Construction of Abridged Life Tables and Indirect Estimation of Some Mortality Measures of Bangladesh in 2005

Md. Rafiqul Islam

Abstract. *The aim of the present study is to estimate some mortality measures such as the age specific death rates (ASDRs), infant mortality rate (IMR) and life table crude death rate (CDR) for male, female and both sexes of Bangladesh in 2005. For this purpose, two abridged life tables, one for male and other for female were constructed using the corresponding secondary data on life expectancy at birth of Bangladesh in 2005 taken from Islam (2003). These were compared to the values in 1991 and it was observed that these rates were showing decreasing trend during 1991-2005. Moreover, a mathematical model was fitted to the number of persons surviving at an exact age x (l_x) only for male of Bangladesh in 2005. Model validation technique, cross validity prediction power (CVPP) and F-test, showed that the mathematical model was valid and hence, fit is well. Instantaneous force of mortality (μ_x) only for male of Bangladesh in 2005 was estimated. And it was found that μ_x exhibited decreasing trend up to age 20-24 and increasing in the remaining age group but rapidly increasing after age 50 years to infinity.*

Keywords: Life expectancy at birth, Linear interpolation, Life table Modeling Cross validity prediction power (CVPP), t-test, F-test, Bangladesh.

1. INTRODUCTION

Bangladesh is located in southern Asia, borders the Bay of Bengal, between Burma and India and occupies a total of 1,44,000 square

kilometers. It is situated latitudinally between $21^0 5'$ and $26^0 4'$ North and longitudinally between $88^0 5'$ and $92^0 5'$ East. The country has a tropical climate with cool, dry weather conditions from October to March. Summers are hot and humid. The monsoon season is characterized by heavy rainfall from June to October. It is sometimes affected by natural disasters such as floods, cyclones, and droughts. Bangladesh is hilly in the Southeast consisting mostly of flat alluvial plains. In Bangladesh, approximately 73 percent of the land is arable. The landscape has an extensive network of rivers that are very important in improving the socioeconomic conditions of the nation. Among them are the Ganges-Padma, Brahmaputra-Jamuna, and the Megna.

Bangladesh is one of the most developing countries with accelerated population growth in the world. In Bangladesh, the population increased from 42 million in 1941 to 129.25 million in 2001 (Bangladesh Bureau of Statistics--BBS, 2001). Bangladesh is one of the most densely populated country containing 129.25 million citizens of which 65.84 million are males and 63.41 millions are females (BBS, 2001). It is the ninth most populous country in the world having population density 834 persons per square km. The gross domestic product (GDP) in the country was $175.5 and the gross national product (GNP) per capita was $360. The population grew at a rate of 1.47 percent per year (Mitra and Associates, 2001).

Like many other developing countries, Bangladesh has not started to complete Vital Registration System (VRS) until now. Though Bangladesh Demographic Health survey (BDHS) collected data on sampling basis but it did not provide sufficient mortality information of the country. Information on mortality was obtained from censuses as well as other sources using some sophisticated indirect techniques. A lot of work on fertility has been carried out but mortality works are done in a very limited scale in Bangladesh. For this reason, an effort has been given to estimate indirectly some mortality measures of Bangladesh from very simple information of data. In this study, firstly, two abridged life tables were constructed. It should be noted that life table is a very sophisticated and mathematical tools in population science, in particular, in demography in the modern age. Life table is used to estimate various demographic parameters such as the IMR, CDR, ASDRS, net reproduction rate (NRR), survival rate, and replacement index. It is also so much applicable to enumerate migration rate. It is also broadly used in life insurance companies.

