LIMITING MAXIMUM DRAG REDUCTION ASYMPTOTE FOR THE MOMENT COEFFICIENT OF AN ENCLOSED ROTATING DISK WITH FINE SPIRAL GROOVES

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Abstract

In this study, the limiting maximum drag reduction asymptote for the moment coefficient of an enclosed rotating disk with fine spiral grooves in turbulent flow region were obtained analytically. Analysis which were based on an assumption for a simple parabolic velocity distribution of turbulent pipe flow to represent relative tangential velocity, was carried out using momentum integral equations of the boundary layer. For a certain K- parameter the moment coefficient results agree well with experimental results for maximum drag reduction in an enclosed rotating disk with fine spiral grooves and drag reduction ratio approximately was 15 %. Additionally, the experimental results for drag reduction on a rotating disk can be explained well with the analytical results.

Keywords: drag reduction, boundary layer, enclosed rotating disk, friction moment

1. Introduction

The awareness of energy consumption in 21st century promotes innovative studies in various engineering fields. One of the studies is efficiency improvement by minimizing energy loss due to wall skin friction by means of reducing laminar shear stress, delaying transition or by controlling turbulent, which is a major fundamental issue to be solved in the field of fluid flow. To enable reducing energy loss due to wall skin friction by those techniques, attempts are being made mainly in North America, Europe and East Asia to understand the phenomena of skin friction drag reduction.

The problem of disk flows has occupied a central position in the field of fluid mechanics in recent years. Disk flows have immediate technical applications (rotating machinery, lubrications, viscometry, heat and mass exchangers, biomechanics, oceanography), but quite apart from their intrinsic interest. Relevant previous research concerned itself almost entirely with infinite-disk flows. The sole reason for this, one suspect, is that similarity transformation, available when the disks are infinite, reduces the number of spatial dimensions of the problem to one. Although it is questionable whether the reduced model approximates to the physical problem of flow between finite disks, the nonlinear ordinary differential equations that define the phenomenon have been the subject of intense analytical and numerical probing. In spite of this, the nature of the basic flow is not well understood and the researchers of infinite disk flows are responsible for one of the long standing controversies of fluid mechanics, which concerns the uniqueness of the basic motion.

It is well known that the problem of reducing energy loss due to skin friction can be analyzed by several different passive techniques, such as polymer additions, surfactant (surface active agent), and compliant walls (highly water repellent), riblets (streamwise microgrooves), and large-eddy breakup devices (LEBUs). From the point of view of the fluid characteristic, the drag reduction can be achieved by using the Tom's effect [1], which is to dissolve a high molecular polymer in a liquid. In general, one of a weakness of this technique is that the additives cannot be added continuously when the fluid equipment is under operation. Consequently, the effort to reduce drag by using another technique, that is, to modify the condition of the surface in a solid body, constitutes an attractive subject for research. Some researchers have studied the use of this technique, namely: with highly water repellent wall coatings [2,3], LEBUs [4] and riblets [5]. The use of water repellent is sufficiently effective for reducing drag in the laminar region, however, if it is subjected to a shearing stress in the long period the effect will fade, and this will reduce the resistance of that water repellent. Because of that, for using the modification the condition of the surface in a solid body, it is proposed that a good surface should be employed which has better durability and resistance characteristics. Surface mounted fine grooves aligned with the local flow direction of the boundary layer have been most successful in reducing the net drag of turbulent boundary layers in spite of a substantial increase in the wetted surface area.

Regarding to a previous study by some researchers that drag reduction occurred in longitudinal microgrooves surface, it is inferred that skin fiction drag also can be reduced in a flow induced by rotating disk in transition laminar to turbulent and turbulent region. Prediction of that condition will happen if a flow angle is alignment with curve angle of groove, geometry of groove is of the same order of the height (thickness) of a viscous sub-layer and a structural scale of a turbulence wall streak.

A new geometrical modification on a disk surface with certain geometry and grooves which is expected to produce a drag reduction effect in an enclosed rotating disk was proposed, based on the idea that a flow can be controlled by grooves in the disk surface. To look whether the drag reduction is occurred or not, study was performed experimentally by measuring total skin friction moment, the results showed that in transition region for Reynolds number $3.3 \times 10^5 < Re < 4 \times 10^5$ occur the maximum drag reduction ratio at about 15 % [6].

