

Karena

$$\frac{(k+i\beta_2)}{(k^2+\beta_2{}^2)} = \frac{(k+i\beta_1)}{(k+i\beta_2)(k-i\beta_2)} = \frac{1}{(k-i\beta_2)}$$

maka persamaan menjadi

$$\textbf{Res } f(k) = \lim_{k \to -i\beta_2} \frac{k^{2+2l}}{(k-i\beta_2)(k^2+\beta_1^2)\left(w - \sqrt{k^2+m_K^2} - \frac{k^2}{2m_N} - m_N\right)}$$

$$= \frac{(-i\beta_2)^{2+2l}}{(-2i\beta_2)(\beta_1{}^2 - \beta_2^2)\left(w - \sqrt{-\beta_2{}^2 + m_K^2} - \frac{-\beta_2{}^2}{2m_N} - m_N\right)}$$

Sehingga residu dari $f(k_8)$ pada *simple pole* $k_8 = -i\beta_2$ adalah

$$= \frac{(-i\beta_1)^{2+2l}}{(-2i\beta_2)(\beta_1{}^2 - \beta_2^2)\left(w - \sqrt{-\beta_2{}^2 + m_K^2} + \frac{\beta_2{}^2}{2m_N} - m_N\right)} \qquad (3.2.1.1.1.8)$$