Ali (1990) reported that two abridged life tables, one for male and other for female had been constructed applying Widowhood Method on marital status data for female and male of 1981 census of Bangladesh. Then,

ASDRs for male, female and both sexes were also calculated from the constructed abridged life tables that were showed the tradition U-shape pattern. In this study the crude death rate (CDR) was also calculated using the age structure and ASDRs. Ali (1994) reviewed the relationship of total separation rates and separation rates due to death with their age variable and found a semi log function of the type $\log_e y = \alpha + \beta x$. In Islam and Ali (2004), it was found that the age specific fertility rates (ASFRs) followed slightly modified biquadratic polynomial model where as forward and backward cumulative ASFRs followed quadratic and cubic polynomial model respectively in the rural area of Bangladesh. The age structure, age specific death rates (ASDRs) and the number of persons surviving at an exact age x (l_x) in life table for male population of Bangladesh in 1991 followed modified negative exponential model, 4th and 3rd degree polynomial model, respectively (Islam, et al, 2003). In (Islam, 2004), it was shown that the ASFRs and proportion of married women of Bangladesh in the reproductive span followed 3rd degree polynomial model where as forward cumulative ASFRs followed quadratic polynomial model respectively of Bangladesh.

There have been changes in mortality levels in Bangladesh. There had no work on mortality since the previously mentioned works.

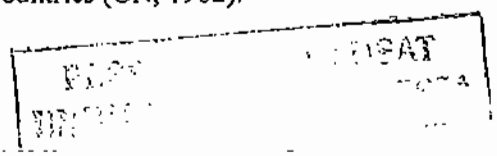
Therefore, the objectives of this study are:

i) to construct life tables and hence to estimate the IMR, CDR, ASDRs of Bangladesh in 2005,
ii) to estimate instantaneous force of mortality (μ_x) only for male of Bangladesh in 2005.

2. DATA AND METHODOLOGY

The life expectancy at birth for male which was 62.05 and for female which was 61.39 were taken from Islam (2003) and used in this study as raw materials.

The linear interpolation $y = f(x) = f(x_1) + \frac{f(x_2) - f(x_1)}{x_2 - x_1}(x - x_1)$ technique (UN, 1983) was applied to estimate lx values for corresponding life expectancy at birth using South Asian Model Life Tables from the United Nations Model Life Tables for Developing Countries (UN, 1982).



Then to construct life table the following mathematical relationships of various continuous functions of life table were used: ${}_nd_x = l_x - l_{x+n}$; $l_{x+n} = l_x\ {}_nP_x$; ${}_nL_x = \int_0^n l(x+t)\,dt$ which can be approximated as ${}_nL_x = \frac{n(l_x + l_{x+n})}{2}$ $(x \neq 2)$ where as $L_0 = 0.3l_0 + 0.7l_1$ and $L_1 = 0.4l_1 + 0.6l_2$, $T_x = \int_0^\infty l(x+t)\,dt$ which can be approximated as $T_x = \sum_{t=0}^{\omega} L_{x+t}$; $e_x^\circ = \frac{T_x}{l_x}$ and $\mu_x = \lim_{t \to 0} \frac{l_{x+t} - l_x}{t \cdot l_x} = -\frac{1}{l_x}\frac{d}{dx}(l_x) = -\frac{d}{dx}(\ln l_x)$ (Biswas, 1988; Keyfitz, 1968; Shryock and Siegel, 1975). Thereafter, the ASDRs were indirectly estimated from the constructed life table using the formula of ASDRs $= \frac{{}_nd_x}{{}_nL_x}$ (Barclay, 1958). The result were presented in Table 1 and Table 2.

Moreover, the ASDRs for both sexes were estimated using the formula $\text{ASDRs} = \frac{{}_nd_x^m + {}_nd_x^f}{{}_nL_x^m + {}_nL_x^f}$ from the constructed life tables and presented in the last column of Table 2. The life table CDR is estimated using the formula:

$CDR = \frac{1}{e_0^0} \bullet 1000$; where e_0^0 is the life expectancy at birth. For this, the expectation of life at birth for both sexes were estimated from the constructed life tables using the formula $e_0^0 = \frac{e_0^f + s\,e_0^m}{1 + s}$; where as e_0^m is the expectation of life at birth for male, e_0^f is the expectation of life at birth for female and s is the sex ratio at birth, in this case, assumed to be 1.05 for developing country. Furthermore, the IMR was also estimated and in this case it is, in fact, q_0.