In practical applications it is necessary to estimate the minimum moment coefficient of a rotating disk in a drag-reducing in an enclosed rotating disk. The analytical method can be used to obtain the minimum values of frictional resistance of a rotating disk by using momentum integral equation in boundary layer. If the velocity profile in pipe flow can be used to obtain the moment coefficient of a rotating disk with maximum drag reduction, the results using the momentum integral equations for a boundary layer on a rotating disk can be used to estimate limiting maximum drag reduction asymptote for the moment coefficient of an enclosed rotating disk with fine spiral grooves

The experimental torque measurement and flow visualization results were used to confirm the validity of the analytical results, and the experimental results for the moment coefficient for maximum drag reduction and the flow pattern could be explained well by the analytical results.

2. Analysis

Figure 1 shows the flow model used in the analysis. The disk is rotated at a constant angular velocity (ω) in a fluid of infinite extent, and there is a boundary layer on the rotating disk surface. The fluid in the boundary layer on the surface of rotating disk will induce or thrown outwards due to the viscosity force. If r , θ and z are cylindrical polar coordinates, u_r and u_θ are the velocity components of the fluid in the directions of r and θ , where τ_r and τ_θ are shearing stress components at the surface, δ is the boundary layer thickness, and ρ is the fluid density and the axial velocity component is neglected, then the momentum integral equations of the boundary layer on the rotating disk are:

$$-\frac{\tau_r r}{\rho} = \frac{d}{dr}\left(r\int_o^\delta u_r^2\, dz\right) - \int_o^\delta u_\theta^2\, dz \qquad (1)$$

$$-\frac{\tau_\theta r^2}{\rho} = \frac{d}{dr}\left(r^2 \int_o^\delta u_\theta\, u_r\, dz\right) \qquad (2)$$

Fig. 2 shows flow model in a enclosed rotating disk with two separate boundary layers. It is well known that the logarithmic velocity profiles in a circular pipe cover a wide range of Reynolds numbers. The formula of the velocity near a smooth wall may be written:

$$u = Kv^*\left(\frac{yv^*}{\nu}\right)^{1/n} \qquad (3)$$

Where v* (friction velocity) = $\sqrt{(\tau_w/\rho)}$, y and τ_W are distance and the pipe wall shearing stress respectively Calculation for turbulent flow range was based on a “one-seventh power law” velocity distribution in the assumption. Within the range of Reynolds numbers covered in the test this power law should given satisfactory result. The velocity profiles are assumed effective over the entire disk surface. In a certain range

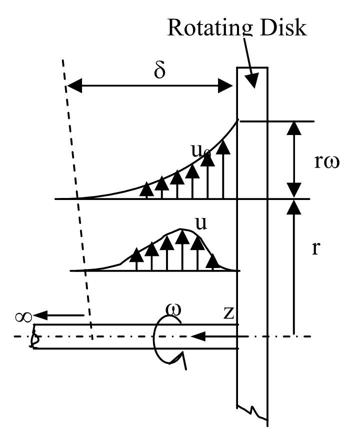


Figure 1. Flow model of velocity profiles in Boundary layer on a rotating disk in a fluid at rest [7]

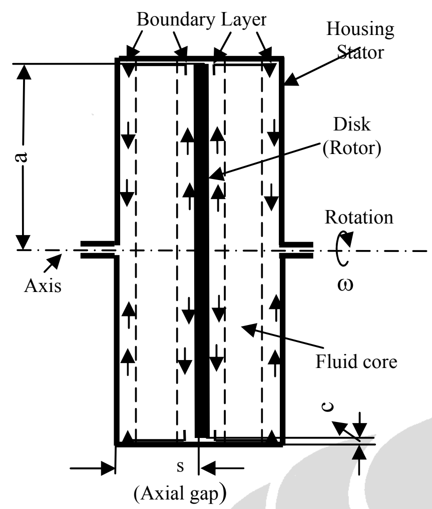
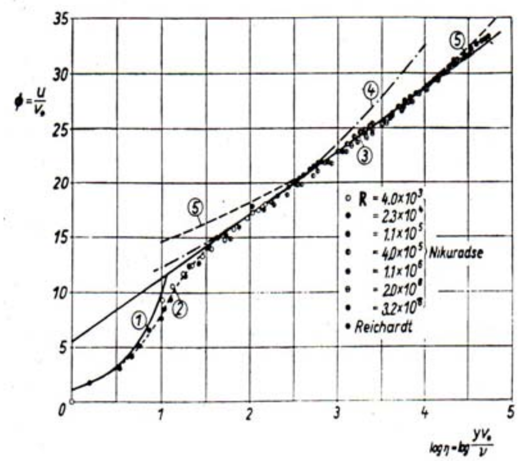