2.1 Model Fitting

It appears from the scattered plot of the number of persons surviving at an exact age x (l_x) for male by age groups (Figure 1) that l_x can be distributed by polynomial model for different ages. Therefore, an n^{th} degree polynomial model was treated and the structure of the model was given by:

$$y = a_0 + \sum_{i=1}^{n} a_i x^i + u$$ (Montgomery and Peck, 1982),

where, x is age group; y is l_x; a_0 is the constant; a_i is the coefficient of x^i (i =1, 2, 3, ..., n) and u is the error term of the model. Here, we had to select a suitable n such that the error sum of square is minimum. Using the software STATISTICA, the mathematical model were estimated.

2.2 Checking Model Validation

To check how much the model is stable, the cross validity prediction power (CVPP), ρ^2_{cv}, is applied here. The formula for CVPP was given by

$$\rho^2_{cv} = 1 - \frac{(n-1)(n-2)(n+1)}{n(n-k-1)(n-k-2)}(1-R^2)$$

where, n is the number of cases, k is the number of predictors in the model and the cross-validated R is the correlation between observed and predicted values of the dependent variable. The shrinkage of the model is the absolute value of the difference of ρ^2_{cv} and R^2. Moreover, the stability of R^2 of the model is equal to 1- shrinkage (Stevens, 1996).

2.3 F-test

The F-test is applied to the model to verify the measure of the overall significance level of the model as well as the significance of R^2. The formula for F-test was stated as

$$F = \frac{R^2 / (k-1)}{(1-R^2) / (n-k)} \quad \text{with (k-1, n-k) degrees of freedom (d.f.)}$$

where k = the number of parameters to be estimated, n is the number of classes and R^2 = the coefficient of determination in the model (Gujarati, 1998).

3. RESULTS AND DISCUSSION

In this study two abridged life tables one for male and other for female were constructed and presented in Table 1 and Table 2. The ASDRs for male and female of Bangladesh in 2005 were estimated and presented in the respective tables. The ASDRs for both sexes of Bangladesh in 2005 were also estimated and presented in the last column of Table 2. If these were plotted in graph paper it can be seen that they followed a traditional pattern or as usual pattern, that was, a U-shape pattern. It should be noted here that traditional pattern of ASDRs is U-shape pattern (Misra, 1995). To study the trend of the ASDRs during 1991-2005, the ASDRs of Bangladesh in 1991 were taken from Islam (2003) and presented in Table 3. It is observed that the ASDRs for male, female and both sexes in 2005 are strictly lower at every ages than the ASDRs for male, female and both sexes in 1991 except for the last age group. This indicates the mortality levels in Bangladesh declined in 1991-2005.

The CDR and IMR for male, female and both sexes of Bangladesh in 2005 were estimated using the information from the estimated abridged life tables and presented in Table 4. To estimate CDR for both sexes in 2005, life expectancy were estimated as 61 years from the constructed life tables. To see the trend of CDR and IMR during 1991-2005, these were been taken from Islam (2003) and presented in Table 4. Here, it was observed that the CDR and IMR for male, female and both sexes in 2005 declined trend during 1991-2005.

Table 1
AN ABRIDGED LIFE TABLE AND ASDRS FOR MALE OF BANGLADESH IN 2005

Age (x)	l_x	d_x	q_x	p_x	L_x	T_x	e_x	ASDRs
0	100000	7954	0.07954	0.92046	94432	6129751	61.29751	0.08423
1	92046	1766	0.019186	0.980814	90986	6035318	65.5685	0.019409
2	90280	862	0.009548	0.990452	89849	5944332	65.84329	0.009594
3	89418	518	0.005793	0.994207	89159	5854483	65.4732	0.00581
4	88900	347	0.003903	0.996097	88727	5765324	64.85179	0.003911
5	88553	777	0.008774	0.991226	440823	5676598	64.10395	0.001763
10	87776	351	0.003999	0.996001	438003	5235775	59.64928	0.000801
15	87425	429	0.004907	0.995093	436053	4797773	54.87872	0.000984
20	86996	510	0.005862	0.994138	433705	4361720	50.13702	0.001176
25	86486	629	0.007273	0.992727	430858	3928015	45.41793	0.00146
30	85857	773	0.009003	0.990997	427353	3497158	40.73235	0.001809
35	85084	1093	0.012846	0.987154	422688	3069805	36.0797	0.002586
40	83991	1643	0.019562	0.980438	415848	2647118	31.51668	0.003951
45	82348	2525	0.030663	0.969337	405428	2231270	27.09562	0.006228
50	79823	4011	0.050249	0.949751	389088	1825843	22.87364	0.010309
55	75812	5870	0.077428	0.922572	364385	1436755	18.95155	0.016109
60	69942	8577	0.12263	0.87737	328268	1072370	15.33228	0.026128
65	61365	11073	0.180445	0.819555	279143	744103	12.12585	0.039668
70	50292	13112	0.260717	0.739283	218680	464960	9.245208	0.05996
75	37180	13096	0.352232	0.647768	153160	246280	6.623991	0.085505
80	24084	10920	0.453413	0.546587	93120	93120	3.866467	0.117268
85	13164	13164	1	0	32910	32910	2.5	0.4

Table 2

AN ABRIDGED LIFE TABLE FOR FEMALE AND ASDRS FOR FEMALE AND BOTH SEXES OF BANGLADESH IN 2005

Age (x)	l_x	d_x	q_x	p_x	L_x	T_x	e_x	ASDRs (f)	ASDRs (b)
0	100000	8767	0.08767	0.91233	93863	6068993	60.68993	.0934	.0888
1	91233	2213	0.024257	0.975743	89905	5975130	65.49307	.0246	.0220
2	89020	1073	0.012053	0.987947	88484	5885225	66.11126	.0121	.0109
3	87947	635	0.00722	0.99278	87630	5796741	65.91175	.0072	.0065
4	87312	416	0.004765	0.995235	87104	5709112	65.38748	.0048	.0043
5	86896	886	0.010196	0.989804	432265	5622008	64.69812	.0021	.0019
10	86010	381	0.00443	0.99557	429098	5189743	60.33883	.0009	.0008
15	85629	524	0.006119	0.993881	426835	4760645	55.59618	.0012	.0011
20	85105	627	0.007367	0.992633	423958	4333810	50.9231	.0015	.0013
25	84478	709	0.008393	0.991607	420618	3909853	46.28249	.0017	.0016
30	83769	872	0.01041	0.98959	416665	3489235	41.65306	.0021	.0019
35	82897	1093	0.013185	0.986815	411753	3072570	37.06491	.0027	.0026
40	81804	1458	0.017823	0.982177	405375	2660818	32.52674	.0036	.0038
45	80346	2034	0.025316	0.974684	396645	2255443	28.07162	.0051	.0057
50	78312	3191	0.040747	0.959253	383583	1858798	23.73579	.0083	.0093
55	75121	4930	0.065627	0.934373	363280	1475215	19.63785	.0136	.0148
60	70191	7455	0.10621	0.89379	332318	1111935	15.84156	.0224	.0243
65	62736	10198	0.162554	0.837446	288185	779618	12.42696	.0354	.0375
70	52538	12883	0.245213	0.754787	230483	491433	9.353849	.0559	.0579
75	39655	14011	0.353322	0.646678	163248	260950	6.580507	.0858	.0857
80	25644	12207	0.476018	0.523982	97703	97703	3.809956	.1249	.1212
85	13437	13437	1	0	33593	33593	2.5	.4000	.4000