Figure 2. Flow Model in a enclosed rotating disk of a large axial clearance



Fifure 3. The universal velocity-distribution law for smooth circular pipes (Schlichting, 1979)

(1) Laminar; $u/v^* = (yv^*/\nu)$;
(2) Transition, after Reichardt, 1951;
(3) Turbulent, all Re, $u/v^* = 2.5 \ln (yv^*/\nu) + 5.5$
or $u/v^* = 5.75\log (yv^*/\nu) + 5.5$;
(4) Turbulent $u/v^* = 8.74 (yv^*/\nu)^{1/7}$ $Re \approx 10^5$;
(5) $u/v^* = 11.5 (yv^*/\nu)^{1/10}$; $v^* = \sqrt{\tau_o/\rho}$

of Reynolds number in the turbulence flow region, when velocity profile subject to the 1⁄7-th power law (Korukawa, 1987), assume that absolute radial velocity on a rotating disk and absolute tangential velocity are:

$$u_r = \alpha\ (1 - K) r\omega \left(1 - \left(\frac{z}{\delta}\right)\right)\left(\frac{z}{\delta}\right)^{1/7} \quad (4)$$

$$u_\theta = r\omega \left\{1 - (1 - K)\left(\frac{z}{\delta}\right)^{1/7}\right\} \quad (5)$$

Where $z = \delta$ is taken as the edge of the boundary layer and $\alpha = \tan^{-1} \left| \frac{u_r}{u_\theta} \right| = -\tan \phi$, α is a constant related to the flow angle ϕ in the surface of rotating disk.

In Eqns. (4) and (5), the boundary conditions for the no-slip condition of fluid at the disk wall, are given as follows: Radial direction:
$z = 0$, $u_r = 0$; $z = \delta$, $u_r = 0$
Tangential direction:
$z = 0$, $u_\theta = r\omega$; $z = \delta$, $u_\theta = Kr\omega$
From Eqn. (4) can obtain:

$$\int_0^\delta u_r^2 dz = \int_0^\delta \left[\alpha\ \ r\omega(1 - K)\left\{1 - \left(\frac{z}{\delta}\right)\right\}\left(\frac{z}{\delta}\right)^{1/7}\right]^2 dz \quad (6)$$

From Eqn. (5) can obtain :

$$\int_0^\delta u_\theta^2 dz = \int_0^\delta \left[r\omega\left(1 - (1 - K)\left(\frac{z}{\delta}\right)^{1/7}\right)\right]^2 dz$$

$$= 0.0278\ \ r^2\omega^2\delta\left(1 + 7K + 28K^2\right) \quad (7)$$

$= 0.207\alpha\, r^2\omega^2\delta(1 - K)^2$
From Eqns. (4) and (5) can obtain:

$$\int_0^\delta u_\theta u_r dz = \int_0^\delta \left[r\omega\left\{1 - (1 - K)\left(\frac{z}{\delta}\right)^{1/7}\right\}\right]\left[\alpha r\omega(1 - K)\left\{1 - \left(\frac{z}{\delta}\right)\right\}\left(\frac{z}{\delta}\right)^{1/7}\right] dz$$

$$= 0.068\ \ \alpha\, r^2\omega^2\delta\,(1 + 4K - 5K^2) \quad (8)$$

Goldstein (1935), assume that if $u = U\ (1\text{-}K)\ (z/\delta)^{1/7}$ and $U = r\omega\ (1+\alpha)^{1/2}$, the velocity profiles close to the disk surface is assume same as a velocity distribution in a flat plate, thus by using an equation for the velocity distribution that derived from Blasius's equation (Blasius,1913) the wall shear stress (τ_W), in the surface is obtained as follow:

$$\tau_w = 0.0225\,\rho\,(1 - K)^{7/4}\,U^2\left(\frac{\nu}{U\delta}\right)^{\frac{1}{4}} \quad (9)$$