Table 3
ASDRS FOR MALE, FEMALE AND BOTH SEXES OF BANGLADESH IN 1991

Age (x)	ASDRs (m)	ASDRs (f)	ASDRs (b)
0	0.09323	0.10766	0.10040
1	0.04843	0.05528	0.05183
2	0.02083	0.02309	0.02195
3	0.00974	0.01036	0.01004
4	0.00566	0.00586	0.00575
5	0.00318	0.00327	0.00323
10	0.00180	0.00191	0.00186
15	0.00159	0.00180	0.00170
20	0.00184	0.00212	0.00200
25	0.00228	0.00250	0.00241
30	0.00305	0.00302	0.00305
35	0.00431	0.00373	0.00403
40	0.00652	0.00480	0.00567
45	0.01044	0.00691	0.00870
50	0.01703	0.01108	0.01403
55	0.02821	0.01845	0.02314
60	0.04220	0.02764	0.03446
65	0.05519	0.03642	0.04503
70	0.08483	0.06542	0.07417
75	0.16745	0.1566	0.16140
80	0.30532	0.31157	0.30861

Table 4
LIFE TABLE CRUDE DEATH RATE (CDR) AND INFANT MORTALITY RATE (IMR) FOR MALE, FEMALE AND BOTH SEXES OF BANGLADESH IN 2005 AND 1991

Year	CDR (per thousand)			IMR (per thousand)		
	Male	Female	Both Sexes	Male	Female	Both Sexes
1991	18.14	17.91	18.03	91	104	98
2005	16.31	16.48	16.39	80	88	84

The fitted model of l_x values for male of Bangladesh in 2005 is:

$$y=93128.04-666.392x+23.81004x^2-0.324233x^3 \quad \ldots \quad (i)$$

t-stat (81.98) (-4.266) (5.04) (-8.58948)

providing coefficient of determination R^2 is 0.991518 and ρ^2_{cv} =0.987829.

The information on model fitting were presented in Table 5. From this table it can be seen that the fitted model is highly cross-validated and its shrinkage is only 0.003689. This implies that the fitted model will be stable for more than 98%. Moreover, from this table, it can be seen from the statistical view that the parameters of the fitted model are highly significant, explaining more than 99% of variance. From t-statistics, it is found that all the parameters of the model are also significant. In this model, the stability of R^2 is more than 99%.

The calculated value of F-test of the model (i) is 701.38 with (3, 18) d.f. where as the corresponding tabulated value is only 5.09 at 1% level of significance. Therefore, it seems from the statistics that the overall measure of the fitted model and its R^2 are highly statistically significant.

From the above equation (i), the rate of change of y with respect to x, i. e. the velocity curve were estimated and sketched in Figure 2 which reveals that the curve increased up to age 25 years and decreased up to last age group.

The instantaneous force of mortality (μ_x) calculated from the fitted model of l_x values for male population of Bangladesh in 2005 were presented in the graph paper shown in Figure 3. From the figure, it can be seen that the force of mortality declined up to age group 20-24 and strictly increased in the whole range but rapidly increased after the age interval 50+.

Table 5
INFORMATION ON MODEL FITTING

Model	n	k	R^2	ρ^2_{cv}	Shrinkage	Parameters	Significant Probability (p)
(i)	22	3	0.991518	0.987829	0.003689		
						a_0	0.000
						a_1	0.000
						a_2	0.00009
						a_3	0.00000