τ_W is taken to be in the same direction as U. Then since τ_r is positive and τ_θ negative, each component is given as Eqn. (10).

$$\left.\begin{aligned} \tau_r &= \frac{\tau_w\ \ \alpha}{\sqrt{1 + \alpha^2}} \\ \tau_\theta &= -\frac{\tau_w}{\sqrt{1 + \alpha^2}} \end{aligned}\right\} \quad (10)$$

(5)

By substitusing Eqns. (6, 7, 9, and 10) into Eqn. (1), the term of Eqn. (10) is obtained as follow :

$$\left(\frac{\tau_w}{\sqrt{1+\alpha^2}}\frac{r}{\rho}\right) = -\left(\frac{r^2\omega^2\delta}{\alpha}\right)\left\{0.621\,\alpha^2(1-K)^2 - 0.0278(1+7K+28K^2)\right\} \tag{11}$$

By substituting . (8, 9, and 10) into Eqn. (2), the term of (10) is obtained :

$$\left(\frac{\tau_w}{\sqrt{1+\alpha^2}}\frac{r}{\rho}\right) = \left(\alpha\ r^2\omega^2\delta\right)\left\{0.272\ (1+4K-5K^2)\right\} \tag{12}$$

By substituting Eqn.(11) into Eqn.(12) can obtain:

$$\alpha = \left\{\frac{(0.0278 + 0.1946K + 0.7784K^2)}{(0.893 - 0.154K - 0.739K^2)}\right\}^{0.5} \tag{13}$$

By substituting Eqns. (9 and 11) into Eqn. (12) can obtain:

$$\delta = 0.137\,r\,\frac{\left\{\frac{(0.9208+0.0406K+0.0394K^2)}{(0.893-0.154K-0.739K^2)}\right\}^{3/10}(1-K)^{1.75}}{\left\{\frac{(0.0278+0.1946K+0.7784K^2)}{(0.893-0.154K-0.739K^2)}\right\}^{4/10}\left(1+4K-5K^2\right)^{8/10}}\frac{1}{Re^{1/5}} \tag{14}$$

Refer to Eqn. (13), it can be considered that in turbulent flow the boundary layer thickness (δ) and the angular velocity ratio of fluid core (K) depend on a disk radius (r).

The moment acting on one side of disk in turbulent flow region is:

$$M = -\int_0^a \tau_\theta\, dA\, r = -\int_0^a \tau_\theta\, 2\,\pi\, r\, dr\, r = -\,2\,\pi \int_0^a \tau_\theta\, r^2 dr$$

By substituting τ_θ from Eqn. (2) into above equation, for r = a, can obtain :

$$M = 2\pi\,\rho\,a^2 \int_0^\delta u_r\, u_\theta\, dz$$

Substitute Eqn. (8) to above equation,
By entering α_2 and δ into above equation can obtain moment coefficient acting on one side of the disk wall as follow:

$$M = 0.0185\pi\rho\omega^2 a^5\left\{\frac{(0.0278+0.1946K+0.7784K^2)}{(0.893-0.154K-0.739K^2)}\right\}^{1/10} x \left\{\frac{(0.9208+0.041K+0.0394K^2)}{(0.893-0.154K-0.739K^2)}\right\}^{3/10}(1-K)^{1.75}\left(1+4K-5K^2\right)^{2/10}\frac{1}{(Re)^{1/5}} \tag{15}$$

$$Cm = \frac{M}{\frac{1}{2}\rho\omega^2 a^5} = 0.037\pi\left\{\frac{(0.0278+0.1946K+0.7784K^2)}{(0.893-0.154K-0.739K^2)}\right\}^{1/10} x \left\{\frac{(0.9208+0.041K+0.0394K^2)}{(0.893-0.154K-0.739K^2)}\right\}^{3/10}(1-K)^{1.75}\left(1+4K-5K^2\right)^{2/10}\frac{1}{(Re)^{1/5}} \tag{16}$$

The analytical results, which exhibit the relationship between C_m and R_e is shown in Fig. 4. In Fig. 4 the parameter of K agrees well with the experimental data in the range $3x10^5<Re<7x\ 10^5$ with the value equals 0.48.

By substituting the value of K = 0.48 into Eqns (13 and 14), the value of the flow angle (α) and boundary layer thickness (δ) are obtained in turbulent flow region as follows:

$$\alpha = 0{,}681$$

$$\delta = 0.042\left(\frac{\nu}{\omega\, r^2}\right)^{1/5}$$

By inserting K = 0.48 into Eqn. (16) the moment coefficient in the turbulent flow of the boundary layer for one side of a disk is given:

$$Cm = \frac{0.0431}{R_e^{1/5}} \tag{17}$$

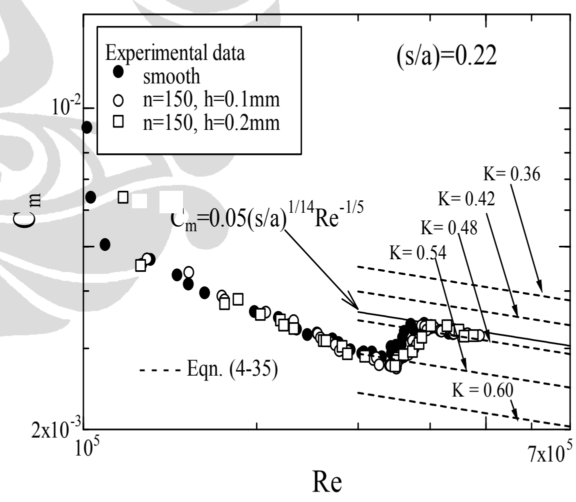


Figure 4. Comparison of Cm between experimental and analytical in turbulent flow

3. Results and Discussion

Due to in analytical work was carried out based on the logarithmic velocity profile of turbulent pipe flow, Goldstein (1935) proposed that for obtaining a value of an exponent n for turbulent flow in an enclosed rotating disk is using an experimental work.

In general the moment coefficient (Cm) of an enclosed rotating disk will decrease at about half if fluid core in the chamber between stator and rotor rotate by a half of the disk angular velocity.

Value of K = 0.8 is obtained from the experimental work, it is mean that rotation of fluid core (Ω) = 0.48 ω. For K = 0.48 value of moment coefficient (Cm) = $0.0431/Re^{1/5}$, compare it with Cm of free disk (K= 0), where Cm = $0.0831/Re^{1/5}$.

Eqn. (17) is equation of limiting maximum drag reduction asymptote for the moment coefficient of an enclosed rotating disk with fine spiral grooves and if it formulated in dimensionless form for friction velocity (u^+=u/v*) and wall distance (y^+= yv*/ ν) can obtain:

$$(u/v^*) = 23.3\ (yv^*/\nu)^{1/9} \qquad (18)$$

Weighardt, K (1946),obtained the value of constant, C and exponents, n as follow:

n	7	8	9	10
C(n)	8.74	9.71	10.6	11.5

where, $((u/v^*) = C(n) \times (yv^*/\nu)^{1/n})$

The constant in Eqn. (18) is a quite large, it is mean that the slope of asymptote line is large and it is assume caused by drag reduction in an enclosed rotating disk with fine spiral grooves.

4. Conclusions

The limiting maximum drag reduction asymptote for the moment coefficient of an enclosed rotating disk was obtained analytically in drag-reducing on disk surface with fine spiral grooves. Analysis was carried out using momentum integral equations of the boundary layer on a rotating disk based on the logarithmic velocity profile of turbulent pipe flow. The analytical results agreed quantitatively with experimental results of maximum drag reduction for disk surface with fine spiral grooves. It can be seen that slope of the asymptote is quite significant if it compare with the asymptote slope of moment coefficient of turbulent pipe flow.

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