Figure 1
OBSERVED AND FITTED LX VALUES FOR MALE OF BANGLADESH IN 2005

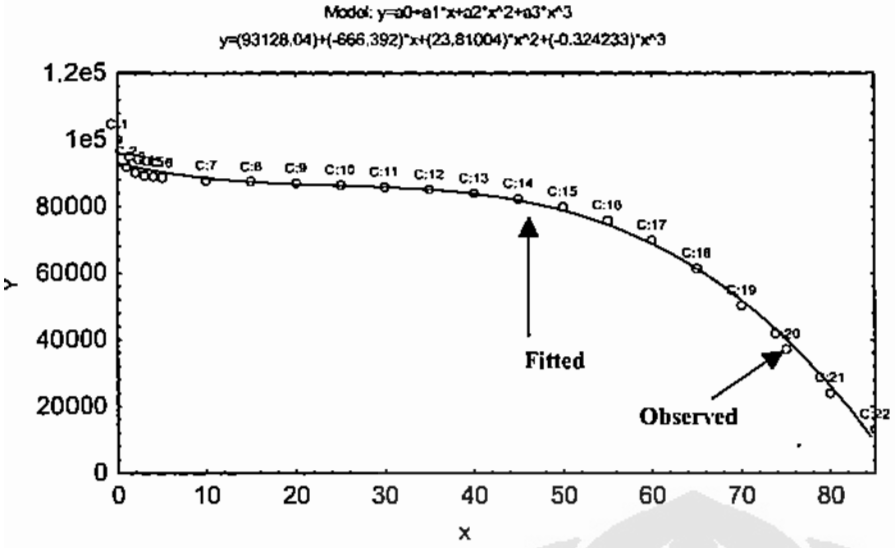


Figure 2
THE VELOCITY CURVE FOR MALE OF LX VALUES OF BANGLADESH IN 2005
(X= AGE GROUP, Y= VELOCITY)

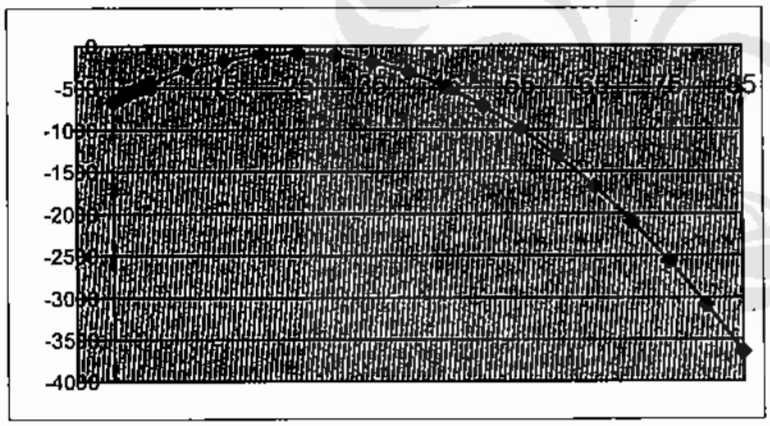
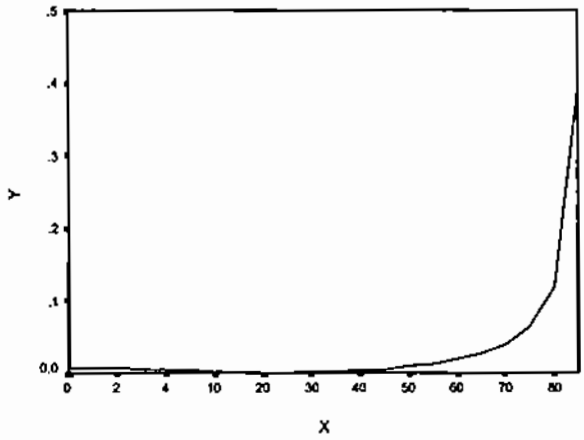


Figure 3
THE FORCE OF MORTALITY FOR MALE OF BANGLADESH IN 2005.
(X= AGE GROUP, Y= FORCE OF MORTALITY)



In this study the ASDRs, life table CDR and IMR for male, female and both sexes of Bangladesh in 2005 were estimated from the constructed abridged life tables. It can be seen that these rates showed decreasing trend over time during 1991-2005. It was found that the l_x values for male followed 3th degree polynomial, i.e. cubic polynomial model. It was observed that force of mortality (μ_x) decreased in the age interval 0 to 24 and increased in the remaining age interval 24 to last age group. It is hoped that the latest findings on mortality would encourage the government and non-government organizations, researchers and planners to plan to boost the socio-economic development and health care program in the country. Life insurance companies may help from the updated information of life tables in this study to bolster their plan of insurance.

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Md. Rafiqul Islam, Associate Professor, Dept. of Population Science and Human Resource Development, Rajshahi University, Bangladesh, E-mail: rafique_pops@yahoo.com